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An Introduction to Temporal Drift

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Abstract

We discuss "temporal drift," my term for the degree to which clocks in a fixed reference frame dis-synchronize with distance as defined by slow clock transport in both pre-relativistic ("classical") and relativistic physics.

Then we demonstrate that the usual expression for length contraction is unaffected by temporal drift, although there is a subtlety in the definition of velocity when temporal drift is present.

Then we demonstrate the invariance of the line element in relativity and show that the off-diagonal terms of the 4-metric are intimately related to the temporal drift when g_{00} is chosen to equal c^2.

Finally, we investigate temporal drift and synchrony in accelerating and rotating frames and find that temporal drift is irreducible in these cases.

Keywords: Temporal Drift, Synchrony, Relativity, Asynchrony, Lorentz Transformation, Time Zones

Introduction

The simplest and most fundamental definition of synchrony of two clocks is through direct comparison. If they are separated by a distance—e.g., two people A and B with wristwatches at opposite ends of a room—A simply walks over to B, makes a comparison and adjusts if necessary. In relativity, there is the caveat that the clock transport must be slow enough that Lorentz time dilatation is negligible in relation to the accuracy of the clocks.

Two things, conceptually, can go wrong with this procedure and do so in relativity, though not in classical theory. One, there may be gravitational time dilatation, which also occurs in an accelerating frame, such that clocks in different gravitational potentials run at different rates and will be later found to be out of synchrony even if synchronized at an earlier time. The second is the Sagnac Effect which occurs in rotating reference frames [1]. In the Sagnac Effect, given three observers, A,Band C, if A and B synchronize watches and B and C synchronize watches, then, upon comparison, A and C will find their watches out of synchrony: synchrony is not transitive, as the mathematicians say. The effect is path-dependent in the sense that the amount by which A and C find their watches out of synchrony depends on the path travelled between A and C: a straight path will give a different result than a curved or crooked path. Whether A and B or B and C are synchronized also depends on the paths (A,B) or (B,C), respectively.

In classical theory there is neither gravitational time dilatation nor a Sagnac effect, so we never have to accept the appearance of temporal drift except if we willfully introduce it by the use of time zones rather than using UTC. For instance, if (X,T) represent the UTC frame and (x,t) the frame with time zones,

$x = X,$

$t = T + \lambda X,$

where λ is the temporal drift. The temporal drift is a discretized function of longitude when using the time zones commonly used throughout the world or is a constant if X refers to longitude when using local solar time.

Temporal Drift in Classical Theory

Suppose we consider two reference frames, Σ and Σ', neither of which possesses temporal drift; then, where V is the relative velocity, they are connected by the Galilean Transformation,

$x = x' + Vt,$

$t' = t$.

Let us introduce temporal drift, λ, in Σ and λ' in Σ'; thus,
$x_1 = x$,
$t_1 = t + \lambda x$,

$x_1' = x'$,
$t_1' = t' + \lambda' x'$.

The relationship between (x_1,t_1) and (x_1',t_1') is then (dropping subscripts):-

$x = (1-\lambda v)\, x' + vt$,

$t' = (1-\lambda v)t + [\lambda' - \lambda(1-\lambda v)]\, x'$,
where, $v := V/1+\lambda V$.

We consider v rather than V to be the velocity of Σ' with respect to Σ because it is equal to $\left.\frac{\partial x}{\partial t}\right|_{x'}$

Note that the velocity of Σ with respect to Σ' is not $-v$ but

$$\left.\frac{\partial x'}{\partial t'}\right|_{x} = (1-\lambda' v)\, v.$$

Consider a clock transported from x to $x+dx$ at velocity v (in classical theory, v need not be small). Suppose that this mobile clock is at rest in Σ' so that the temporal drift, λ', in Σ' does not affect it, since x' is fixed. (Here we are considering synchronization by direct comparison or [slow] clock transport, as in the introduction.).

Now, $dx'=0$, so $dx=vdt$ and $dt' = dt-\lambda vdt = dt-\lambda dx$. Thus, a clock transported from x to x+dx disagrees with the local clocks fixed in Σ by amount $-\lambda dx$; that is, in the language of the opening paragraph of the introduction, A finds that his watch is out of synchrony with B and that we have the equivalent of time zones.

This is why we call λ the temporal drift: it measures the amount of dis-synchrony between two spatially separated clocks as a function of distance.

Finally in this section, let us calculate the apparent length contraction of a moving rod. Suppose that the rod is at rest in Σ', then it matters not if there is a non-zero dt' between marking the two ends of the rod in Σ', because it is stationary, but in Σ we must measure the two ends of the rod simultaneously because it is moving; that is, we need $dt-\lambda dx=0$ (not dt=0 unless λ=0). Then we've, $dx=(1-\lambda v)\, dx'+v\lambda dx$, or $dx = dx'$ and there is no length contraction, the usual result for classical theory.

Temporal Drift in Relativity.

Suppose that initially neither Σ nor Σ' have any temporal drift; then they are connected by the usual Lorentz Transformation,

$$x = \sqrt{1-V^2/c^2}\, x' + Vt,$$
$$t' = \sqrt{1-V^2/c^2}\, t - \frac{V}{c^2} x'.$$

Now introduce temporal drift λ in Σ and λ' in Σ' using the same relations as in the classical case. Then, with some manipulation, we find, dropping subscripts,

$$x = \sqrt{(1-\lambda v)^2 - v^2/c^2}\, x' + vt,$$
$$t' = \sqrt{(1-\lambda v)^2 - v^2/c^2}\, t + [\lambda' - \lambda(1-\lambda v) - v^2/c^2]x'.$$

where, as in the classical case, $v=V/(1+\lambda V)$.

Note that when v is small and v^2/c^2 is negligible, the modified Lorentz transformation is identical to the modified Galilean transformation that we found in section 2 and thus the dis-synchrony in a slowly transported clock is once again $-\lambda dx$.

Let us once again calculate the apparent length contraction of a rod at rest in Σ' using the same technique as in the classical case (v need no longer be small). We arrive at,

$$dx = \sqrt{1 - \frac{v^2/c^2}{(1-\lambda v)^2}}\, dx'.$$

This result is identical to the usual Lorentz contraction factor $\sqrt{1-V^2/c^2}$ when we make the substitution $v = V/1+\lambda V$.

Lastly in this section, with some manipulation, we may verify that $d\sigma'^2 = d\sigma^2$, where,
$d\sigma^2 := c^2\,(dt - \lambda dx)^2 - dx^2$.

This quantity is clearly the invariant line element, where the components of the four-metric, g, are,

$g_{00} = c^2$,
$g_{01} = -c^2\,\lambda$,
$g_{11} = -1+c^2\,\lambda^2$.

In the case of three spatial dimensions with curvilinear coordinates u_m, we would find three components of the temporal drift corresponding to $dt-\lambda_m du_m$ (where the repeated indices are summed 1 to 3), and, when g_{00} was chosen to equal c^2, we'd find that,

$g_{0m} = -c^2\,\lambda_m$,
$g_{mn} = -g_{mn} + c^2\,\lambda_m\,\lambda_n$,
where g_{mn} is the spatial metric.This last formula is in conformity with Tolman's result [2],
$g_{mn} = -g_{mn} + (g_{0m}\, g_{0n})/g_{00}$.

Physical Interpretation.

We have seen in section 3 that the interpretation of the off-diagonal components of the four-metric (when g_{00} is chosen to equal c^2 so that the coordinate u_0 is the reading on a clock, t, in the given reference frame) is as ($-c^2$ times) the temporal drift, while the components g_{mn} are a combination of the spatial metric and the temporal drift. Thus, we have arrived at a physical interpretation of the four metric, which was formerly considered vaguely just to be some sort of gravitational "potential." Note that, when $g_{00}=c^2$, the coordinates (t,u) have a direct physical meaning (t is the reading on a clock and u are the curvilinear spatial coordinates in the given reference frame), belying the currently popular opinion (originated by Einstein himself) that the coordinates in general relativity have no direct physical meaning but are merely convenient overlays on the physical problem at hand [3].

Note, that when authors such as Misner, Thorne and Wheeler attempt to interpret the physical meaning of the coordinate u_0 when $g_{00} \neq c^2$ they are misguided as such a coordinate is merely a convenient transformation of the physically meaningful variables, vaguely akin to the techniques of Hamilton-Jacobi theory [4].

Note that we can always redefine the fourth coordinate such that $g_{00} = c^2$ by choosing,

$$t = \int \frac{\sqrt{g_{00}}}{c} du_0,$$

where the integration is carried out assuming that the spatial coordinates, u, are constants.

Just as in the classical case, we can redefine a system of synchronization by writing t=t'+μ(u). Note that μ depends only on the spatial coordinates and ∂μ/∂t=0 so that the t clock and the t' clock run at the same rate, they are just synchronized differently and have different temporal drifts. We can choose the t clock to be universally synchronized by slow clock transport if the temporal drift of the t^' clock is independent of time and obeys $\partial\lambda_m / \partial u_n = \partial\lambda_n / \partial u_m$, because then the system of partial differential equations, $\partial\mu/\partial u_m = \lambda_m$, can be integrated to find a μ which is independent of time, as then $dt' + \lambda_m du_m$ is the perfect differential of t'+μ(u).

If $\partial\lambda_m / \partial t \neq 0$ then that is an example of gravitational time dilatation since clocks at different spatial positions run at different rates, while if $\partial\lambda_m / \partial u_n \neq \partial\lambda_n / \partial u_m$, we have the Sagnac effect whereby synchronization by slow clock transport is path-dependent: two spatially separated clocks may be judged to be in or out of synchrony depending upon the spatial trajectory followed between them [1].

If neither gravitational time dilatation nor a Sagnac effect are present—and we shall see presently that this happens only in inertial reference frames—then slow clock transport leads to the same result as Einstein synchronization by exploiting the constancy of the speed of light in an inertial frame, because they both lead to the standard Lorentz transformation (once all temporal drifts have been integrated out by the foregoing procedure) as we saw in section 3.

Constant Acceleration.

In my paper, I showed that the velocity field of a constantly accelerating rigid body with respect to an arbitrary but fixed inertial reference frame (without temporal drift) is,

$$v = \frac{at}{1 + ax/c^2},$$

(see reference, where however the convention c=1 is used as well as the origin of x is shifted, so that v=t/x and it is unclear that this is just v=at in the classical limit) [4,5].

In our notation, where t* is mathematical time coordinate, they derive in the accelerating frame,

$$d\sigma^2 = c^2(1 + ax/c^2)^2 dt^{*2} - dx^2 - dy^2 - dz^2.$$

Let us make the transformation $t = (1+ax/c^2)t^*$, then using the rules of section 4, we find that, because now $g_{00} = c^2$,

$$\lambda_x = \frac{at/c^2}{1+ax/c^2}, \lambda_y = \lambda_z = 0.$$

Note that since (∂λ_x)/∂t≠0, we have an instance of gravitational time dilatation in which clocks at different gravitational potentials (values of x) run at different rates [4].

Constantly Rotating Sphere

In the inertial reference frame,
$d\sigma^2 = c^2dt^2 - dr^2 - r^2\, d\theta^2 - r^2 \sin^2\theta\, d\varphi^2$.

Let us represent rotation by, $t^* = t,\ r' = r,\ \theta' = \theta,\ \varphi' = \varphi - \omega t$. Then, in the rotating frame, dropping primes,

$$d\sigma^2 = (c^2 - \omega^2 r^2 \sin^2\theta) dt^{*2} - 2\omega r^2 \sin^2\theta\, d\varphi dt^* - dr^2 - r^2\, d\theta^2 - r^2 \sin^2\theta\, d\varphi^2.$$

Write, therefore, $t = t^*\sqrt{1 - \omega^2 r^2 \sin^2\theta / c^2}$, so that $g_{00} = c^2$,
we find, using the rules of section 3,

$$\lambda_r = -\frac{\omega^2 r t \sin^2\theta / c^2}{1 - \omega^2 r^2 \sin^2\theta / c^2},$$

$$\lambda_\theta = -\frac{\omega^2 r t \sin\theta \cos\theta / c^2}{1 - \omega^2 r^2 \sin^2\theta / c^2},$$

$$\lambda_\varphi = +\frac{\omega r^2 \sin^2\theta}{\sqrt{1 - \omega^2 r^2 \sin^2\theta / c^2}}.$$

Thus, we see that λ_r and λ_θ exhibit gravitational time dilatation as they depend on t, while λ_θ and λ_φ exhibit a Sagnac effect because $\partial\lambda_\theta / \partial\varphi = 0 \neq \partial\lambda_\varphi / \partial\theta$.

Conclusion

This research arose out of an attempt to define a unique synchronization of clocks in accelerating or rotating reference frames. As we've seen, this is not possible and thus in non-inertial reference frames living with the temporal drift concept is not optional and even in inertial reference frames we can introduce it with the use of time zones even in the classical case.

We have seen, moreover, that when g_{00} is chosen to equal c^2, the coordinates have a direct physical meaning: u_0 is the reading on a clock, t, and u are the curvilinear spatial coordinates in the given reference frame. The 4-metric itself also is decomposed into the temporal drift and the spatial metric and its rather vague physical meaning is unveiled. It is not merely a gravitational potential or a metric in a Minkowskian space-time. Unpublished research of mine indicates that gravity is a force (defining a force, as Newton did, as a deviation from a spatial metric, not a space-time metrics). which has a vector potential equal to $c^2 \lambda_m$ in a non-Eucidean 3-space defined by metric g_{mn}.

We find no justification for the static Minkowski block space-time picture in which all events past, present and future are aid out deterministically for all time. I propose an alternate picture of a dynamic, evolving space-time which call "space with time" or "temporal geometry", which is more consonant with our intuition about nature: it is dynamic and evolving and not static, like the space-time of Minkowski or the geometries of Euclid or Riemann.

We could conceive of space-times connected by transformations between reference frames other than the Lorentzian or Galilean. Then, as is shown in, the line element $d\sigma^2$ is no longer invariant and does not serve as a distance in a Minkowskian space time; i.e., the space time manifold of Minkowski falls apart and we no longer have a unified 4-metric g_μν but must introduce the temporal drift and the spatial metric as separate entities [6]. Ignatowski also showed that in such a non-Lorentzian space-time that there would be a privileged reference frame in the sense that only the Lorentz transformation forms a group: that is the Lorentz transformation of a Lorentz transformation is another Lorentz transformation (the Galilean transformation is a special Lorentz transformation in which $c \rightarrow \infty$). We might think that it is not necessary to state the Laws of Physics in any reference frame other than the privileged frame, but we do not live in the privileged frame and so a theory of transformations remains necessary.

Moreover, a slightly non-Lorentzian transformation almost certainly applies in nature because the Lorentz transformation is only one single infinitesimal point in an infinite spectrum of possible transformations and nothing in nature is exact to an arbitrary degree of precision: nature is too chaotic and messy, so we must use temporal geometry with a separate temporal drift and spatial metric rather than the unified 4-metric of Minkowski.

I have tried repeatedly to find an elegant treatment of a temporal geometry in which the coefficients, μ, γ, and φ of the general transformation,

$$x = \mu(v)\, x' + vt,$$
$$t' = \gamma(v)\, t + \varphi(v)\, x',$$

depend on the reference frame in which the transformation to other reference frames of velocity *v* holds, but with little success. I leave this research to future mathematicians.

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