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Correction and Extension of Quantum Statistics

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Abstract

A comparative analysis of the Classical Statistics of Maxwell-Boltzmann and the Quantum Statistics of Fermi-Dirac and Bose-Einstein revealed some inaccuracies and fundamental errors in them. Fundamental errors are related to the fact that the factor of the Planck formula was discarded, which eliminates the non-physical divergence of PROBABILITY! Two options are considered for actually restoring the probabilistic nature of Quantum Statistics, originally incorporated into it by Planck himself. The Planck Statistics gives a correct description of the distribution of Bose particles and the transition to the Maxwell-Boltzmann Statistics, which made it possible to identify and eliminate the phenomenological errors of the Theory of Superconductivity, which makes it possible to explain the discovered high-temperature superconductivity as "normal". The restored Planck Statistics also makes it possible to predict the existence of high-temperature superconductivity in new materials and artificial structures.

Keywords: Probability, Quantum Statistics, Planck Statistics, Superconductivity, Coherent Resonant Waves.

It's very good that we're still feeling bad!" - there is an opportunity to move to the best. Unless, of course, there is strength and desire, and not only the Author of the Idea, but also the Scientific Community. But the purely consumer attitude of the Society towards Science hinders the development of the Fundamental Ideas in it, and the Scientific Community itself lowers it to the philistine level, within the framework of which their "scientific" career is built. But ETHICS is the heart of Science, without which ITS LIFE stops. At the same time, Science degenerates into a DEAD Game of the Mind, fundamentally unable to improve the Description even of NON-LIVING Nature.

And Boltzmann's report at the Royal Academy, even at the peak of his fame, was publicly spat upon, and not tried to be understood. AND! he hanged himself. Here is the Discovery in the Theory of Sets by the German mathematician Ernst Zermelo, Russell first called a mistake, and then appropriated it as Russell's PARADOX. And Zermelo, believing him, considered that the Logician had no right to make a mistake, and committed philosophical suicide - he left Science. The great Planck also returned the mathematician from non-existence, and having calculated two World Constants, he first assigned the name of the spat upon Boltzmann. And, although Planck himself was temporarily suspended from work and deprived of his professorship for rejecting the mathematical distortions of the Quantization he introduced (until he agreed to accept the Nobel Prize), nevertheless, he returned the name of Boltzmann in Science from a special case of the canonical distribution of Gibbs to the founders Classical Maxwell-Boltzmann Statistics.

$$n^{M-B} = Exp\left[-\frac{E_i}{k_B T}\right] \qquad 1$$

If we digress a little from the actual Statistics, then we can also recall the name of the senior telegrapher Heaviside, whom they "ashamed" to attribute to Electrodynamics, written down in the final form by Maxwell according to Heaviside. In general, he was never recognized as a scientist, and his Electrodynamic Theory of Gravity was not given any importance.

But returning to Statistics, after the expiration of time it is f.(1) Richard Feynman calls the Pinnacle of Statistical Mechanics and rewrites it in operator form to construct Quantum Statistics - to determine the mean, normalized to the statistical sum, of the mathematical expectation of a state with energy E_i [1]

$$\langle A\rangle = \sum_i \langle i|A|i\rangle \cdot Exp\left[-\frac{E_i}{k_B T}\right] \qquad 2$$

As Feynman said about Statistical Mechanics, "All the rest of its content is either a descent from the top, when the basic Principles are applied to particular questions, or an ascent to it, when the basic relationships are derived and the concepts of thermal equilibrium and temperature are refined.” But even on the descent, Physics got lost. Without fully understanding the Elementary Oscillator and moving the founders of the Planck Quantization and Einstein away from the Primary Quantization, physicists built virtually mystical structures from the formulas of the Secondary Quantization [2-8]. Well, until recently, people thought about the connection between Pontryagin's Dualism and coherence [9, 10, 11].

But then, immediately after Planck's discovery, everyone was in a hurry to comprehend Planck's quanta and declare themselves. And for the QUANTUMS introduced by Planck, the Quantum Statistics of Fermi-Dirac and Bose-Einstein were built

$$n_i^{F-D} = \frac{g_i}{Exp\left[\frac{E_i-\mu}{k_B T}\right]+1}, \quad n_i^{B-E} = \frac{g_i}{Exp\left[\frac{E_i-\mu}{k_B T}\right]-1}$$

where g_i is the level degeneracy factor, μ is the chemical potential, k_b is the Boltzmann constant, T is the absolute temperature. In what follows, we will use the given values

$$E^* = \frac{E_i}{k_B T}, \mu^* = \frac{\mu}{k_B T}$$

These statistics were called quantum statistics and continue to be used. But they describe the distribution over quasi-continuously located energy levels of electrons. And unlike the Maxwell-Boltzmann Classical Statistics, for which the total integral over all states is strictly equal to one (the PROBABILITY of finding an electron at some allowed energy level is strictly equal to 1), “quantum statistics” do not fully take this into account. However, considering the Pauli Principle, instead of the initial concentration in the Maxwell-Boltzmann statistics, the Fermi-Dirac statistics use the assumption that Fermi electrons are "scooped" from the initial volume - from the reservoir at energies less than chemical. potential. This is exactly how the Fermi-Dirac statistics, with the multiplicity of level degeneracy equal to 2, qualitatively describes the probabilities of filling the levels with Fermi electrons for all their allowed energy values and was quite correctly used as the first approximation (Fig. 1). And it is in the one shown in Fig. 1 form The Fermi-Dirac Statistics, which is, in fact, laid down in the FOUNDATION of Quantum Solid State Physics, has received numerous confirmations.

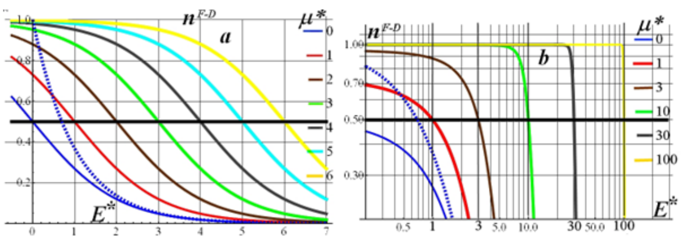

Figure 1: Dependence of the Statistical Fermi-Dirac Distribution on the reduced energy for different values of the reduced chemical. potential: a - on a linear scale for small values of chemical. potential, b - in a double logarithmic scale for large values of chemical. potential (dashed curve shows, for comparison, the Statistical Distribution of Maxwell-Boltzmann at zero chemical potential).

The second, Bose-Einstein, was called, because of the unlimited degeneracy of energy levels, Statistics with a reservation, with a non-conserved number of particles. But, strictly speaking, it cannot in principle be called the statistics of the equilibrium distribution of electrons. It gives non-physical (not probabilistic) infinities near chemical. potential and, having retained only in the denominator from Planck's formula "-1", does not pass the test for probability and entirely - its integral over all energies is greater than chemical. potential, is divergent.

If we divide the spectral energy density of the Planck radiation by the energy of the Planck QUANTUM at a given frequency, then we get the spectral density for a particular case, bosons, for the number of photons:

$$\frac{u_\nu(\nu,T)}{h\nu} = \frac{8\pi\nu^2}{c^3}\frac{1}{e^{\frac{h\nu}{k_B T}}-1} = \frac{8\pi}{c^3 h^2}\left(k_B T\right)^2\left[\left(\frac{h\nu}{k_B T}\right)^2\frac{1}{e^{\frac{h\nu}{k_B T}}-1}\right]$$

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in red, which depends only on the photon energy reduced to thermal energy. And the total number of quanta - photons, and at an infinitely degenerate energy level, of course, is not infinite - their number is determined precisely by the first factor that increases quadratically with temperature (highlighted in blue in Form 4). And this first factor, like the entire Planck function, naturally vanishes at zero temperature and completely determines their "non-conservation" when the temperature changes. The second factor of the Planck function (solid lines in Fig. 2) was actually used to construct the "Statistics" of Bose - Einstein (dashed lines in Fig. 2), but the numerator that eliminates the divergences was completely discarded from the "Statistics". Nevertheless, it was Bose-Einstein's "Statistics" that laid the foundation for the Theory of Superconductivity. At the same time, the chemical potential was included in the Bose-Einstein statistics conditionally - it cannot be associated with an infinite number of bosons in any way, but simply sets the Fermi level as the energy reference point for the generated bosons. Those. chem. the potential simply sets a certain binding of the distribution of Bose particles to a certain energy in the distribution of Fermi particles. But the form of the Bose-Einstein function this “chem. potential”, diverging even if not at zero, but at this energy level, does not change in any way - the divergences remain (dashed lines in Fig. 2).

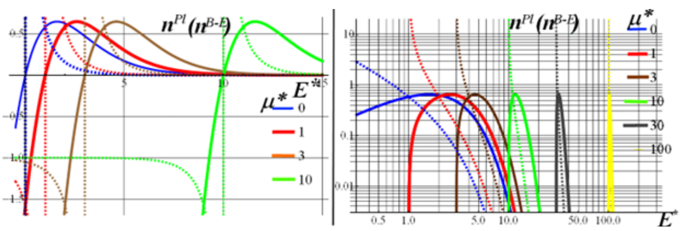

Figure 2: The dependence of the Statistical Planck Distribution on the reduced energy for different values of the reduced chemical. potential: a - on a linear scale for small values of chemical. potential, b - in a double logarithmic scale for large values of chemical. potential (dashed curve shows, for comparison, the "Distribution" of Bose-Einstein).

At the same time, the graphs in Fig. 2 consider that the quantization is carried out by the vibrational part of the energy - resonant surface waves do not depend on the "depth of the vessel" if their amplitude is much less than the depth, the Fermi level can also be used for the very original Planck function from formula 4, rewriting it statistical factor in reduced energies:

$$n^{Pl} = \frac{\left(E^* - \mu^*\right)^2}{e^{E^* - \mu^*} - 1} \qquad 5$$

$$\frac{h\nu_{\max}}{kT} = \alpha, \quad \alpha = 2{,}821439... \Rightarrow \left(E^*\right)_{\max} = \mu + \alpha \qquad 6$$

And based on the equality of the total probability of finding a particle at any level above the chemical level. potential, one can also obtain the normalized Planck Statistics for the photons described by him:

$$n_i^{Pl} = \frac{1}{2Zeta[3]} \cdot \frac{\left(\mathrm{E}^*\right)^2}{e^{\mathrm{E}^*} - 1} = 0.41595368629035373 \cdot \frac{\left(\mathrm{E}^*\right)^2}{e^{\mathrm{E}^*} - 1} \qquad 7$$

But for simplicity, we did not use this numerical correction for the curves shown in Fig. 2, since there are not so much numerical as fundamental problems in using Bose-Einstein statistics.

After all, the singularity of this “Statistics”, strictly speaking, is the error of the Bose-Einstein mathematical function used near the chemical energy. potential, was used to describe superconductivity. Although, the step on the current-voltage characteristic of the metal-superconductor contact (the Giaver step) shows that the Bose-Einstein "distribution" could only be used as a correction to the Classical Distribution. The roughness of the Theory of Superconductivity, built on the basis of the "distribution" of Bose-Einstein, was the reason that the mentioned Theories gave only Nobel Prizes, and the increase in temperature beyond the transition in metal alloys gave the Empirical Matthias Rules. And high-temperature superconductors were discovered not at all according to the Theory of Superconductivity, but by chance, when a Soviet physicist at the Girikond enterprise measured the temperature characteristics of serial ceramic high-resistance ones! resistance. True, for this discovery, he received only a reprimand at work and left for the USA (where he and his discovery were simply used to receive the Nobel Prize).

Because

And the statement in some textbooks, which has also been transferred to WIKIPEDIA, that at zero chem. Potential Bose-Einstein statistics goes into Maxwell-Boltzmann statistics FALSE:

$$n_i^{B-E} = \frac{g_i}{Exp\left[\frac{\mathrm{E}_i}{k_B T}\right] - 1} \neq g_i Exp\left[-\frac{\mathrm{E}_i}{k_B T}\right] = n_i^{M-}$$

$$\frac{1}{Exp\left[\frac{\mathrm{E}_i}{k_B T}\right] - 1} \xrightarrow[\mathrm{E}_i \to 0]{B-E} \infty, \qquad Exp\left[-\frac{\mathrm{E}_i}{k_B T}\right] \xrightarrow[\mathrm{E}_i \to (]{M-E} \qquad 8$$

In addition, the Bardeen-Cooper-Schrieffer (BCS) theory constructed for superconductivity assumed of the presence of a density of states for Cooper pairs, which, under the assumption that their matrix elements near the Fermi level are constant, determines the energy of their binding through the Debye energy

$$N\left(K, \mathrm{E}^*\right) \Rightarrow \Delta = 2\hbar\Omega_D \left(e^{-N\left(K,\mathrm{E}^*\right)\cdot|F|} - 1\right) \qquad 9$$

Although, strictly speaking, the Statistics constructed by Planck himself proceeded from the "continuous" statistics of Maxwell-Boltzmann. So originally, she used the continuity of energy change. But at the same time, it does not consider the fundamental discovery of Planck himself - Discreteness (Quantum) of the allowed values of energy levels. But when using a quasi-continuous spectrum of allowed states for Cooper pairs, its negative values below the Fermi level of normal electrons can, in principle, be associated with additional "evaporation" of normal electrons due to the binding energy of the Cooper pair. But at the same time, it is necessary to consider the correction for energy, namely, that their chem. the potential of Bose particles below the Fermi level on the binding energy of the Cooper pair.

And so, considering the above, the use of Bose-Einstein's "Statistics" for Cooper pairs of Fermi electrons, purely qualitatively, can only be justified by the ASSUMPTION that the entire superconducting electron fluid is concentrated at the same level near the Fermi energy of normal electrons. In fact, this means that either there is no dispersion of Bose electrons, or their second allowed level lies at a distance much greater than the average thermal energy. Which, in turn, means that the statistics of the bosons themselves are actually not considered in any way. But superconductivity, as an analogue of superfluidity, also arises as a transfer not of a single pair of electrons, but of the (whole) mass of electrons. So the production of Bose electrons apparently concerns a large number of Fermi electrons located at the energy levels of the entire band of free (or quasi-free, as in high-temperature superconductors) normal electrons. And this, in an implicit form, was also laid down in the BCS.

So, there are many unresolved questions of quantum statistics. But it is necessary to analyze and correct the Theory of Superconductivity on the basis of Planck's Statistics, and not on the Bose-Einstein surrogate.

Planck Statistics works, of course, only for values greater than one - for a large number of integer quanta, and it, in contrast to the "statistics" of Bose Einstein (f.7), gives the correct limiting transition to the Maxwell-Boltzmann statistics.

But the General Statistics, which generalizes both the classical and the quantum, because of the neglect of the original Planck's Quantum Statistics, which was correctly constructed for a particular case, has not been, and is not built now. To build the General Statistics, the Planck function apparently needs to be decomposed differently than it was done above

$$\frac{u_\nu(\nu, T)}{h\nu} = \frac{8\pi\nu^2}{c^3} \frac{1}{e^{\frac{h\nu}{k_B T}} - 1} = \frac{8\pi k_B}{c^3 h}(T\nu)\left|\frac{h\nu}{k_B T} \frac{1}{e^{\frac{h\nu}{k_B T}} - 1}\right| \qquad 10$$

Then for chem. zero potential, we get the correct transition from Quantum Statistics to Classical

$$\frac{\mathrm{E}_i}{k_B T} \frac{1}{Exp\left[\frac{\mathrm{E}_i}{k_B T}\right] - 1} \xrightarrow[\mathrm{E}_i \to 0]{(Planck)^*} 1 \qquad 11$$

This statistical factor, as shown in Fig. 3, is similar to the Maxwell-Boltzmann Statistics and plausibly describes the distribution of bosons at energies above chemical. potential.

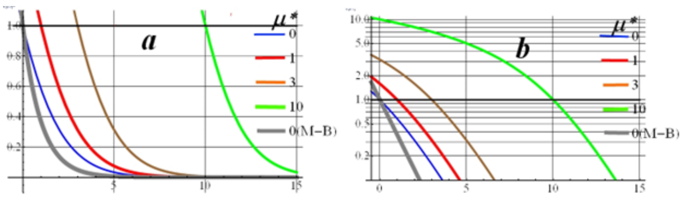

Figure 3: Modified, similar to the Maxwell-Boltzmann Statistics (gray line) Planck Quantum Statistics for different values of the given chemical. potential.

However, the Planck formula itself, from which the two shown modifications of statistical factors are extracted, does not consider the destruction of Cooper pairs above the critical temperature. In principle, they tried to take this into account with the help of an additional factor in the BCS (formula 8). But the statistical factors themselves admit their existence with high energies for Cooper pairs (as for photons). But this, apparently, contradicts the destruction of Cooper pairs by microwave (IR) radiation. So for bosons in the General form, the Statistics itself requires further clarification. Moreover, the Little-Parks size effect indicates a coherent state of Cooper pairs - a collective, and not one partial effect. This apparently explains the rather high frequencies of radiation - IR, necessary for the destruction of superconductivity, corresponding to temperatures much higher than the super transition temperature.

So, to answer the fundamental question: Is there an increase in the concentration of Cooper pairs as the temperature rises below the super transition temperature, it is necessary to experimentally investigate the dependence of the frequency of their destruction by IR radiation on temperature, which can be qualitatively investigated by the shape of the plasma reflection of the superconductor. This, in principle, will help to explain the difference between very high conductivity, for which, ideally, a purely ballistic transfer of individual Cooper pairs should be observed, from superconductivity, for which it is necessary to consider the synchronous displacement of all Cooper pairs, described by the dispersion law of a coherent wave. But this apparently also requires refinement of the Planck function itself, obtained for diffuse radiation, and not for coherent radiation - a pure resonant wave has no parasitic harmonics and, thus, has a minimum energy for diffuse radiation quanta. At the same time, the degree of wave coherence determines, as for plasma oscillations, its maximum “allowed” wave vector and the “zero temperature” of the ideal wave itself. And this, in turn, is the very possibility of laser cooling, the fact that the wave itself is “cold” and does not heat.

But such an approach reverses the very role of Statistics of bosons, which are enough in a Solid Body even without Cooper pairs – at a non-zero temperature, both photons and phonons always exist. Bosons just violate the coherence of an ideal wave, and as the temperature rises, when the maximum frequency of the Planck function reaches its critical frequency, it is completely destroyed at the critical temperature of the super transition, as in microwave destruction. So Planck's statistics simply determines the degree of diffuseness of a coherent wave, including for superconductivity. And in this regard, superconductivity itself is a kind of modification of the ballistic transfer of electrons over potential barriers. And episodic observations of high-temperature superconductivity over the diamond surface, in which there are few intrinsic photons and phonons up to the high Debye temperature, are quite understandable. Moreover, from these positions it is possible to explain high-temperature superconductivity in ceramics and it can be expected in monoatomic layers of graphite on solid ideal substrates and the existence of polar superconductivity in structures of p-n junctions [12].

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