ISSN: 3067-8307
Review Article

World Journal of Applied Mathematics and Statistics

Perturbed FLRW Metric Explains the Difference in Measurements of the Hubble Constant

W. B. Belayev

Center for Relativity and Astrophysics, Saint-Petersburg, Russian Federation

***Corresponding author:** W. B. Belayev, Center for Relativity and Astrophysics, Saint-Petersburg, Russian Federation.

Submitted: 11 April 2025 **Accepted:** 21 April 2025 **Published:** 30 April 2025

https://doi.org/10.63620/MKWJAMS.2025.1011

***Citation:** W. B. Belayev. (2025). Perturbed FLRW Metric Explains the Difference in Measurements of the Hubble Constant. Wor Jour of Appl Math and Sta, 1(2), 01-06.*

Abstract
The expansion of the universe yields two consistent differing values of a Hubble constant depending on the methods of measurement. The aim of this work was to explain the Hubble tension by properties of space cor- responding to a radially inhomogeneous metrics. A cosmological model described by this perturbed FLRW metric has zero pressure and exponen- tially expanding space. But the observed space appears to be compressed in the radial direction. We consider the dependence of the change rate of photon energy on the direction of its motion. It is found applying the prin- ciple of the photon's energy integral extremum and the relationship be- tween it and the energy of a material particle obtained using Lagrange me- chanics. The change rate of a photon's energy varies depending on whether it moves in a radial direction or has an angular component. The metric co- efficients are determined by the difference in measurements of the Hubble constant using gravitational lensing and a distance ladder.

Keywords: Perturbed FLRW metric, Einstein Equations, Particle Lagrangian, Hubble Tension, Gravitational Lensing; Distance Ladder

Introduction

In the ΛCDM (Lambda Cold Dark Matter) cosmological model, based on the FRLW (Friedmann-Lemaˆıtre-Robertson-Walker) metric, matter moves synchronously with the expanding space. Data from the JWST telescope allowed the identi-fication of galaxies with a redshift of z > 10, which led to a departure from the existing Big Bang scenario [1-4]. The perturbed FLRW metric, describing small deviations from the background space-time, can also serve as the basis for certain cosmological models. Deviations from standard cosmology are investigated by considering flat FLRW space-time in Painleve´–Gullstrand coordinates [5-9]. With this coordi- nate choice the space is not expanding, although the distance between galaxies is certainly increasing. In Refs. is studied a distorted stereographic projection of hyperconical universes, which are 4D hypersurfaces with linear expansion of space embedded into 5D Minkowski spacetime. This model takes the perspective of explaining Hubble tension [10-12]. We propose model, which is described by perturbed FLRW metric with spatial inhomogeneity in radial direction.

Determination of the Hubble constant using the cosmic distance ladder, and gravitational lensing, yields a difference of about 1/10 of its value. We consider the dependence of the change rate of photon energy on the direction of its motion. It is found applying the principle of the extremum of the photon's energy integral and the relationship between it and the energy of a material particle obtained using Lagrange mechanics [13-21].

The Solution of the Einstein Equations for the per- Turbed Metric

The most common presentation of the spatially flat FLRW cosmology is in terms of the explicit line element

$$ds^2 = c^2 d\acute{t}^2 - a(\acute{t})^2[dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2)], \quad (1)$$

where t´ is the time measured by an observer associated with the 4-velocity vec- tor ui = (1, 0, 0, 0). A generalization of this metric is obtained by transforming the time interval

$$cd\acute{t} = cdt - f(t)rdr \text{ ,(2)}$$

where function f (t) has dimension inverse length. The linear element takes the following form:

$$ds^2 = c^2dt^2 - 2cf(t)rdrdt - (a(t)^2 - f(t)^2r^2)dr^2 - a(t)^2r^2(d\theta^2 + \sin^2\theta d\varphi^2).$$

(3)

For small r, this metric is approximated by the metric of flat expanding space- time. The components of the Einstein tensor [22] are given by

$$G^{11} = \frac{1}{a^6}\left\{3a^4\dot{a}^2 - 6a^3f\dot{a} + r^2\left[2(a^2f^2 - a^3f\dot{a})\dot{f} + 2af^3\dot{a} - 4a^2f^2\dot{a}^2\right] + r^4\left[f^4\dot{a}^2 + 2af^3\dot{a}\dot{f}\right]\right\}, \quad (4)$$

$$G^{12} = G^{21} = \frac{fr}{a^6}\left[\left(2af\dot{a} + f\dot{a}^2\right)r + \left(2a\dot{f} + 2f\dot{a} - 3a\dot{a}^2\right)a\right], \quad (5)$$

$$G^{22} = \frac{1}{a^6}\left\{-2a^3\ddot{a} - a^2\dot{a}^2 + 2a^2\dot{f} + 2af\dot{a} + r^2\left(2a f \dot{a} \dot{f} + f^2\dot{a}^2\right)\right\}, \quad (6)$$

$$G^{33} = G^{44}\sin^2\theta = \frac{1}{a^6r^2}\left\{-2a^3\ddot{a} - a^2\dot{a}^2 + 2a^2\dot{f} + 2af\dot{a} + r^2\left(a\ddot{a}f^2 - f^2\dot{a}^2 + fa^2\ddot{f} + a^2\dot{f}^2 + a f\dot{a}\dot{f}\right)\right\}, \quad (7)$$

where ($\dot{}$) denotes differentiation with respect to x1 = ct.

The components of the contravariant energy-momentum tensor correspond to the energy density, momentum density, energy flux through unit surfaces per unit time and stresses [23]. The energy-momentum tensor of matter in contravariant form is given by

$$T^{ij} = \left(c^2\rho + p\right)u^iu^j - g^{ij}p. \quad (8)$$

The non-zero contravariant coefficients for the metric (3) will be as follows:

$$g^{11} = \frac{a^2 - f^2r^2}{a^2},\ g^{12} = g^{21} = -\frac{fr}{a^2},\ g^{22} = -\frac{1}{a^2},\quad g^{33} = -\frac{1}{a^2r^2},\ g^{44} = -\frac{1}{a^2r^2\sin^2\theta}. \quad (9)$$

In the absence of angular momentum of matter (u3= u4 = 0), the non-zero components of the energy-momentum tensor (8) are given by

$$T^{11} = \left(c^2\rho + p\right)\left(u^1\right)^2 - \left(1 - \frac{f^2r^2}{a^2}\right)p, \quad (10)$$

$$T^{12} = T^{21} = \left(c^2\rho + p\right)u^1u^2 + \frac{fr}{a^2}p, \quad (11)$$

$$T^{22} = \left(c^2\rho + p\right)\left(u^2\right)^2 + \frac{1}{a^2}p, \quad (12)$$

$$T^{33} = T^{44}\sin^2\theta = \frac{1}{a^2r^2}p. \quad (13)$$

Let's consider a model with a cosmological constant Λ = 0. We will seek a so- lution for a region where the radial velocity u2 is small, and the term of com- ponent T 22 containing its square is negligible and can be disregarded. Next,
we will justify this assumption. Also, we assume that the terms of the Ein- stein tensor components (4)-(7) containing r2 in the numerator are small and can be neglected. Given the satisfaction of this condition and for g11, Einstein equations Gij = χTij + gij Λ with the energy-momentum tensor components (10)-(13), yield

$$\frac{3a\dot{a}^2 - 6f\dot{a}}{a^3} = \chi c^2\rho, \quad (14)$$

$$\frac{2a\dot{f} + 2f\dot{a} - 3a\dot{a}^2}{a^3}fr = \chi a^2\left(c^2\rho + p\right)u^1u^2 + \chi frp, \quad (15)$$

$$\frac{-2a^2\ddot{a} - a\dot{a}^2 + 2a\dot{f} + 2f\dot{a}}{a^3} = \chi p. \quad (16)$$

The absence of additional energy and momentum influx implies the equal- ity of the corresponding densities T 12, T 21 to zero, i.e., the right-hand side of equation. From its left-hand side we obtain

$$2a\dot{f} + 2f\dot{a} - 3a\dot{a}^2 = 0. \quad (17)$$

Solving this equation for f under the condition a(t0) = 1 we find

$$f(t) = \frac{1}{a}\left(f_0 + \frac{3}{2}c\int_{ct_0}^{ct}\dot{a}^2a\,dt\right) \quad (18)$$

with f (t0) = f0. Substituting this into equation (14) under the condition

$$\rho = \frac{\rho_0}{a(t)^3}, \quad (19)$$

with average density of the Universe at the present time ρ0 = ρ(t0) yields

$$3a\dot{a}^2 - \frac{6f_0\dot{a}}{a} - \frac{9\dot{a}}{a}c\int_{ct_0}^{ct}\dot{a}^2a\,dt = \chi c^2\rho_0. \quad (20)$$

Expressing the integral from here and differentiating it with respect to x1, from the obtained equality we find

$$\chi c^2\rho_0\left(1 - \frac{a\ddot{a}}{\dot{a}^2}\right) = -3\dot{a}^2a + 3\ddot{a}a^2. \quad (21)$$

This equation has a solution

$$a(t) = e^{cA_1(t - t_0)}, \quad (22)$$

where A1 is a constant.

From equations (16) and (17), we obtain the pressure

$$p = -\frac{2}{c\chi}\frac{d}{dt}\left(\frac{\dot{a}}{a}\right), \quad (23)$$

which, due to (22), turns out to be p = 0. In this case, from (15) under the con- dition of the absence of energy and momentum influx from the outside (17), we find the radial component of the matter velocity in the comoving reference frame.

$u^2 = 0$. (24)

Thus, it turns out to be a valid assumption that the radial velocity in the re- gion under consideration is small and may not be considered in the component of the energy-momentum tensor (12).

The function (18), with the scale factor of space (22), takes the form

$$f(t) = e^{-cA_1(t-t_0)} f_0 + \frac{1}{2} A_1 \left(e^{2cA_1(t-t_0)} - e^{-cA_1(t-t_0)} \right) \quad (25)$$

and equation (20) yields

$$3{A_1}^2 - 6 f_0 A_1 = \chi c^2 \rho_0 . \quad (26)$$

Change in Photon Energy

Let's determine how the energy of a photon changes in the considered space- time applying the principle of the extremum of the photon's energy integral. The Euler-Lagrange equations for the covariant components of the energy-momentum vector of a light-like particle of unit energy

$$\frac{dp_\lambda}{d\mu} - F_\lambda = 0 \quad (27)$$

with

$$F_\lambda = \frac{1}{2u_1 u^1} \frac{\partial g_{ij}}{\partial x^\lambda} u^i u^j, \quad (28)$$

corresponding to the energy, take the form

$$\frac{dp_1}{d\mu} = -\frac{1}{u_{1ph} u^1_{ph}} \left[\dot{f} r u^1_{ph} u^2_{ph} + (\dot{a} a - \dot{f} f r^2)(u^2_{ph})^2 + \dot{a} a \Omega^2 \right] \quad (29)$$

under the designation

$$\Omega^2 = r^2 \left[((u^3_{ph})^2 + \sin^2\theta (u^4_{ph})^2 \right]. \quad (30)$$

From the equality of interval (3) to zero, we obtain

$$u^2_{ph} = \frac{-f r u^1_{ph} + \sigma \sqrt{a^2 (u^1_{ph})^2 - (a^2 - f^2 r^2)\Omega^2}}{a^2 - f^2 r^2}, \quad (31)$$

where σ takes on values of ±1 depending on the direction of the light. For small deviations from radial motion:

$$\Omega^2 << (u^1_{ph})^2 \quad (32)$$

we find

$$u^2_{ph} = \frac{1}{f r + \sigma a} u^1_{ph} - \frac{\sigma \Omega^2}{2 a u^1_{ph}}. \quad (33)$$

The expression for the time component of the covariant velocity vector is as follows:

$$u_{1ph} = u^1_{ph} - f r u^2_{ph} = \frac{a}{a + \sigma f r} u^1_{ph} + \frac{\sigma f r \Omega^2}{2 a u^1_{ph}}. \quad (34)$$

Further we will consider f r, a˙/a, f˙/f to be small quantities. After substituting
the components u2 and u1ph into equation (29) without accounting for higher
order small quantities, it is rewritten in the form

$$\frac{dp_1}{d\mu} = -\frac{\sigma \dot{a} + \dot{f} r}{f r + \sigma a}. \quad (35)$$

Since the covariant and contravariant temporal components of the generalized momenta of a light-like particle

$$p_\lambda = \frac{u_\lambda}{u^1 u_1}, \quad (36)$$

$$p^\lambda = \frac{u^\lambda}{u^1 u_1} \quad (37)$$

are related by

$$p^1 = \frac{u^1_{ph}}{u_{1ph}} p_1 = \left(1 + \sigma \frac{f r}{a} \right) p_1 , \quad (38)$$

the rate of energy change can be written in the form

$$\frac{dp^1}{d\mu} = \left(1 + \sigma \frac{f r}{a} \right) \frac{dp_1}{d\mu} + \sigma \frac{1}{a^2 u^1_{ph}} \left[r(\dot{f} a - \dot{a} f) u^1_{ph} + f a u^2_{ph} \right]. \quad (39)$$

Substituting u2 from equation (33) and dp1/dμ from equation (35) into this

expression, we find

$$\frac{dp^1}{d\mu} = -\frac{\dot{f} r + \sigma \dot{a}}{\sigma a} + \frac{1}{\sigma a^2} \left[r\left(\dot{f} a - \dot{a} f \right) + \frac{f a}{f r + \sigma a} - \frac{\sigma f \Omega^2}{2(u^1_{ph})^2} \right]. \quad (40)$$

Assuming that the terms in this equation containing rare small in the region under consideration, we can write

$$\frac{dp^1}{d\mu} = -\frac{\dot{a}}{a} + \frac{f}{\sigma a^2} - \frac{f \Omega^2}{2a^2 (u^1_{ph})^2}. \quad (41)$$

In the case of the motion of light emitted towards the observer, we choose σ =−1.
Then, upon substitution of (22) and (25), this expression is transformed to
become

$$\frac{dp^1}{d\mu} = -A_1 \left(\frac{3}{2} - \frac{1}{2} e^{-3cA_1(t-t_0)} \right) - f_0 e^{-3cA_1(t-t_0)} - \frac{\Omega^2}{2(u^1_{ph})^2} \left[\frac{1}{2} A_1 \left(1 - e^{-3cA_1(t-t_0)} \right) + f_0 e^{-3cA_1(t-t_0)} \right] . \quad (42)$$

In the region under consideration, the energy of a radially moving photon is given by

$$E_{ph} = h\nu_0 \left(1 - r \left. \frac{dp^1}{d\mu} \right|_{t=t_0} \right), \quad (43)$$

where ν0 is the value of the photon frequency at the observation point. Since
in the cosmological model described by an orthogonal metric, the Hubble con- stant is expressed as

$$\breve{H} = c \frac{\dot{a}(t_0)}{a(t_0)}, \quad (44)$$

from equation (26), when comparing it with the Friedman equation

$$\chi c^2 \rho = \frac{3}{a^2}\left[k + (\dot{a})^2\right] - \Lambda, \qquad (45)$$

it follows that A1 can be a quantity of the same order as the Hubble parameter. After substituting the function f (25) without small quantities, equation (42) takes the form

$$\left.\frac{dp^1}{d\mu}\right|_{t=t_0} = -A_1 - f_0 - \frac{f_0 \Omega^2}{2(u^1_{ph})^2}. \qquad (46)$$

Since in the region under consideration the spacetime metric (3) approximates the Minkowski metric, the affine parameter will be close to μ = ct.

Change in the Energy of a Material Particle in the Co-Moving Frame

The Lagrangian of a material particle [17] in space-time (3) is given by

$$L = cm\Big\{(u^1)^2 - 2f(t)ru^1u^2 - [a(t)^2 - f(t)^2r^2](u^2)^2 - a(t)^2r^2((u^3)^2 + \sin^2\theta(u^4)^2)\Big\}^{1/2}. \qquad (47)$$

The motion of a material particle in a gravitational field, like that of a photon, in accordance with equation (27) at μ = s can be represented [17, 21] as the result of the action of generalized forces

$$F_\lambda = \frac{\partial L}{\partial x^\lambda} = \frac{1}{2}cm\frac{\partial g_{ij}}{\partial x^\lambda}u^iu^j, \qquad (48)$$

which, however, do not generally form a first-order tensor. The components of the vector of generalized forces associated with

$$F^k = \frac{1}{2}cmg^{k\lambda}\frac{\partial g_{ij}}{\partial x^\lambda}u^iu^j \qquad (49)$$

are related to gravitational forces Fk = cFk. The equations of motion (27) for the contravariant momentum are transformed into the form

$$\frac{d\bar{p}^k}{d\mu} = \frac{dp^k}{d\mu} + \frac{d\overset{\leftrightarrow}{p}^k}{d\mu} = F^k, \qquad (50)$$

where

$$\frac{d\overset{\leftrightarrow}{p}^k}{d\mu} = g^{k\lambda}\frac{\partial g_{\lambda i}}{\partial x^j}u^jp^i \qquad (51)$$

represents the energy imparted to the gravitational field.

Let's consider a particle of matter in a comoving frame, in which it is mo- tionless (24). The component of the force vector (49) corresponding to energy, F1, becomes zero. The rate of change of energy transferred to the gravitational field (51) will be

$$\frac{dp^1_m}{ds} = cm\frac{f\dot{f}r^2}{a^2}. \qquad (52)$$

Due to (50), the energy of the material particle changes at a rate of

$$\frac{dp^1_m}{ds} = -cm\frac{f\dot{f}r^2}{a^2}. \qquad (53)$$

The derivative of the function f (t) has the form

$$\dot{f} = -A_1f_0e^{-cA_1(t-t_0)} + A_1^2\left(e^{2cA_1(t-t_0)} + \frac{1}{2}e^{-cA_1(t-t_0)}\right). \qquad (54)$$

Now we can express the change in energy of the material particle at the present time as

$$\left.\frac{dp^1_m}{ds}\right|_{t=t_0} = cmA_1\left(f_0 + \frac{3}{2}A_1\right)f_0r^2. \qquad (55)$$

In the considered region, the energy of the material particle will be

$$E_m = c\left(cm - r\left.\frac{dp^1_m}{ds}\right|_{t=t_0}\right). \qquad (56)$$

Cosmological Redshift in the Radial Motion of Pho- tons

The equation (26) is rewritten in the form

$$3(A_1 + f_0)^2 - 12f_0A_1 - 3{f_0}^2 = \chi c^2\rho_0. \qquad (57)$$

We assume that f0 is of the same order of magnitude as A1 and, consequently, with the Hubble constant. Since in the considered region the quantity f r is small, considering the expressions for the rate of change of particle energies (46) and (55), the change in the energy of a material particle will be small in comparison with the change in the equivalent energy of a photon (43) because of

$$\left(\left.\frac{dp^1_m}{ds}\right|_{t=t_0}\right)\Big/\left(h\nu_0\left.\frac{dp^1}{d\mu}\right|_{t=t_0}\right) = o(fr), \qquad (58)$$

where the differentiation parameters are

s ≈ μ ≈ ct. (59)

The observed redshift of photon energy is considered as a result of the change in the ratio of their energy (43), (46) to the energy of atoms (56). In a small neighborhood without small higher-order quantities due to (58), this will be

$$\frac{E_{ph}}{E_m} = \frac{h\nu_0}{c^2m}[1 - r(A_1 + f_0)], \qquad (60)$$

from which the value of the Hubble constant at present time
$H_0 = c\,(A_1+f_0)$, (61)
can be deduced, obtained as a result of registering radially moving photons.
In this case, equation (57) takes the form

$$3\left(\frac{H_0}{c}\right)^2 - 12\frac{H_0}{c}f_0 + 9{f_0}^2 = \chi c^2 \rho_0 . \qquad (62)$$

Observed Length Element
The length element

$$dl^2 = \left(\frac{g_{1p}g_{1q}}{g_{11}} - g_{pq}\right)dx^p dx^q \qquad (63)$$

with p, q = 1, 2, 3 in the space-time (3) is given by

$$dl = a(t)\sqrt{dr^2 + d\theta^2 + \sin^2\theta\, d\varphi^2}. \qquad (64)$$

That is, the space is isotropic in proper frame.

The radical elements of the length of the metric (3) at constant time

$$dl' = \sqrt{a(t)^2 - f(t)^2 r^2}\,dr \qquad (65)$$

is less than obtained from (64) with dθ = dϕ = 0 and approach it with decreas- ing r. The observed space in the coordinate frame appears to be compressed in the radial direction. This will cause distant objects to appear smaller in size than in isotropic space. A similar effect was discovered when observing galax- ies with large z [4]. However, for precise adjustment of the considered model parameters for such distances, a solution is required without the assumption about the insignificance of some terms in the tensor Gij .

Difference in Hubble Constant Measurements
Let's consider how the energy of a photon will change in the presence of grav- itational lensing. We denote the change in the energy of the photon when moving along the path Ll , which includes gravitational lensing, as

$$\delta p_l^1 = \int_{L_l}\frac{dp^1}{d\mu}d\mu \qquad (66)$$

and along the radial path as

$$\delta p_r^1 = \int_{L_r}\frac{dp^1}{d\mu}d\mu. \qquad (67)$$

We assume that the coordinate distance Δr from the beginning of the paths to the observer is the same. Under the gravitational lensing, neglecting small quantities, the Hubble constant is determined as follows:

$$H_0 = -\frac{c\left(\delta p_l^1 - \delta p_r^1\right)}{\delta\mu_l - \delta\mu_r}, \qquad (68)$$

where δµl and δµr are the change in the affine parameter along each of the paths.

The coordinate system is chosen such that when the photon moves along the path Ll the condition θ = π is satisfied. If we consider only the influence of the deviation from radial motion, then the difference in the time taken for the photon to travel along both paths will be

$$\delta t = \frac{1}{c}\int_0^{\Delta r}(1-\cos(\alpha r))dr, \qquad (69)$$

where in the first approximation we consider

$$\cos(\alpha(r)) = -\frac{u_{ph}^2}{\sqrt{(u_{ph}^2)^2 + (r u_{ph}^4)^2}}. \qquad (70)$$

Due to condition (32), the integral (69) transforms into the form

$$\delta t = \frac{1}{c}\int_0^{\Delta r}\frac{(r u_{ph}^4)^2}{2(u_{ph}^2)^2}dr. \qquad (71)$$

In the considered region, we have

$$\delta\mu_l - \delta\mu_r = c\,\delta t. \qquad (72)$$

By substituting the corresponding value from (46) into (66) and (67), we find the additional difference in energy change caused by the discrepancy in the rate of its during radial and angular motion

$$\Delta(\delta p_l^1 - \delta p_r^1) = -\int_0^{\Delta r}\frac{f_0 (r u_{ph}^4)^2}{2(u_{ph}^1)^2}dr. \qquad (73)$$

The change in the value of the Hubble constant when light moves along a curved path will be

$$\Delta H_0 = -\frac{c\Delta\left(\delta p_l^1 - \delta p_r^1\right)}{\delta\mu_l - \delta\mu_r}. \qquad (74)$$

Since for the considered motion of the photon, the relations

$$u_{ph}^1 \approx -a u_{ph}^2 \approx -u_{ph}^2 \qquad (75)$$

hold, by substituting the values (72) and (73) into (74), we obtain
$\Delta H_0 = c f_0$. (76)
Let's estimate the values of A1 and f0 using the results of obtaining the Hub- ble constant with and without gravitational lensing. The Hubble constant was found through gravitational lensing of

the cosmic microwave background:
68.3±1.5 km s−1 Mpc−1, Ref. [15], and of radiation from the supernovae: 66.6+4.1 km s−1 Mpc−1, Ref. [16]. The values obtained through the extragalactic distance ladder to supernovae are 73.5±1.1 km s−1 Mpc−1, Ref. [13], and 75.4+3.8 km s−1 Mpc−1, Ref. [14]. These results yield the value ΔH0 = cf 0 = −7+2.4 km s−1 Mpc−1, and due to relation (61) corresponding to the radial motion of the photon we obtain
cA1 = 81.5±2.4 km s−1 Mpc−1.

When the value of f0 is negative, the photon moving towards observer from the star additionally loses energy compared to the motion tangential to the circle from the center of observation or in the opposite direction. Also, the negativity of the function f means that space is expanding faster than matter is scattering. The dependence of the spacetime properties on the observer's position is similar to the manifestation of the spacetime curvature in the non- flat FLRW model.

Conclusions

Unlike the ΛCDM cosmology, in the presented model the observed space ex- panded not synchronously with movement of matter. In region corresponding to small perturbations of the FLRW metric the Universe expands with zero
pressure while conserving energy and momentum. Solution is obtained that relates exponential proper expansion of space for a model with a zero cosmo- logical constant. The considered metric can also be applied to a cosmological model with a nonzero Λ-term.

In addition to the space expansion, the movement of matter in the radial
direction is also established in coordinate frame. The observed space in this frame appears to be compressed in the radial direction. This will cause distant objects to appear smaller in size than in isotropic space.

The redshift in the spectra of galaxies is determined using Lagrange me- chanics in its application to the principle of the extremal integral of photon energy. In the considered spacetime, from the observer's standpoint, a photon loses energy faster when moving radially towards them compared to its mo- tion with a tangential component. The light used to obtain redshift through the ladder distance travels along a radial path, unlike gravitational lensing, where its pass has a tangential component. Based on the difference in value of the Hubble constant obtained using gravitational lensing and distance ladder the metric coefficients were found in a local region in which their perturbations are small.

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