

# The Influential Impact of Tensor Decompositions on Biomedical Sciences: A Review

Ismail A. Mageed\*

Sheffield Institute of Education Charles St, Sheffield City Centre, Sheffield S1 2LX

\*Corresponding author: Ismail A Mageed, Sheffield Institute of Education Charles St, Sheffield City Centre, Sheffield S1 2LX.

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## Abstract

Examining several methods for Tensor Decompositions and their possible usefulness in the biomedical sciences is the main goal of this work. The goal is to use Tensor Decompositions to improve research outcomes. Additionally, the review points up unsolved problems and suggests future lines of inquiry for this area of study. In essence, by exploring a broader vision for a higher-level performance of biomedical sciences, these suggested open questions would give the research community more opportunities to express themselves, innovate, and offer more practical applications to advance the state of the art. When considering the broader picture, this also implies that Tensor Decompositions may be used to transform the current space AI sector and machine learning technologies.

**Keywords:** Tensor Decompositions (TDS), Tensor Train Networks (TTNS), Tucker Decomposition (TUD), Canonical Polyadic Decomposition (CPD), Polyadic Decomposition (PD), Biomedical Sciences

## Introduction

There are well-known techniques for super-resolution, which employ interpolation techniques [1-5]. TDs approaches have received a lot of interest from researchers in various domains of computer science and engineering [2-5]. Fig. 1 depicts TTNs.

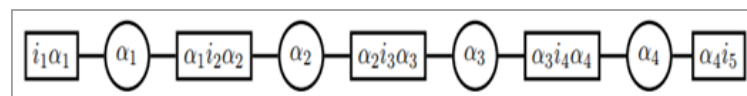


Figure 1: A schematic of TTNs [6].

These methods necessitate TT, TUD, and CPD. Furthermore, there are several characteristics that determine which decompositions function well. The TUD [5], for instance, works well for compressing low-dimensional arrays that contain samples of Green's function and related integrals [7-12]. However, for com-

pressed high-dimensional data, it performs poorly because of the curse of dimensionality [13, 14]. In place of traditional PD, TTN is frequently used when high precision is required. Fig. 2 shows a CPD array that is three-way.

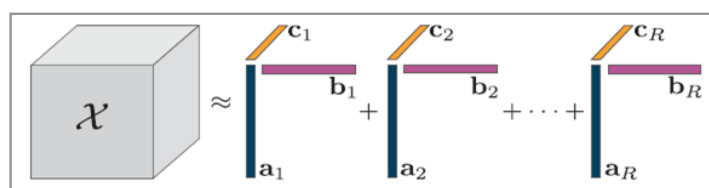


Figure 2: A three-way array's CP decomposition [14].

## Tucker Representation (TR)

$\tau$  defines a three-dimensional array,  $n_1, n_2, n_3$ , namely  $\tau \in \mathbb{C}^{n_1 \times n_2 \times n_3}$ . Thus, this array's TR has a core tensor  $\text{Mc} \in \mathbb{C}^{n_1 \times n_2 \times n_3}$  of low rank combined with a set of factor matrices  $M_i \in \mathbb{C}^{n_i \times r_i}$ ,  $i=1,2,3$ . These rewrite  $\tau$  to the form

$$\tau = C x_1 M^1 x_2 M^2 x_3 M^3 \quad (1)$$

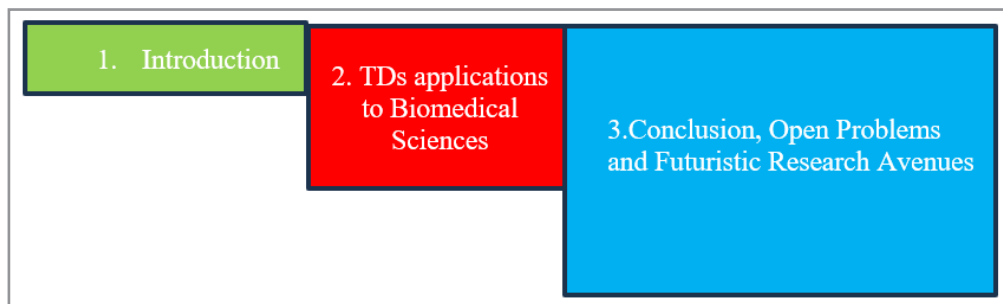
Notably,  $x_i$  and  $r_i$  serve as  $i$ -mode matrix-tensor multiplication

and multilinear rank pertaining to  $i^{\text{th}}$  dimension respectively. More interestingly,  $\tau$  is defined to be highly compressible if

$$(r_1 r_2 r_3) + \sum_{i=1}^3 n_i r_i \ll n_1 n_2 n_3 \quad (2)$$

$C$  and  $M^i$  (c.f., (1), (2)) are calculated by singular value decomposition (SVD) approach [4,15,16] for a given tolerance,  $\text{tol}$ .

The path map for this study is.

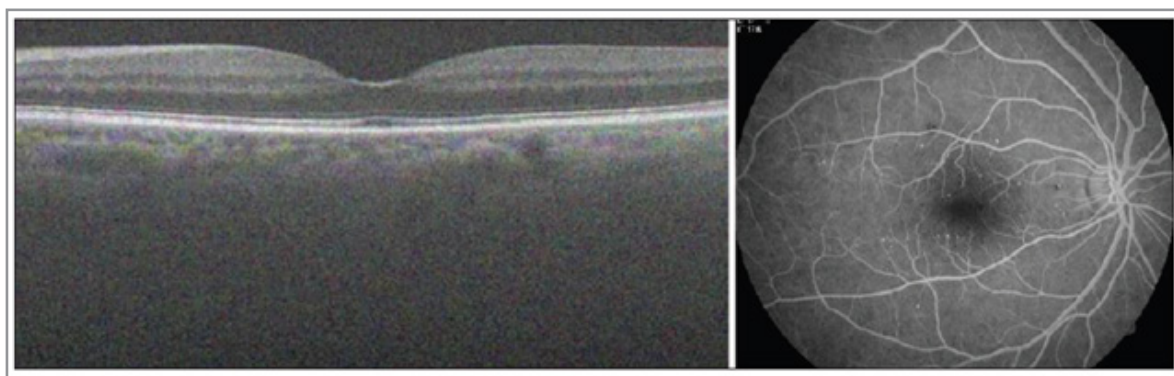


The second section covers the applications of TDs in the biomedical sciences. Section three presents the next phase of research along with some significant new open problems and closing thoughts.

## TDs Applications to Biomedical Sciences

The usefulness of matrix-based techniques in analysing multidimensional datasets is greatly diminished since matrices are unable to preserve the multidimensional correlation of elements in higher-order datasets. Besides this, tensor-based methods have shown promising results. Researchers were driven to switch from matrices to tensors by these factors taken together [17].

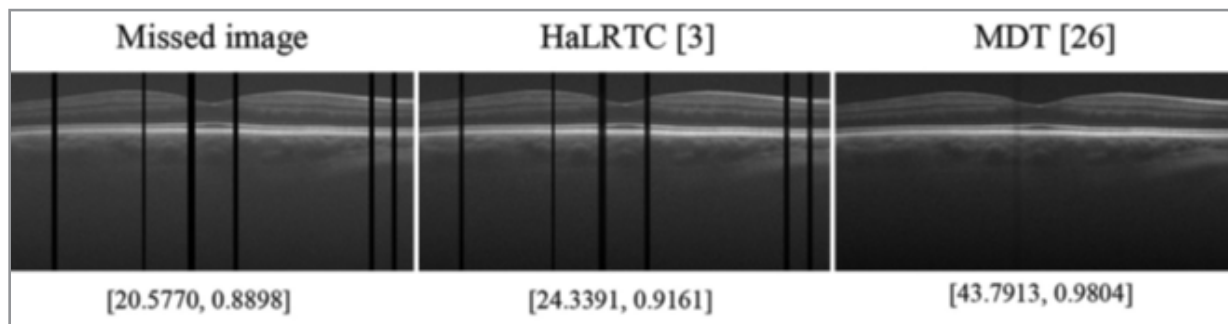
Biomedical image and signal analysis are one of the most important applications of signal and image processing. This is because reliable information must be extracted from biomedical databases, which have a direct impact on patients' health [17]. Furthermore, it is common for many datasets to be simultaneously recorded from a patient. One typical example is taking a patient with schizophrenia's electroencephalogram (EEG) and functional magnetic resonance imaging (fMRI) [17]. In this context, tensors appear to be one of the best techniques for the simultaneous exploitation of. Fig.3 shows a selection of biomedical image examples [17].



**Figure 3:** Biomedical Imaging, for Instance. An Illustration of An Optical Coherence Tomography Image and A Fundus Fluorescein Angiography Are Shown from Left to Right.

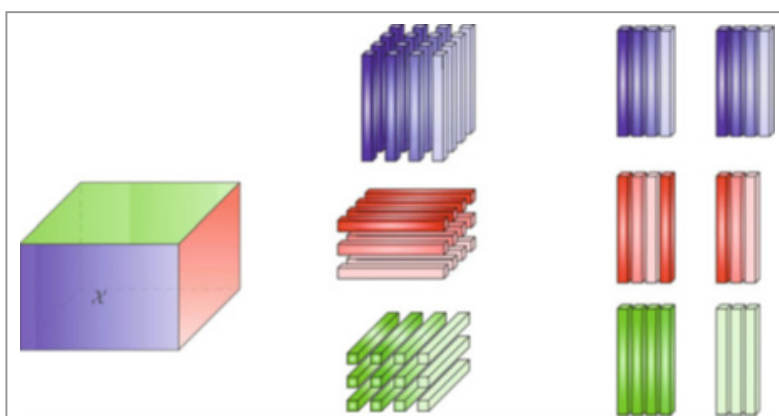
A more thorough comparison of low-rank-based and decomposition-based approaches is shown in Fig.4 [18]. These graphic compares two approaches for recovering missing slices of an OCT image: a tensor decomposition method and a low-rank-based method. This picture illustrates how the approach of known as MDT was able to recover the lost slices of a three-dimensional (3D) OCT image of the dataset, but HaLRTC (nuclear norm minimisation method for low rankness) was unable to do so [17, 18]. The generated images calculated peak signal to noise ratio (PSNR) and structural similarity index (SSIM) further support the tensor decomposition method's superior performance in recovering lost slices.

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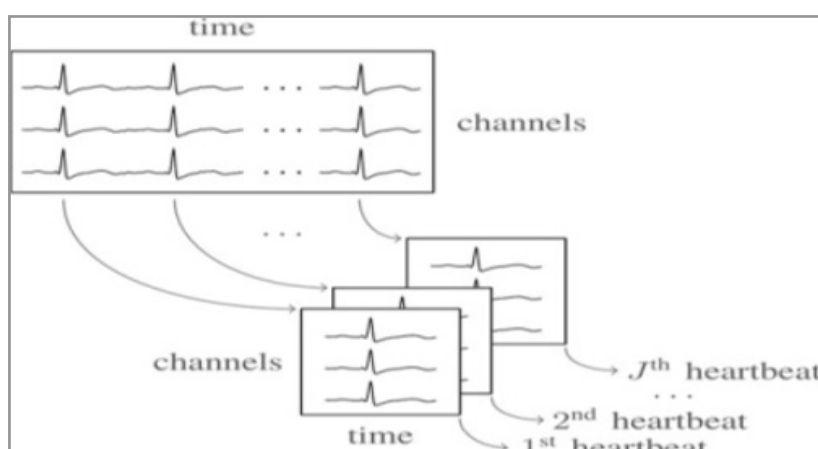
**Figure 4:** Comparison of the HaLRTC and MDT algorithms for reconstructing a sample B-scan of a dataset [18] with missing slices. The findings demonstrate that whereas MDT (decomposition-based approach) was able to recover the image, HaLRTC (low rank-based approach employing nuclear norm minimisation) was unable to recover the missing slices. Below each image, the PSNR and SSIM have been reported.

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**Figure 5:** A third-order ( $3 \times 4 \times 4$ ) tensor's mode-1, mode-2, and mode-3 matricizations, flattenings, or unfoldings.

The tensorization process is shown in Fig.6 where a higher-order tensor is created from the two-dimensional multilead ECG data containing J beats [20].



**Figure 6:** To provide tensor tools that enable the extraction of additional information, tensorization approaches generate a tensor from a data matrix in a meaningful manner. Here, we segment a multilead ECG data matrix using a technique known as segmentation, yielding a third-order tensor of modes channels  $\times$  time  $\times$  beats.

Time and space are the two dimensions that describe multilead ECG signals. ECG data is a good fit for tensor-based analysis since it is comparatively noise-free and has inherent structure, unlike other biological signals like EEG.

Tensor approaches have been applied in a small number of ECG applications in recent years.

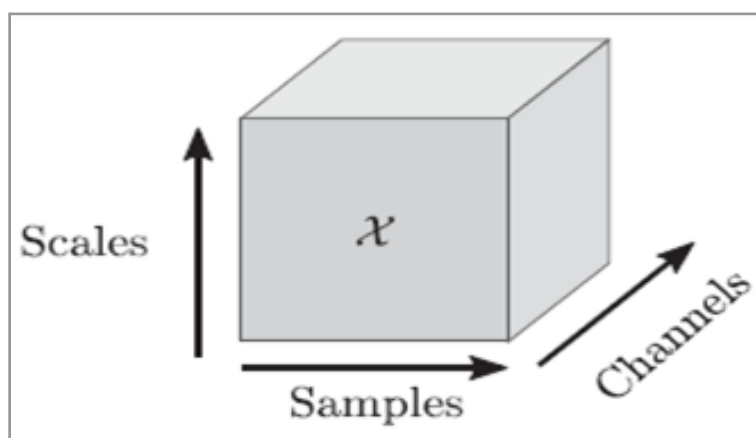
This section provides a summary of five common ECG processing issues where tensors have been effectively applied to produce pertinent outcomes:

- Compression of ECG data
- Myocardial infarction detection and localisation
- Classification of irregular heartbeats
- Identification and measurement of T-wave alternans
- Examination of alterations in heartbeat morphology.

Wireless electrocardiograms (WECG) allow for continuous heart monitoring of patients in their homes, but movements

can create unwanted noise, called motion artifacts, that interfere with the heart signals. The authors proposed using a method called tensor decomposition to combine data from WECG and motion sensors to filter out these artifacts. By analyzing the data from healthy subjects performing various movements, they can improve the quality of the heart signal measurements and assess their method's effectiveness using statistical measures [21-28].

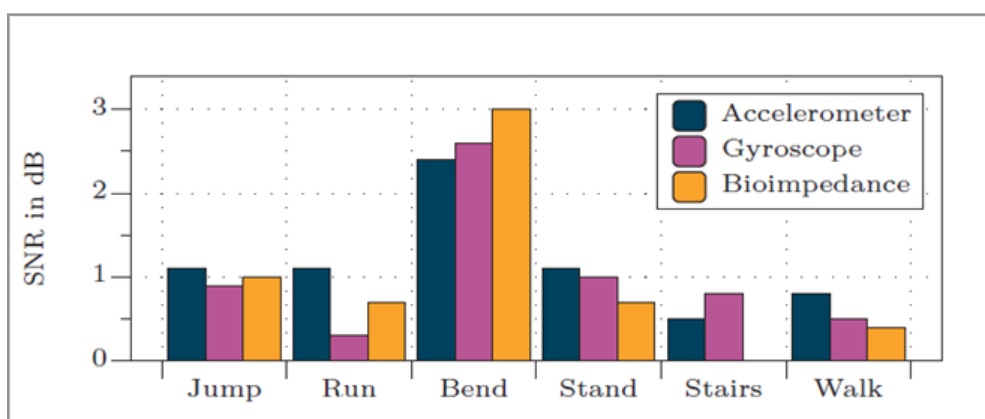
The Stationary Wavelet Transform (SWT), also known as the Maximal Overlap Discrete Wavelet Transform (MODWT), is used to analyze ECG (electrocardiogram) and motion reference data by breaking them down into different frequency components over time. Unlike the regular Discrete Wavelet Transform (DWT), the SWT keeps the same number of data points at each level of analysis, allowing for consistent comparisons [29]. By using the Haar wavelet, the data is transformed into a two-dimensional matrix that represents how different frequency bands change over time, helping to identify important features in the signals, see Fig. 7



**Figure 7:** Using the WECG and a reference sensor, the three-dimensional tensor is constructed.

The CPD-model effectively reduces motion artifacts in ECG signals, improving the Signal-to-Noise Ratio (SNR) from -5 dB to at least 0.5 dB across various movements [29]. It performs best during the "Bending forward" movement and less effectively when "Climbing stairs." Additionally, the model successfully

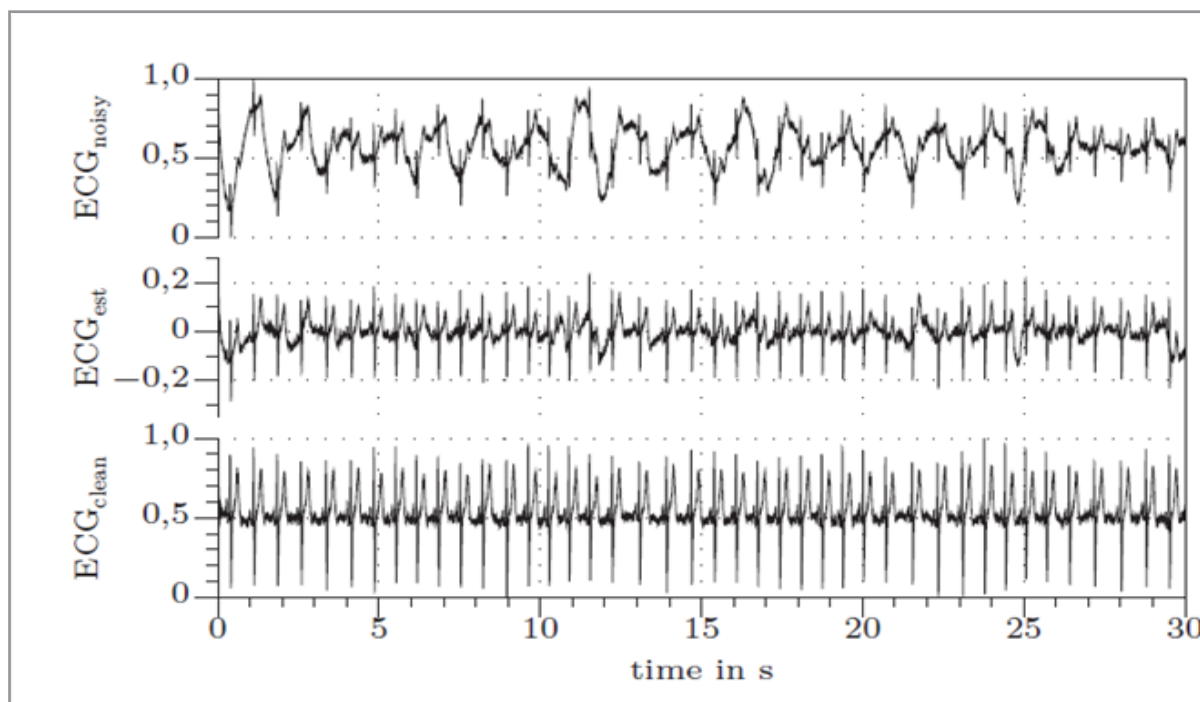
decreases the correlation between the ECG and the accelerometer data, indicating its usefulness in cleaning up ECG signals, and the researchers also tested the model using other sensors like gyroscopes and bioimpedance, as shown by Fig.8 [29].



**Figure 8:** The CPD-model's performance for several reference sensors.

Fig. 9 shows the results of a method used to clean up an electrocardiogram (ECG) signal while a subject bends forward for 30 seconds. The original noisy ECG signal (ECG<sub>noisy</sub>) is heavily affected by motion artifacts, making it hard to see important fea-

tures like the QRS complexes, which represent heartbeats. After applying a tensor-based approach, most of the noise is removed, allowing the QRS complexes and even the smaller P-waves to be clearly seen [29].



**Figure 9:** Implementation of the CPD motion artifact reduction for a single subject bending

## Conclusion

### Open Problems and Futuristic Research Avenues

To substantiate the impact of TDs on the advancement and transformation of biomedical sciences, an explanation is provided. This triggers the following open problems:

- The goal of was to examine and classify several tensor-based biomedical image processing techniques. The algorithms have been grouped according to the tensor decomposition techniques they have employed and their applications [17]. It has been demonstrated that traditional TD techniques have been extensively employed for many purposes, but a small number of studies on biological image analysis have examined TT and TR decompositions. Additionally, tensor approaches are less frequently applied to other biomedical pictures and are mostly focused on MRI images. All these point to the necessity for greater focus on applying tensor-based techniques—particularly tensor networks—for various biomedical image processing applications [17].
- Based on, it is a still an open problem to employing advanced tensor methods like Block Term Decomposition (BTD) and TUD techniques to analyse complex data structures, such as the multi-dimensional data from ECG readings [29]. These methods can enhance the process of removing motion artifacts because they allow for more flexibility in how the data is broken down, which can lead to better results. By using these approaches, researchers hope to improve the accuracy of the ECG readings while still effectively eliminating unwanted noise from movement. The next phase of research includes finding possible solutions to the provided open

problems, as well as the exploration of more TDs applications in other interdisciplinary fields of human knowledge.

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