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New Innovations in Engineering Fields and Basic Sciences in the Analytical Solving of Complicated Nonlinear Differential Equations and Integrals by New Approaches IAM, Wolf,a and AYM

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Abstract

In this article, we want to introduce three new innovations and discoveries in the field of application of mathematics in engineering and basic sciences. And also, we want to prove that these new innovations are very powerful in solving nonlinear differential equations analytically in engineering and basic science fields. It is a fact that a new scientific innovation in engineering sciences can create great changes in the progress of industry and scientific works, most innovations and creativity originate from the channel of mathematics, in this article two new types of innovation by Mohammad Reza Akbari for Researchers and students of the world are introduced. And by presenting this article, we want to reduce the concerns of scientists and researchers in the analytical solution of very complex differential equations. In this article, we present three innovative methods called ***Integral Akbari Method (IAM)****,* ***Woman Life Freedom****,* ***abkari method (Wolf, a)*** *and* ***Akbari Yasna Method (AYM).*** *In this article, we want to solve equations like the following in the fields of engineering and basic sciences as:*

$$u'' = \left\{ \sqrt[m]{e^{\sqrt{\varepsilon u!}}} \right\}^{-\beta u'}$$

$$\begin{cases} u'' = \varepsilon(u! + v!)! \\ v'' = \eta(v!\,u!)! \end{cases} , \quad \begin{cases} u'' = \varepsilon \sqrt[p]{\cos(v!)\sin(u''!)} \\ v'' = \sqrt[q]{\beta \sin(v!\,u!)} \end{cases}$$

$$I = \iint \sqrt[p]{\varepsilon\left(\sqrt[\eta]{x^{(xy)!}}\ \sqrt[\beta]{y^{(xy)!}} \right)!}\ dydx$$

And partial nonlinear differential equations as:

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2} + \varepsilon u^q \left(\frac{\partial u}{\partial x} \right)^p$$

These virgin methods can be successfully applied in various engineering fields such as petroleum industry (solid, fluid, mass and heat transfer, dynamic, quantum physic, chemical reactors, electronics, applied sciences, economics, and etc.)

Keywords: New Method, Woman-Life-Freedom, Akbari Method (WOLF,a), Akbari-Yasna-Method (AYM), Integral Akbari Method (IAM), Nonlinear Differential Equation (PDE), Innovation in Engineering Mathematics Fields.

Introduction

The scientific power and abilities in a new innovation in the field of engineering can create tremendous changes in industry and practical work. In this manuscript, we want to introduce new innovations in the field of nonlinear differential equations, which are very powerful in the analysis and design of engineering problems. In the paper, our aims introduce of accuracy, capabilities and power for solving complicated non-linear differential in the engineering fields and basic sciences. IAM and WoLF,a method can be successfully applied in various engineering fields and all application areas, and also in the basic sciences (physics), economics and so on. It is worth noting that these methods are convergent at any form of differential equations, including any number of initial and boundary conditions. During the solution procedure, it is not required to convert or simplify the exponential, trigonometric and logarithmic terms, which enables the user to obtain a highly precise solution. As all experts know most of engineering actual systems behavior in practical are nonlinear process and analytical scrutiny these nonlinear problems are difficult or sometimes impossible. Our purpose is to enhance the ability of analytical solving the mentioned nonlinear differential equations and analytical solving of complicated integral in the all areas with two methods of simple and innovative approach which entitled "IAM" and "WoLF,a". He's Amplitude Frequency Formulation method which was first presented by Ji-Huan He gives convergent successive approximations of the exact solution and Homotopy perturbation technique HPM [1, 5]. It is necessary to mention that the above methods do not have this ability to gain the solution of the presented problem in high precision and accuracy so nonlinear differential equations such as the presented problem in this case study should be solved by utilizing new approaches like AGM method [6-13]. that created by Mohammadreza Akbari (in 2014). In recent years, analytical methods in solving nonlinear differential equations have been presented and created by Mohammadreza Akbari, these methods are called and AKLM (Akbari Kalantari Leila Method) and ASM [14-16]. (Akbari Sara's Method) and AYM (Akbari Yasna's Method) and IAM (Integral Akbari Method) [20]. These example somehow can be considered as complicated cases to deal with for all of the existed analytical methods especially in the design slides engineering, which means old methods cannot resolve them precisely or even solve them in a real domain [17-19].

Mathematical Formulation of the Problem

Analytical solution of nonlinear partial differential equations by IAM, 'WoLF,a' and AYM methods (Integral Akbari Method, Woman Life Freedom, akbari and Akbari Yasna Method)

Example1, for "IAM" Solution Process (Integral Akbari Method)

We consider a set of nonlinear differential equation as a nonlinear phenomenon in the engineering field and basic sciences as follows:

$$\begin{cases} u'' = \varepsilon(u! + v!)! \\ v'' = \eta(v!\, u!)! \end{cases} \tag{1}$$

The boundary conditions are:

$$bc:\ u(0) = u1,\ u(L) = u2,\ v(0) = v1,\ v(L) = v2 \tag{2}$$

The following boundary values have been used for the physical parameters of this problem as:

$$u1 = 0,\ u2 = 0\ ,\ v1 = 1,\ v2 = 0 \tag{3}$$

IAM Solution Process (Integral Akbari Method)

The output answer set of nonlinear differential equations the Eq. (1) according to boundaries conditions Eqs.(2) by IAM method is obtained as follows.

$$\begin{aligned} u(x) = -\frac{x\varepsilon}{16}\Bigg\{ & 3L\left(1+\left(\frac{1}{2}\right)!\right)! + 2L\left(1+\left(\frac{3}{4}\right)!\right)! - \\ & x\left(1+\left(\frac{L-x}{L}\right)!\right)! - 3x\left(1+\left(\frac{2L-x}{2L}\right)!\right)! - \\ & 2x\left(1+\left(\frac{4L-x}{4L}\right)!\right)! + 6L - 4x\Bigg\} \end{aligned} \tag{4}$$

And also

$$\begin{aligned} v(x) = -\frac{1}{16L}\Bigg\{ & 3\eta x L^2\left(\left(\frac{1}{2}\right)!\right)! + 2\eta x L^2\left(\left(\frac{3}{4}\right)!\right)! \\ & - Lx^2\eta\left(\left(\frac{L-x}{L}\right)!\right)! - 3Lx^2\eta\left(\left(\frac{2L-x}{2L}\right)!\right)! \\ & - 2Lx^2\eta\left(\left(\frac{4L-x}{4L}\right)!\right)! + 3L^2\eta x - 2x^2\eta L - 16L + 16x\Bigg\} \end{aligned} \tag{5}$$

By selecting the physical values for the Eqs. (1) and considering the physical properties of systems in engineering phenomena, we consider the following hypothetical values for design as:

$$\varepsilon = 0.2,\ \eta = 0.3\ ,\ L = 1$$

Comparing the Achieved Solutions by Numerical Method Order Runge-Kutta and IAM Method (Integral Akbari Method), According the Physical Values Eqs. (6) as follows:

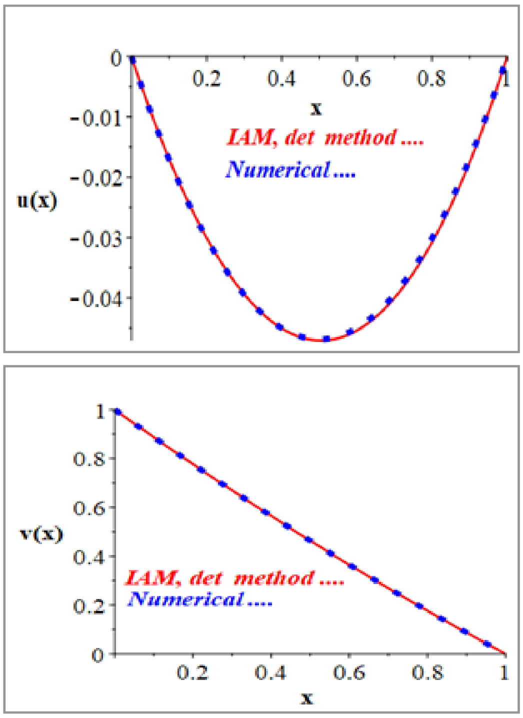


Fig1: A comparison between IAM and Numerical solution.

Example2, for "WoLF,a" Solution Process (Woman Life Freedom, akbari)

We consider a set of nonlinear differential equation as a nonlinear phenomenon in the engineering field and basic sciences as follows:

$$\begin{cases} \dfrac{d^2u}{dx^2} = \varepsilon \sqrt[p]{\cos(v!)\sin(u'!)} \\ \dfrac{d^2v}{dx^2} = \sqrt[q]{\beta \sin(v!\,u!)} \end{cases}$$

The boundary conditions are:

$$bc:\; u(0) = u1,\, u(L) = u2,\; v(0) = v1,\, v(L) = v2$$

The following boundary values have been used for the physical parameters of this problem as:

$$u1 = 1,\, u2 = 0 \quad,\, v1 = 0,\, v2 = 1$$

'WoLF,a' Solution Process (Woman Life Freedom, akbari)

The output answer set of nonlinear differential equations the Eq. (1) according to boundaries conditions Eqs. (2) by **'WoLF,a'** method is obtained as follows.

$$u(x) = \frac{(x-L)}{6p\,L\cos(1)}\left\{x\varepsilon \sqrt[p]{\ln\left[\cos(1)\sin\left(-\frac{1}{L}\right)!\right]}\,[3pL\cos(1) + (\gamma x + L\gamma)\sin(1)] - 6p\cos(1)\right\}$$

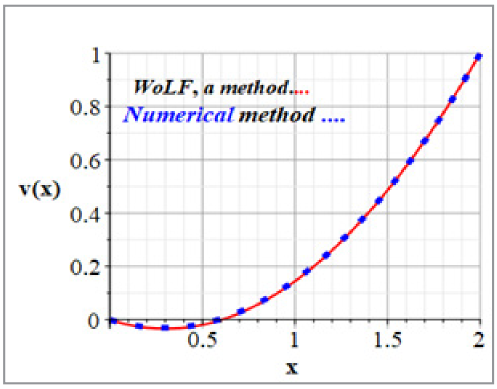
and

$$v(x) = \frac{x}{6q\,L\sin(1)}\left\{\sqrt[q]{\ln(\beta\sin(1))}\,(L-x)[3qL\sin(1) - \cos(1)\,(L+x)] + 6q\sin(1)\right\}$$

By selecting the physical values for the Eqs. (1) and considering the physical properties of systems in engineering phenomena, we consider the following hypothetical values for design as:

$$\varepsilon = 0.2,\, \beta = 0.3,\, L = 2,\, p = 3,\, q = 5,\; \gamma = 0.5772156649$$

Comparing the achieved solutions by Numerical Method order Runge-Kutta and 'WoLF,a' method (Woman Life Freedom, akbari), according the physical values Eqs.(6) as:



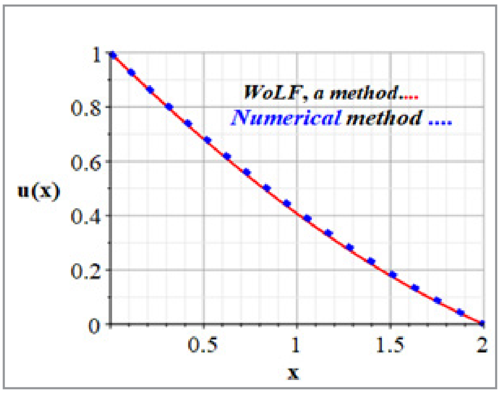


Fig2: A comparison between 'WoLF,a' and Numerical solution.

Example3, for "WoLF,a" Solution Process (Woman Life Freedom, akbari)

We consider a complicated nonlinear differential equation as a nonlinear phenomenon in the engineering field and basic sciences as follows:

$$u'' = \left\{\sqrt[m]{e^{\sqrt{\varepsilon u!}}}\right\}^{-\beta u'} \tag{1}$$

The boundary conditions are:

$$bc:\; u(0) = u1,\, u(L) = u2 \tag{2}$$

The following values have been used for the physical parameters of this problem as:

$$u1 = 1,\, u2 = 0 \tag{3}$$

'WoLF,a' Solution Process (Woman Life Freedom, akbari)

The output answer nonlinear differential equations the Eq. (1) according to boundaries Eqs. (2) by 'WoLF,a' method is obtained as follows.

$$u(x) = -\frac{x}{12m\,L^2}\left\{e^{-\frac{\beta\sqrt{\varepsilon}}{m^2}}(L-x)\left[\sqrt{\varepsilon}\,\gamma\beta L + \sqrt{\varepsilon}\,\gamma\beta x + 6m\,L^2\right] - 12m\,L\right\} \tag{4}$$

By selecting the physical values for the Eqs. (1) and considering the physical properties of systems in engineering phenomena, we consider the following hypothetical values for design as:

$$\beta = 0.2, \; L = 2, \; m = 5 \qquad (5)$$

Comparing the achieved solutions by Numerical Method order Runge-Kutta and 'WoLF,a' method (Woman Life Freedom, akbari), according the physical values Eqs. (5) as:

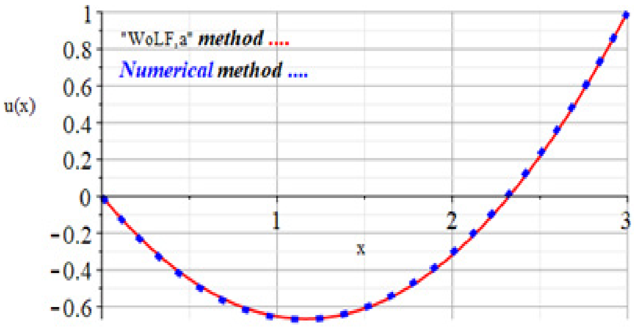


Fig3: A comparison between 'WoLF,a' and Numerical solution.

Example4, for "WoLF,a" Solution Process (Woman Life Freedom, akbari)

We consider a complicated partial nonlinear differential equation as a nonlinear phenomenon in the engineering field and basic sciences as follows:

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2} + \varepsilon u^q \left(\frac{\partial u}{\partial x}\right)^p \qquad (1)$$

The boundary and initial conditions are:

$$u(0, t) = 0 \;, \; u(L, t) = 0 \;, u(x, 0) = uo \qquad (2)$$

"WoLF,a" Solution Process (Woman Life Freedom, akbari)
The output answer partial nonlinear differential equations the Eq. (1) according to boundaries and initial conditions Eqs. (2) by "WoLF,a" method is obtained as follows.

$$u(x, t) = \sum_{n=1}^{\infty} \left\{ A \sin\left(\frac{n\pi x}{L}\right) \times e^{-\frac{1}{5L^2}\left[5\pi^2 \alpha n^2 t + L^2 \,\mathrm{LambertW}\left(\Delta e^{-\frac{5\pi^2 \alpha n^2 t}{L^2}}\right)\right]} \right\} \qquad (3)$$

$$\Delta = \frac{-16}{21L^2} A^5 \pi n t \varepsilon \left(1-(-1)^n\right) \quad A = \frac{2uo}{n\pi}\left\{1-(-1)^n\right\} \qquad (4)$$

We consider physical values as following:

$$\alpha = 0.02, \; \varepsilon = 0.005, \; L = 1, \; uo = 1 \,, p = 2 \,, q = 4 \qquad (5)$$

Comparing the achieved solutions by Numerical Method order Runge-Kutta and "WoLF,a" method (Woman Life Freedom, akbari), according the physical values Eqs. (5) as:

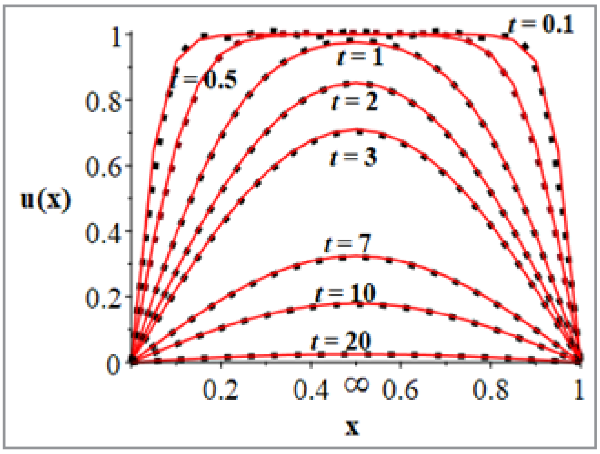


Fig4: A comparison between "WoLF,a" and Numerical solution.

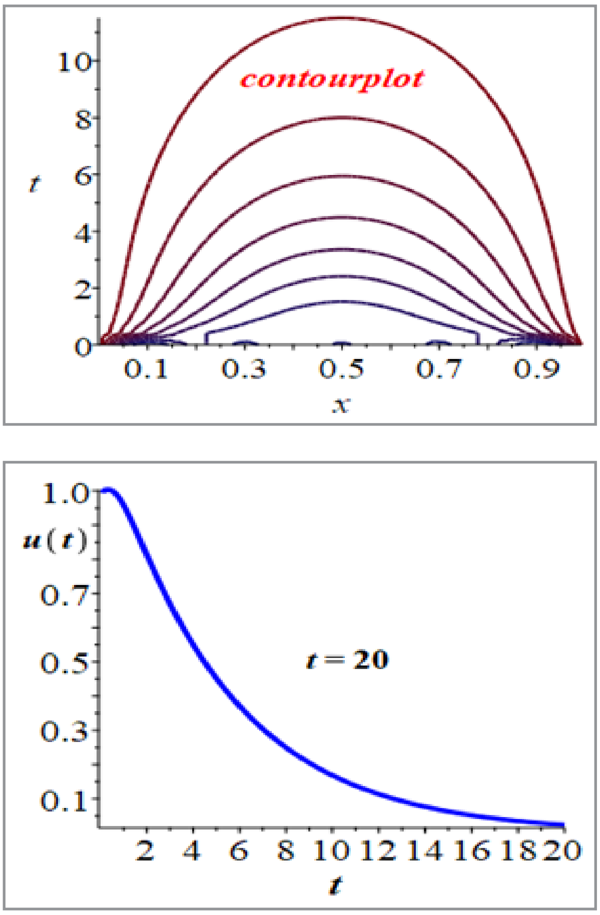


Fig5: Diagram the contour plot and time.

Example5, for "IAM" Solution Process (Integral Akbari Method)

We consider a complicated double integral as a nonlinear phenomenon in the engineering field and basic sciences as follows:

$$I = \iint \sqrt[p]{\varepsilon\left(\sqrt[\eta]{x^{(xy)!}} \; \sqrt[\beta]{y^{(xy)!}}\right)!} \; dy dx \qquad (1)$$

$$x \in [0, b] \,, y \in [0, d]$$

IAM Solution Process (Integral Akbari Method)

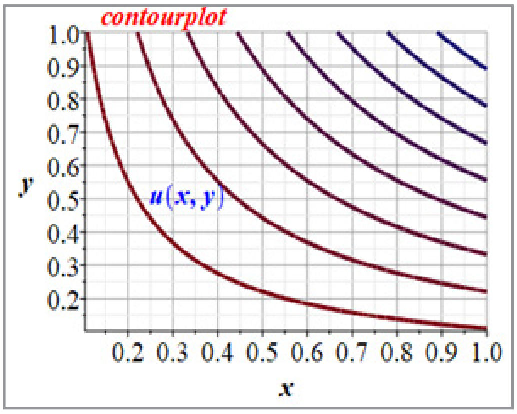
The output answer set of nonlinear differential equations the Eq. (1), by **IAM** method is obtained as follows.

$$I(x,y)=\frac{xy}{4}\left\{\left[\frac{3}{2}\sqrt[p]{\varepsilon}+\sqrt[p]{\varepsilon\left(\sqrt[\eta]{x^{\left(\frac{xy}{2}\right)!}}\sqrt[\beta]{\left(\frac{y}{2}\right)^{\left(\frac{xy}{2}\right)!}}\right)!}\right.\right.$$
$$-\frac{1}{2}\sqrt[p]{\varepsilon\left(\sqrt[\eta]{x^{(xy)!}}\sqrt[\beta]{y^{(xy)!}}\right)!}$$
$$\left.\left.+\left(\sqrt[p]{\varepsilon}+\sqrt[p]{\left(\varepsilon\left(\sqrt[\eta]{x^{(xy)!}}\sqrt[\beta]{y^{(xy)!}}\right)\right)!}\right]\right\}$$

We consider physical values as:

$$\varepsilon=0.6,\ a=1,\ d=1,\ p=5,\ \beta=3,\ \eta=2$$

Comparing the achieved solutions by Numerical Method and IAM method (Integral Akbari Method), according the physical values Eqs. (3) as:



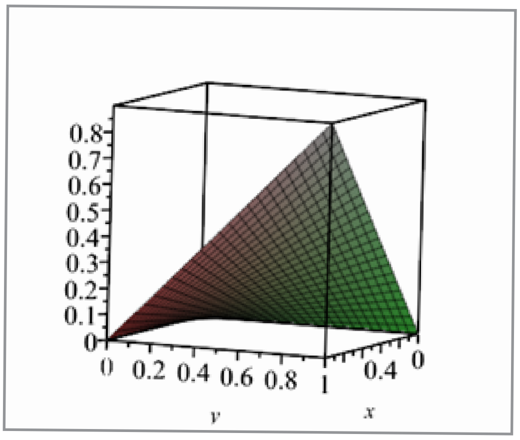


Fig2: A contour plot IAM method solution.

Conclusions

In this article, we proved conclusively that new methods (AYM,'Wolf,a' and AYM) can all kinds of complicated practical problems related to nonlinear partial and ordinary differential equations in the engineering field and basic sciences an they can be easily solved analytically. Obviously, most of the phenomena of physically are nonlinear, so it is quite difficult to study and analyze nonlinear mathematical equations in this area, also we wanted to demonstrate the strength, capability and flexibility of the new methods. This method is newly created and they can have high power in analytical solution of all kinds of industrial and practical problems in engineering fields and basic sciences for complicated nonlinear differential equations.

History of AGM , ASM , AYM , AKLM , MR.AM and IAM, WoLF,a Methods

AGM (Akbari-Ganji Methods), ASM (Akbari-Sara's Method), AYM (Akbari-Yasna's Method) AKLM (Akbari Kalantari Leila Method), MR.AM (Mohammad Reza Akbari Method) and IAM (Integral Akbari Methods), WoLF,a method (Women Life Freedom, akbari), have been invented mainly by Mohammadreza Akbari (M.R. Akbari) in order to provide a good service for researchers who are a pioneer in the field of nonlinear differential equations.

- AGM method Akbari Ganji method has been invented mainly by Mohammadreza Akbari in 2014. Noting that Prof. Davood Domairy Ganji co-operated in this project.
- ASM method (Akbari Sara's Method) has been created by Mohammadreza Akbari on 22 of August, in 2019.
- AYM method (Akbari Yasna's Method) has been created by Mohammadreza Akbari on 12 of April, in 2020.
- AKLM method (Akbari Kalantari Leila Method) has been created by Mohammadreza Akbari on 22 of August, in 2020.
- MR.AM method (MohammadReza Akbari Method) has been created by Mohammadreza Akbari on 10 of November, in 2020.
- IAM method (Integral Akbari Method) has been created by Mohammadreza Akbari on 5 of February, in 2021.
- WoLF,a method (Women Life Freedom, akbari) has been created by Mohammadreza Akbari on 5 of February, in 2022.

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