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The Study (i) through 6 Major Cases, of the Conditions to Satisfy the Mathematical Property of the Compatibility between the Equivalence Binary Relation of the Equality of 2 "Njiki's Relative Fractions", Defined by $\frac{b}{a}=\frac{d}{c}\Leftrightarrow\exists k\in\mathbb{Z}/\frac{b}{a}=k\bullet\frac{d}{c}\vee\begin{cases}b=kd\\a=kc\end{cases}$, in $\mathbb{Q}_{nve}=\mathbb{Q}\cup\left\{\frac{b}{a}/b\in\mathbb{Z}\right\}$, and by $\frac{b}{a}=\frac{d}{c}\Leftrightarrow bc=ad$, in $\mathbb{Q}$, and the "Njiki's Fundamental Additive Operation or Operation of Addition", Marked $+_{1,1}$, in $\mathbb{Q}_{nve}$, and Defined by $\frac{b}{a}+_{1,1}\frac{d}{c}=\frac{b+d}{a+c}$, and in the Appendix (ii) of their two best applications (i) $k\bullet\frac{b}{a}=\frac{kb}{ka}$ in the Special Case of the "Njiki's $\mathbb{Q}_{nve}$ Sequences or Suites $\left(\frac{b}{a}\right)_{\lambda\in\mathbb{N}}$" and (ii) the Njiki's suite $S_n=\frac{n+1}{2}=n+1,1$, Leading to the term of the "Njiki's Property's Application – Definition"

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Abstract

The present paper aims at making the "Njiki's relative fractions", from the "Njiki's Z-module or Q-vector plane $(\mathbb{Q}_{nve},+_{1,1},\bullet)$",)", or, $(\mathbb{Q}_{nve},+_{\alpha,\beta}))$", with $+_{\alpha,\beta}$ being one of the generalized version of $+_{1,1}$ and where $(\alpha,\beta)\in\mathbb{Z}^2$, to be found in "The Njiki's Fundamental Theorem-Definition on Fractions in the Mathematical Set Q and by Extension in R and C, for the Purpose of Leading to the Construction of Some Algebraic Structures as Its Theoretical Applications and for the Practical Ones" [1], well known to the mathematical community and other mathematics' users all over the world. In fact, the mathematical property of the compatibility invoked in the above title does constitute a very important issue about the "Njiki's relative fractions" and because it is not always satisfied whatever the cases or in all cases. And that also specially because of the "Njiki's property's application-definition", $k\bullet\frac{b}{a}=\frac{kb}{ka}$, that follows from it and provable by induction over k, among other means or ways of proof. About their study, it is defined a "plural" sum mean or average with 16 forms, from 4 elements, of $\mathbb{Q}_{nve}$, similar in form to 4 others and 2 by 2, and it is concluded 120 cases of conditions able to be grouped into some ways, according to some criteria or characteristics, like that of belonging to the same "Njiki's sequences or suites $\left(\frac{b}{a}\right)_{\lambda\in\mathbb{N}}$", of them $(120=C_{16}^2=\frac{16!}{2!(16-2)!})$. Nevertheless, that defined "plural" sum mean or average over the 16 forms set could have also been with 4 = 2x2 or 9 = 3x3 forms derived from the previous 16 = 4x4 and respectively in $6=C_4^2$ and $4=C_4^3$ cases and leading mainly to the same conclusion. However, to make the paper as complete as possible some tutorials will be added to it in the Appendix. And specially when it comes to work with the "Njiki's double equality $\frac{b}{a}=\frac{kb}{ka}=k\bullet\frac{b}{a}$" $\forall k\in\mathbb{Q}\wedge\forall(a,b)\in\mathbb{Z}^2$ which besides bears the above application, $k\bullet\frac{b}{a}=\frac{kb}{ka}$, known as the "Njiki's property's application – definition n°1"; and about the definition therein, that of the $\mathbb{Q}_{nve}$ external multiplication $\bullet$ associated to the "Njiki's fundamental additive operation or operation of addition" $+_{1,1}$, and extendable, by Euclidian division, to $k\bullet\frac{b}{a}=\frac{kb}{ka}=\frac{b}{a}=q+\frac{r}{a}$, with $0\le r<a$ to be known as the "Njiki's Neo-Euclidian main relative fractions' property". The fact that in this present paper nearly every concept, property and definition is prefixed with the author's name, Njiki, is due to the mathematical innovation brought by him in mathematics and that since his previous paper [1]. And just as it is of the half plane of H. Poincaré , the Lie's groups, the Galois' extensions, the Roll's formula, the Cauchy's suites, the Peano's suite, the Riemann's integral, the L.R. Ford's circles, the Chasle's relation, the theorem of Thalès studied by RudolfBkouche, the Pythagore's theorem generalized by Al Kashir, the Boole's algebra which got universalized into discrete mathematics before or after being followed by the double Boole algebra which is not another one, algebra, but just one of its main or major mathematical property, the dunce's or the fool's addition clarified by the Farey's numbers, due to the Prince Louis Augustin Cauchy, and they themselves being generalized by the "Njiki's fundamental additive operation $+_{1,1}$", etc. And those 3 last ones showing that nothing can remain unchanged. Evolution being something that always must be awaited for and as far as sciences are concerned, in general [2-15].

Keywords: The Mathematical Property of Compatibility, Binary Relation, Additive Operation or Operation of Addition, Number-Vector or Vector-Number, Number-set or set-Number, Vector-Set or Set-Vector, the "Njiki's Double Dual" or the "Njiki's Triple Nature" of the "Njiki's Relative Fractions", Non-Fractions or Horizontal or Euclidian Relative Fractions, Proper or Vertical or Neo-Euclidian or "Njiki's Relative Fractions", the "Njiki's Number-Vector-set Approach" or the "Njiki's Triple Approach" of the "Njiki's Relative Fractions", the First neo-Euclidian Geometry arithmetic, the Njiki's Vector Plane.

0. General Introduction

The Preliminaries or Prolegomena

01. The recall of the definition of the compatibility of an equivalence binary relation R over a set E with an internal composition law * over E.

Given a set E, an internal composition law * over E (that is to say an application of E × E into E noted (a,b)↦a*b), and an equivalence binary relation R over E; that equivalence binary relation R over that set E is said to be compatible with the internal composition law * over the very set E if for all elements a, a', b, b' of E the following logical propositional condition is satisfied: If $a\mathcal{R}a'$ and $b\mathcal{R}b'$, then $(a*b)\mathcal{R}(a'*b')$ (0)

An example of study of such a mathematical property of compatibility and as it is below: with E=$\mathbb{Q}_{nve}$, $\mathcal{R}=\text{"}\frac{b}{a}=\frac{d}{c}\text{"}$ and $*=+_{1,1}$, is there any compatibility between R and *? And that's what that example below is all about.

The mathematical property of compatibility as above so defined is a fundamental property in algebra leading to the theory of quotient groups or quotient group theory, for example. More of facts, in this paper, it enables, by one of its two hereby mentioned applications, to give a proof to the definition of the $\mathbb{Q}_{nve}$ external multiplication ●, by induction. And then hence the origin of the term of the "Njiki's property's application-definition".

0_2. About the equality sign "=" in the mathematical expression "$\frac{b}{a}=\frac{d}{c}$" in the "Njiki's Z-module or Q-vector plane $(\mathbb{Q}_{nve},+_{1,1},\bullet)$" or $(\mathbb{Q}_{nve},+_{\alpha,\beta})$", with $+_{\alpha,\beta}$ being one of the generalized versions of +_1,1 and where $(\alpha,\beta)\in\mathbb{Z}^2$

The equality sign "=", in the mathematical expression "$\frac{b}{a}=\frac{d}{c}$" in the "Njiki's Z-module or Q-vector plane $(\mathbb{Q}_{nve},+_{1,1},\bullet)$" or $(\mathbb{Q}_{nve},+_{\alpha,\beta})$", with $+_{\alpha,\beta}$ being one of the generalized versions of $+_{1,1}$ and where $(\alpha,\beta)\in\mathbb{Z}^2$, can be taken either as one of the declined versions, $=,=_{1,1},=_{\alpha,1},=_{1,\beta},=_{\alpha,\beta}$ with $=\,=\,=_{1,1}\,=\,=_{\alpha,1}==_{1,\beta}==_{\alpha,\beta}$, of the "Njiki's fundamental additive operation or operation of addition" marked $+_{1,1}$ and as shown in the logo below, or, either as an equivalence binary relation defined by $\frac{b}{a}=\frac{d}{c}\Leftrightarrow\exists k\in\mathbb{Z}/\frac{b}{a}=k\bullet\frac{d}{c}\vee\begin{cases}b=kd\\a=kc\end{cases}$, in $\mathbb{Q}_{nve}$. In this article, it is going to be considered as in the last or later case and exceptionally as in the first one when it comes to prove the "Njiki's property's application-definition n°1" $k\bullet\frac{b}{a}=\frac{kb}{ka}$, by induction, and both at its initialization stage and at its generalization one, and as it is shown in the Part 4 below.

0_3. The logo showing the "Njiki's fundamental addition" $+_{1,1}$, at the circle center (0), on one hand, and its generalizations $+_{\alpha,1}$, $+_{\alpha,\beta}$, $+_{1,\beta}$ (1), declinations $=,=_{1,1},=_{\alpha,1},=_{1,\beta},=_{\alpha,\beta}$ with $=_{\alpha,\beta}\,=\,=_{1,1},\,=\,=_{\alpha,1}\,=\,=_{1,\beta}==(2)$ and associated external multiplication ●(3), on the circle of the very corresponding center (0) of $+_{1,1}$, on the other hand; and circle namely the "Njiki's circle" of (1), (2), (3) from (0)

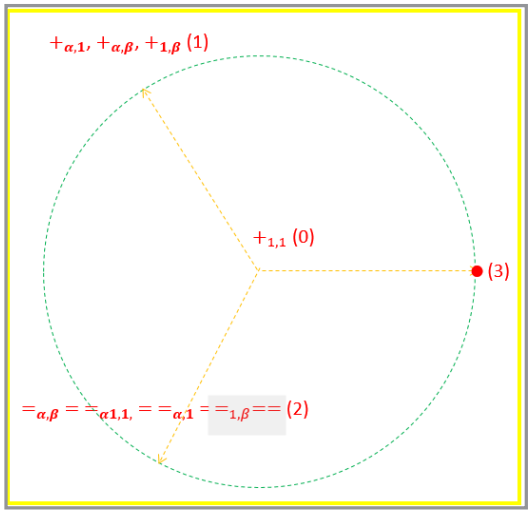

Figure 1

0_4. Some subsidiary studies to go through

04.1. The study of the generalization of the use of the "Njiki's fundamental additive operation or operation of addition" marked $+_{1,1}$ and including both its generalized versions $+_{\alpha,\beta}$, $+_{\alpha,1}$ or $+_{1,\beta}$, and, declined ones $=,=_{1,1},\,=_{\alpha,1},=_{1,\beta},=_{\alpha,\beta}$ with $=_{\alpha,\beta}\,=\,=_{1,1},\,=\,=_{\alpha,1}\,=\,=_{1,\beta}==,$, from the "Njiki's Z-module or Q-vector plane $(\mathbb{Q}_{nve},+_{1,1},\bullet)$" including the "Njiki's sequences or suites $\left(\frac{b}{a}\right)_{\lambda\in\mathbb{N}}$ " " to the overall $\mathbb{R}^n$, vector spaces of dimension $n\in\mathbb{N}$, mathematical branch or domain; taking that so over any other symbol so far used at or in its place, and the linear combination $\alpha\bullet u+_{1,1}\beta\bullet v$ therein being equal to $u+_{\alpha,\beta}v(\alpha\bullet u+_{1,1}\beta\bullet v=u+_{\alpha,\beta}v)$ And in terms of the vector coordinates, given $u=\begin{pmatrix}u_1\\u_2\\\cdot\\\cdot\\u_n\end{pmatrix}$ and $v=\begin{pmatrix}v_1\\v_2\\\cdot\\\cdot\\v_n\end{pmatrix}$ in the $\mathbb{R}^n$canonical basis:

$$\alpha \bullet u +_{1,1} \beta \bullet v = \alpha \bullet \begin{pmatrix} u_1 \\ u_2 \\ \cdot \\ \cdot \\ u_n \end{pmatrix} +_{1,1} \beta \bullet \begin{pmatrix} v_1 \\ v_2 \\ \cdot \\ \cdot \\ v_n \end{pmatrix} = \begin{pmatrix} \alpha u_1 + \beta v_1 \\ \alpha u_2 + \beta v_2 \\ \cdot \\ \cdot \\ \alpha u_n + \beta v_n \end{pmatrix} = \begin{pmatrix} u_1 v_1 \\ u_2 v_2 \\ \cdot \\ \cdot \\ u_n v_n \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

And that so obtained n×2 matrix $\begin{pmatrix} u_1 v_1 \\ u_2 v_2 \\ \cdot \\ \cdot \\ u_n v_n \end{pmatrix}$ being able to help define a linear application g from $\mathbb{R}^2$ into $\mathbb{R}^n$as follows:

$g:R^2 \rightarrow R^n$

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} \mapsto \begin{pmatrix} u_1 v_1 \\ u_2 v_2 \\ \cdot \\ \cdot \\ u_n v_n \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = (u \ v) \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \alpha \bullet u +_{1,1} \beta \bullet v$$

. And that last one being a linear combination of u and v, reciprocally.

0_4.2. Imagine and study a case of an antidisplacement f, from $\{1\}\times R^2$ into R^2, according to the above 0_4.1

For the tip about it, given a 2×3 matrix $\begin{pmatrix} d & b-a \\ c & a & b \end{pmatrix}$ such that $a^2+b^2=1$, take f as it follows:

$f: \{1\}\times R^2 \rightarrow R^2$

$$\begin{pmatrix} 1 \\ \alpha \\ \beta \end{pmatrix} \mapsto \begin{pmatrix} d & b-a \\ c & a & b \end{pmatrix} \begin{pmatrix} 1 \\ \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} d+\alpha b-\beta a \\ c+\alpha a+\beta b \end{pmatrix}$$

0_4.3. The meaning or signification of α and β in the symbol $+_{\alpha,\beta}$ as a very useful information for mathematics' users α and β, in the sign $+_{\alpha,\beta}$ of the generalised "Njiki's fundamental additive operation or operation of addition" $+_{1,1}$, describe some coefficients of linearity, on the theoretical level, and on the practical one, they, α and β, can be, oh no, they are to be taken or considered as numerical or literal units of measure in which the other two numbers attached to them are expressed. And their direct ratio, $\frac{\alpha}{\beta}=k$, or indirect or reverse one, $\frac{\beta}{\alpha}=\frac{1}{k}$, giving the magnitude of the measure' units between both.

0_4.4. About fractions per se and according to Jean Claude Njiki

According to Jean Claude Njiki, fractions in the field (Q,+,×) are referred to as horizontal and that is to say non-fractions or Euclidian fractions and all things corresponding to what is known about the classical or traditional fractions. Now, when being in the "Njiki's Z-module or Q-vector plane $(\mathbb{Q}_{nve}, +_{1,1}, \bullet)$", or, $(\mathbb{Q}_{nve}, +_{\alpha,\beta})$", they are referred to as vertical, synonym of proper fractions or fractions in proper, Neo-Euclidian fractions or of Njiki.

0_4.5. About the "Njiki's relative fractions" and their associated new "Njiki's fundamental additive operation or operation of addition", $+_{1,1}$ Fractions are usually referred to as "rational". And since my last paper [1], that adjective of rational has been permutated with that other of "relative" of the integers and for the purpose of the cause. And hence the justification of the "Njiki's relative fractions", and because it is due to me Njiki. And now, about their associated new and second universal established mathematical "Njiki's fundamental additive operation or operation of addition", it is that marked $+_{1,1}$ and from Njiki of course as well and in complement of the classical or traditional one, +; so far used in arithmetic and not so well used in geometry out of the vector spaces of dimension 1, but the Njiki's fundamental one $+_{1,1}$.

The primary goal in this article is to study the mathematical property of compatibility between the equivalence relation for these "Njiki's relative fractions" and their associated new mathematical "Njiki's fundamental additive operation or operation of addition" $+_{1,1}$.

0_4.6. About the assertion that a "Njiki's relative fraction" is simultaneously a number, a vector, and a set leading to a clear and defined terminology such as the (i) "Njiki's number-vector-set approach" or the "Njiki's triple approach", (ii) "the Njiki's vector-number approach", "the Njiki's number-set approach", etc., or the "Njiki double approaches" of the "Njiki's relative fractions"

That assertion is formally defined or justified, among, other ways of reasoning including that of the latterly following 16 forms' set, by the double equality $\frac{b}{a}=\frac{kb}{ka}=k\bullet\frac{b}{a}$ where (i)$\frac{b}{a}$ in $\frac{b}{a}$ stands for a total number, (ii) $\frac{b}{a}$in $\frac{kb}{ka}=\frac{k}{k}\times\frac{b}{a}$, that is an average or a mean number in $\mathbb{Q}_{nve}$, describes a Q equivalence class or class of equivalence or a $\mathbb{Q}_{nve}$ "Njiki's sequence or suite" which is a set, and (iii) $\frac{b}{a}$ in $k\bullet\frac{b}{a}$ holds for a vector in $\mathbb{Q}_{nve}$. And hence then the explanations of the above combined terms in (i) 3, three[3], or, (ii) 2, two[5]. 0_4.7. About the "neo-Euclidian fractions" or the "Njiki's sequences or suites" The "neo-Euclidian fractions" the "Njiki's…" and the "Njiki's sequences or suites", $\left(\frac{b}{a}\right)_k=\frac{kb}{ka}=k\bullet\frac{b}{a}$, are synonyms of each another, and that so in honour to Euclid by Njiki.

1. Specific Introduction

The present current ongoing paper, connected to linear algebra or vector geometry and with application to a Njiki's arithmetic of mean or average numbers, wants itself to be an educational mathematical paper and then so to be very simple and easy to digest.

That being so said, if the compatibility between the equivalence binary relation of the equality of two "Njiki's relative fractions", defined by $\frac{b}{a}=\frac{d}{c}\Leftrightarrow bc=ad$, in Q, and the classical or traditional additive operation or operation of addition, marked +, is easily established, it is not so that or so so for the compatibility of the equivalence binary relation of the equality of two "Njiki's relative fractions", defined by $\frac{b}{a}=\frac{d}{c}\Leftrightarrow\exists k\in\mathbb{Z}/\frac{b}{a}=k\bullet\frac{d}{c}\vee\begin{cases}b=kd\\a=kc\end{cases}$, in $\mathbb{Q}_{nve}=\mathbb{Q}\cup\left\{\frac{b}{0}/b\in\mathbb{Z}\right\}$, with the "Njiki's fundamental additive operation or operation of addition" of two "Njiki's relative fractions" $\frac{b}{a}+_{1,1}\frac{d}{c}=\frac{b+d}{a+c}$, in $\mathbb{Q}_{nve}$. In order to show that fact, it is going to be proceeded by the following four occasional properties over the "Njiki's relative fractions".

2. The "Njiki's Relative Fractions" Properties of the Occasion

2.1 Property 1: a *"Njiki"*s relative fraction" $\frac{b}{a}$, in $\mathbb{Q}_{nve}$, is both a number, belonging to the set $\mathbb{Q}\subset\mathbb{Q}_{nve}$, and, a vector, belonging to $\mathbb{Q}_{nve}$ properly, and that is to say a number-vector or a vector-number and in order to make them be one or interpenetrate into each other, and that both through (i) the calculation of a sum mean or average, and (ii) the definition of the external multiplication ● resulting from the *"Njiki"*s fundamental additive operation or operation of addition" $+_{1,1}$ in $\mathbb{Q}_{nve}$, and all things leading to the "Njiki's number-vector or vector-number approach" of the "Njiki's relative fractions" as a new branch of mathematics, created or founded by him Njiki and for the purpose of the cause. And the term of the *"Njiki"*s number-vector or vector-number approach" of the *"Njiki"*s relative fractions" standing or holding

from the fact that the algebraic structure $(\mathbb{Q}_{nve}, +_{1,1}, \bullet)$, said to be that of Njiki is the seat both of the calculation of the numbers' sums means or averages and of the *"Njiki's* Z-module or Q-vector plane $(\mathbb{Q}_{nve}, +_{1,1}, \bullet)$".

2.2 Property 2: any *"Njiki's* relative fraction" $\frac{b}{a}$ belongs to an equivalence class in Q denoted $\frac{b}{a}$, or to a sequence or suite of Njiki in $\mathbb{Q}_{nve}$ denoted $\left(\frac{b}{a}\right)_{\lambda\in\mathbb{N}}$. And that is to say that

$$\frac{b+d}{a+c}-\frac{1}{2}\left(\frac{b}{a}+\frac{d}{c}\right)=\frac{(a-c)(bc-ad)}{2ac(a+c)} \quad (1)$$

or

$$\frac{\alpha b+\beta d}{\alpha a+\beta c}-\frac{1}{2}\left(\frac{b}{a}+\frac{d}{c}\right)=\frac{(\alpha a-\beta c)(bc-ad)}{2ac(\alpha a+\beta c)} \quad (2)$$

2.3 Property 3: any classical or traditional sum total, in Q, of two "Njiki's relative fractions" $\frac{b}{a}$ and $\frac{d}{c}$, $\frac{b}{a}+\frac{d}{c}$, is unique. And because of the proof below:

$$\forall\left(\frac{b}{a},\frac{d}{c}\right)\in\mathbb{Q}^2\wedge\forall(\alpha,\beta)\in\mathbb{Z}^{*2}/\frac{b}{a}=\frac{\alpha b}{\alpha a}\wedge\frac{d}{c}=\frac{\beta d}{\beta c},\ \frac{b}{a}+\frac{d}{c}=\frac{\alpha b}{\alpha a}+\frac{\beta d}{\beta c}=\frac{bc+ad}{ac}. \quad (3)$$

2.4 Property 4: any sum mean or average, of Njiki, in $\mathbb{Q}_{nve}$, of two "Njiki's relative fractions" $\frac{b}{a}$, and from the numbers' theory view point $\frac{b}{a}=\frac{\alpha b}{\alpha a}=\frac{\frac{b}{1}}{\frac{a}{1}}=\frac{\alpha\frac{b}{\alpha}}{\alpha}$, and, $\frac{d}{c}$, and from the numbers' theory point of view $\frac{d}{c}=\frac{\beta d}{\beta c}=\frac{\frac{d}{1}}{\frac{c}{1}}=\frac{\beta\frac{d}{\beta}}{\beta}$, with $(\alpha,\beta)\in\mathbb{Z}^2$, and whose standard or generic default form is $\frac{b}{a}+_{1,1}\frac{d}{c}$, is plural or declinable and by "Njiki's conjecture" considered as a number-set or a set-number, or, a vector-set or a set-vector, and more precisely as a representative of set and like it is in the case of Q equivalence classes or classes of equivalence, or, in that of $\mathbb{Q}_{nve}$"Njiki's sequences or suites" to be found in [1], noted $\frac{b}{a}+_{1,1}\frac{d}{c}$ to just as much or by any other of the remaining 15 from the Cartesian product of the Part 6. below, and abbreviated n-s or s-n, or, v-s or s-v on the theoretical level, and n-s or s-n, or, v-s or s-v representing on the practical level a real toolbox or a real dissection kit of which 2 modular forms, $\frac{ab+d}{\alpha a+c}$ and $\frac{b+\beta d}{a+\beta c}$, a natural or granulometric mean or average, $\frac{b+d}{a+c}$ an average or mean simple arithmetic, $\frac{1}{2}\left(\frac{b}{a}+\frac{d}{c}\right)$, 2 weighted arithmetic mean, $\frac{ab+\beta d}{\alpha a+\beta c}$ and $\frac{1}{(\alpha+\beta)}\left(\alpha\frac{b}{a}+\beta\frac{d}{c}\right)$, and any 10 others $\frac{bc+d}{(a+1)c}$, $\frac{ab+d}{(\alpha a+1)c}$, $\frac{b+ad}{a(1+c)}$, $\frac{b+\beta ad}{a(1+\beta c)}$, $\frac{ab+ad}{a(\alpha+c)}$, $\frac{ab+\beta ad}{a(\alpha+\beta c)}$, $\frac{abc+ad}{ac(\alpha+1)}$, $\frac{bc+\beta d}{(\alpha+\beta)c}$, $\frac{abc+\beta d}{(\alpha a+\beta)c}$, $\frac{bc+\beta ad}{ac(1+\beta)}$ and whose identities are to be studied, for all useful purposes and within the framework of Tutorials. And all 16 mean or average forms in total obtained in Part 6, by a square Cartesian product of 4x4 and thus defined in extension. And in comprehension as or by:

$$\left\{\frac{f}{e}\in\mathbb{Q}_{nve}/\frac{f}{e}\in\frac{b}{a}+_{1,1}\frac{d}{c}\right\}$$

And that is to say that

$$\forall\left(\frac{b}{a},\frac{d}{c}\right)\in\mathbb{Q}_{nve}^2,$$

$$\frac{b}{a}+_{1,1}\frac{d}{c}=\left\{\frac{ab+d}{\alpha a+c},\frac{b+\beta d}{a+\beta c},\frac{b+d}{a+c},\frac{1}{2}\left(\frac{b}{a}+\frac{d}{c}\right),\frac{ab+\beta d}{\alpha a+\beta c},\frac{bc+d}{(a+1)c},\frac{ab+d}{(\alpha a+1)c},\frac{b+ad}{a(1+c)},\frac{b+\beta ad}{a(1+\beta c)},\frac{ab+ad}{a(\alpha+c)},\frac{ab+\beta ad}{a(\alpha+\beta c)},\right.$$
$$\left.\frac{abc+ad}{ac(\alpha+1)},\frac{1}{(\alpha+\beta)}\left(\alpha\frac{b}{a}+\beta\frac{d}{c}\right),\frac{bc+\beta d}{(a+\beta)c},\frac{abc+\beta d}{(\alpha a+\beta)c},\frac{bc+\beta ad}{ac(1+\beta)}\right\}$$
$$=\left\{\frac{f}{e}\in\mathbb{Q}_{nve}/\frac{f}{e}\in\frac{b}{a}+_{1,1}\frac{d}{c}\right\} \quad (4)$$

The above properties 3 and 4 can be summarized as follows: the language of calculating the sum total of two "Njiki's relative fractions" is unique meaning that it does not depend upon the representatives of the Q classes of equivalence, and that of calculating their sum mean or average and of Njiki is plural and because depending upon the representatives of the Q equivalence classes or upon the $\mathbb{Q}_{nve}$ sequences or suites of Njiki.

3. About the 2 times or the double dual nature of the *"Njiki's* relative fractions" and as in the following 3 cases in particular a) equivalence classes in or of Q, b) sequences or suites of Njiki in $\mathbb{Q}_{nve}$, and c) n-s or s-n, v-s or s-v of Njiki in $\mathbb{Q}_{nve}$.

It is therein (1) (i) numerical, and, (ii) vectorial or linear, and, (2) (i) elementary and (ii) set-theoretic. And to make it short and simple, it is of a number-vector-set one and to mean it triple; and the number therein being of a mean or an average one or of a total one, and depending on what additive operation, of $+_{1,1}$ or +, is used between $\frac{b}{a}$ and $\frac{d}{c}$ respectively, the vector being of 2 coordinates and the set being of cardinal = 16 And now, let us shift to the major part of the study.

4. The study (i) of 6 major cases, out of a total of 120, and be it $\frac{6}{120}=\frac{1}{20}$ in terms of the probabilistic language, of 6 major cases of the conditions to satisfy the mathematical property of the compatibility between the equivalence binary relation of the equality of 2 "Njiki's relative fractions", defined by $\frac{b}{a}=\frac{d}{c}\Leftrightarrow\exists k\in\mathbb{Z}/\frac{b}{a}=k\bullet\frac{d}{c}\vee\begin{cases}b=kd\\a=kc\end{cases}$, in $\mathbb{Q}_{nve}=\mathbb{Q}\cup\left\{\frac{b}{0}/b\in\mathbb{Z}\right\}1$, , and by $\frac{b}{a}=\frac{d}{c}\Leftrightarrow bc=ad$, in Q , and the "Njiki's fundamental additive operation or operation of addition", marked $+_{1,1}$, in $\mathbb{Q}_{nve}$, and defined by $\frac{b}{a}+_{1,1}\frac{d}{c}=\frac{b+d}{a+c}$, through or using the North=West/South-East diagonal of the "Njiki's 4 x 4 square matrix or Cartesian product additive table or table of addition". It holds from the following results:

$$\frac{b+d}{a+c}=\frac{ab+\beta d}{\alpha a+\beta c}\Leftrightarrow\alpha=\beta\vee\frac{b}{a}=\frac{d}{c}$$
$$\frac{b+d}{a+c}=\frac{1}{2}\left(\frac{b}{a}+\frac{d}{c}\right)\Leftrightarrow a=c\vee\frac{b}{a}=\frac{d}{c}$$
$$\frac{b+d}{a+c}=\frac{1}{(\alpha+\beta)}\left(\alpha\frac{b}{a}+\beta\frac{d}{c}\right)\Leftrightarrow ac=\beta a\vee\frac{b}{a}=\frac{d}{c}$$
$$\frac{ab+\beta d}{\alpha a+\beta c}=\frac{1}{2}\left(\frac{b}{a}+\frac{d}{c}\right)\Leftrightarrow\alpha a=\beta c\vee\frac{b}{a}=\frac{d}{c}$$
$$\frac{ab+\beta d}{\alpha a+\beta c}=\frac{1}{\alpha+\beta}\left(\alpha\frac{b}{a}+\beta\frac{d}{c}\right)\Leftrightarrow\alpha\beta=0\vee a=c\vee\frac{b}{a}=\frac{d}{c}$$
$$\frac{1}{2}\left(\frac{b}{a}+\frac{d}{c}\right)=\frac{1}{\alpha+\beta}\left(\alpha\frac{b}{a}+\beta\frac{d}{c}\right)\Leftrightarrow\alpha=\beta\vee ac=0\vee\frac{b}{a}=\frac{d}{c}$$

5. The proof of the "Njiki's property's application-definition"[2] n°1" $k\bullet\frac{b}{a}=\frac{kb}{ka}$, by induction

First step: initialization Let us set k=1, then $1\bullet\frac{b}{a}=\frac{1\times b}{1\times a}$, and by one vector spaces' property and one arithmetic's property $\frac{b}{a}=\frac{b}{a}$. And therefore the result.

Second step: generalization Let us set $k\bullet\frac{b}{a}=\frac{kb}{ka}$ and let us prove that $(k+1)\bullet\frac{b}{a}=\frac{(k+1)b}{(k+1)a}$ $(k+1)\bullet\frac{b}{a}=k\bullet\frac{b}{a}+_{1,1}1\bullet\frac{b}{a}$, according to the distributive property of the external multiplicative law $\bullet$ over the internal additive law $+_{1,1}$, in $\mathbb{Q}_{nve}$

$=\frac{kb}{ka}+_{1,1}\frac{b}{a}$, according to the hypothesis and according to the first step's result.

$=\frac{kb+b}{ka+a}$, according to the definition of the internal additive law $+_{1,1}$, in $\mathbb{Q}_{nve}$

$=\frac{(k+1)b}{(k+1)a}$, by the factorization by b and a

And therefore $(k+1)\bullet\frac{b}{a}=\frac{(k+1)b}{(k+1)a}$ which is the result.

6. The "Njiki's 4 x 4 square matrix or Cartesian product additive table or table of addition" representing the number-set or the set-number, the vector-set or the set-vector $\frac{b}{a}+_{1,1}\frac{d}{c}$ for the purpose of studying the mathematical property of compatibility of the equivalence binary relation of the equality of two "Njiki's relative fractions", $\frac{b}{a}=\frac{d}{c}$, in $\mathbb{Q}_{nve}$, and the "Njiki's fundamental additive operation or operation of addition" $+_{1,1}$ defined by $\frac{b}{a}+_{1,1}\frac{d}{c}=\frac{b+d}{a+c}$ in $\mathbb{Q}_{nve}$

$+_{1,1}$	$\frac{d}{c}$	$\frac{\beta d}{\beta c}$	$\frac{d}{\frac{c}{1}}$	$\frac{\beta\frac{d}{c}}{\beta}$	From number theory point of view
$\frac{b}{a}$	$\frac{b+d}{a+c}$	$\frac{b+\beta d}{a+\beta c}=\frac{b}{a}+_{1,\beta}\frac{d}{c}$	$\frac{bc+d}{(a+1)c}$	$\frac{bc+\beta d}{(a+\beta)c}$	From number and vector theories point of view
$\frac{\alpha b}{\alpha a}$	$\frac{\alpha b+d}{\alpha a+c}=\frac{b}{a}+_{\alpha,1}\frac{d}{c}$	$\frac{\alpha b+\beta d}{\alpha a+\beta c}=\frac{b}{a}+_{\alpha,\beta}\frac{d}{c}$	$\frac{\alpha b+d}{(\alpha a+1)c}$	$\frac{\alpha bc+\beta d}{(\alpha a+\beta)c}$	
$\frac{\frac{b}{a}}{1}$	$\frac{b+ad}{a(1+c)}$	$\frac{b+\beta ad}{a(1+\beta c)}$	$\frac{1}{2}\left(\frac{b}{a}+\frac{d}{c}\right)$	$\frac{bc+\beta ad}{ac(1+\beta)}$	
$\frac{\alpha\frac{b}{a}}{\alpha}$	$\frac{ab+ad}{a(\alpha+c)}$	$\frac{ab+\beta ad}{a(\alpha+\beta c)}$	$\frac{abc+ad}{ac(\alpha+1)}$	$\frac{1}{(\alpha+\beta)}\left(\alpha\frac{b}{a}+\beta\frac{d}{c}\right)$	
From number theory point of view	From number and vector theories point of view				↷

Table or Matrix 1

And when number and vector go together, walk hand in hand, get along well, etc., the "Njiki's number-vector or vector-number approach" or the "Njiki's double approach" of "his relative fractions" follows in $\mathbb{Q}_{nve}$.

7. The study of the remaining 120-6=114 equalities, inducing correspondent equivalences giving relationships between or values on $_{\alpha,\beta,a,c,}\frac{b}{a}$ and $\frac{d}{c}$ and like it is in the Part 4 and that from the above "Njiki's additive table or matrix" of the Part 6 and that for the purpose of establishing the conditions of the compatibility between the equivalence binary relation "$\frac{b}{a}=\frac{d}{c}$", in $\mathbb{Q}_{nve}$, and the "Njiki's fundamental additive operation" $+_{1,1}$, in $\mathbb{Q}_{nve}$

The content of this part of the present paper is left as tutorials and for it is of a so very simple and ease aspect. And in the sense that it just can be made on the above Part 4 pattern.

8. Discussion or Conclusion

From the results of the study in the Part 4, the mathematical property of the compatibility between the equivalence binary relation of the equality of 2 "Njiki's relative fractions", defined by $\frac{b}{a}=\frac{d}{c}\Leftrightarrow\exists k\in\mathbb{Z}/\frac{b}{a}=k\bullet\frac{d}{c}\vee\begin{cases}b=kd\\a=kc,\end{cases}$ in $\mathbb{Q}_{nve}=\mathbb{Q}\cup\left\{\frac{b}{a}/b\in\mathbb{Z}\right\}1$, and by $\frac{b}{a}=\frac{d}{c}\Leftrightarrow bc=ad$, in Q , and the "Njiki's fundamental additive operation or operation of addition", marked +_1,1, in $\mathbb{Q}_{nve}$, and defined by $\frac{b}{a}+_{1,1}\frac{d}{c}=\frac{b+d}{a+c}$, is satisfied (1) in the 3 following cases when the referential set $\mathbb{Q}_{nve}$, referred to an orthogonal vectors system, is restricted into its sub-sets (i) the Njiki's suites or sequences $\left(\frac{b}{a}\right)_{\lambda}=\left\{\frac{d}{c}\in\mathbb{Q}_{nve}/\frac{d}{c}=\lambda\bullet\frac{b}{a}=\frac{\lambda b}{\lambda a}\right\}$, (ii) the Njiki's stricto-vector line $\left\{\frac{b}{0}/b\in\mathbb{Z}\right\}$, and (iii) the Njiki's tangentially or co-tangentially respectively vertical or horizontal line $\left\{\frac{b}{a}/b\in\mathbb{Z}\right\}$, for any given arbitrary fixed $a\in\mathbb{Z}$, (2) in the 4 following other ones when $+_{1,1}$ is declined into (i) $+_{\alpha,\alpha}$ or $+_{\beta,\beta}$, (ii)$+_{0,\beta}$, (iii) $+_{\alpha,0}$, (iv) $+_{0,0}$, in $\mathbb{Q}_{nve}$ and with $(\alpha,\beta)\in\mathbb{Z}^2$, and (3) in the combinations of the above both, (1) and (2), to be explicated in the further research works. And meaning that the present article or paper is not a dead-end study. More of facts, if the sub-sets of $\mathbb{Q}_{nve}$ mentioned in (1), (ii) and (iii), are denoted $\mathbb{Q}_{(nve)0}$ and $\mathbb{Q}_{(nve)a}$, then $\mathbb{Q}_{(nve)0}=\left\{\frac{b}{0}/b\in\mathbb{Z}\right\}$ and $\mathbb{Q}_{(nve)a}=\left\{\frac{b}{a}/b\in\mathbb{Z}\right\}$, and where saying that $\mathbb{Q}_{(nve)a}$ generalizes $\mathbb{Q}_{(nve)0}$ or that $\mathbb{Q}_{(nve)0}$ specifies $\mathbb{Q}_{(nve)a}$ makes sense.

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10. Appendix

10.1. The study of the best application $k\bullet\frac{b}{a}=\frac{kb}{ka}$ of the above studied mathematical property of compatibility between the equivalence binary relation "$\frac{b}{a}=\frac{d}{a}$" and the "Njiki's fundamental additive operation or operation of addition" "$\frac{b}{a}+_{1,1}\frac{d}{c}=\frac{b+d}{a+c}$" in the main text and in the major sub-set of the "Njiki's fundamental sequences or suites $\left(\frac{b}{a}\right)_{\lambda\in\mathbb{N}}$" of $\mathbb{Q}_{nve}$ of the partial case when forms are considered individually, and each against others for comparisons and by eliminations.

That so said best application $k\bullet\frac{b}{a}=\frac{kb}{ka}$, which is also given by the definition of the $\mathbb{Q}_{nve}$ external multiplication, marked $\bullet$ and associated to the "Njiki's additive operation or operation of addition" $+_{1,1}$ in $\mathbb{Q}_{nve}$, can equally be shown, and it is done in the Part 5, by induction over k≥2 where the initialization step k=2 exactly clearly makes bring out the veritable application of the compatibility. And not the least about the generalization one. And then the justification of the term "Njiki's property's application-definition". Now, given a "Njiki's relative fraction" $\frac{b}{a}$, it does present according to the following extendable "Njiki's Neo-Euclidian relative fractions main property" (i) a vector or vertical branch held by the "Njiki's Z-module or Q-vector plane $(\mathbb{Q}_{nve},+_{1,1},\bullet)$", or, $(\mathbb{Q}_{nve},+_{\alpha,\beta})$" with $(\alpha,\beta)\in\mathbb{Z}^2$, and (ii) a number or horizontal one held by the classical or traditional field (Q,+,×) properties. And both being not to be cut at knife. Diagrammatically speaking, it can be represented as follows.

$$\underbrace{k\bullet\frac{b}{a}=\frac{kb}{ka}}_{(\mathbb{Q}_{nve},+_{1,1},\bullet)\ \text{or}\ (\mathbb{Q}_{nve},+_{\alpha,\beta})}=\overbrace{\frac{kb}{ka}=\frac{b}{a}}^{(\mathbb{Q},+,\times)}=\underbrace{ba^{-1}=q+\frac{r}{a}}_{(\mathbb{Q},+,\times)}$$

with $0\leq r<a$

The "Njiki's double equality" $k\bullet\frac{b}{a}=\frac{kb}{ka}=\frac{b}{a}$ declines into $C_3^2=3$ simple equalities and of which 2 geometric and 1 arithmetic directly attached to 1 of the 2 geometric, the remaining 1 geometric leading to another 1 arithmetic into a form of a disjunction of 1 arithmetic and 1 geometric equalities or expressions.

The 2 geometric simple equalities derived from the "Njiki's double equality" $k\bullet\frac{b}{a}=\frac{kb}{ka}=\frac{b}{a}$ are the respective vector homothetic $k\bullet\frac{b}{a}=\frac{kb}{ka}$ directly attached to its associated arithmetic $\frac{kb}{ka}=\frac{b}{a}$ and which is the 3rd simple equality invoked earlier, and, the similarity relation $k\bullet\frac{b}{a}=\frac{b}{a}$ associated to the following disjunction of 1 arithmetic and 1 geometric equalities or expressions $k=1\vee\frac{b}{a}=\frac{0}{0}$. And the which $k=1\vee\frac{b}{a}=\frac{0}{0}$ being indeed the particular values that obey the introductory "Njiki's double equality" $k\bullet\frac{b}{a}=\frac{kb}{ka}=\frac{b}{a}\ \forall k\in\mathbb{Q}\wedge(a,b)\in\mathbb{Z}^2$.

Still about the "Njiki's double equality" $\frac{b}{a}=\frac{kb}{ka}=k\bullet\frac{b}{a}$ in the "Njiki's Z-module or Q-vector plane $(\mathbb{Q}_{nve},+_{1,1},\bullet)$" once much more time, Euclidian on the left side where, attached to arithmetic, $\frac{b}{a}=\frac{kb}{ka}$, and, Neo-Euclidian or of Njiki on the right one and where, linked to geometry and through an homothetic linear application $\frac{kb}{ka}=k\bullet\frac{b}{a}$, or, a similarity relation $\frac{b}{a}=k\bullet\frac{b}{a}$, it must always henceforth, and that is to say from now onwards, be taken, considered or referred to as the "fundamental characteristic property" of the "Njiki's relative fractions" or of fractions in the "Njiki's Z-module or Q-vector plane $(\mathbb{Q}_{nve},+_{1,1},\bullet)$", or, $(\mathbb{Q}_{nve},+_{\alpha,\beta})$, with $+_{\alpha,\beta}$ being one of the generalized version of $+_{1,1}$ and where $(\alpha,\beta)\in\mathbb{Z}^2$. And the very "Njiki's Z-module or Q-vector plane" being within the "Njiki's number-vector-set approach" or the "Njiki's triple approach" of the "Njiki's relative fractions" leading to the creation of the "Njiki's International and Neo-Euclidian Mathematical Intelligent School", and in short the NINEMIS. And the which NINEMIS showing that geometry and arithmetic interpenetrate or enter into each another. Else, and for example, over the total 3 cases $\left(3=C_3^2=\frac{3!}{2!(3-1)!}=\frac{3!}{2!}\right)$: $\frac{b}{a}=k\bullet\frac{b}{a}\Leftrightarrow\frac{b}{a}-k\bullet\frac{b}{a}=0\Leftrightarrow(1-k)\bullet\frac{b}{a}=0\Leftrightarrow1=k\vee\frac{b}{a}=\frac{0}{0}$ And $k=1$ or $\frac{b}{a}=\frac{0}{0}$ which are only 2 particular cases of the general one $k\bullet\frac{b}{a}=\frac{kb}{ka}=\frac{b}{a}\ \forall k\in\mathbb{Q}\wedge(a,b)\in\mathbb{Z}^2$.

The NINEMIS is a school of education to numbers, that is to say to arithmetic, the domain of numbers' properties studies, and to vectors, meaning to vector geometry or linear algebra, the areas of vectors' studies. And all that by the way of fractions.

10.2. Eurêka, Eurêka

There are 2 major, standard and fundamental types of universal systems of operations in mathematics so far: the first being held by the classical or traditional addition marked +, and defined in N, Z, Q, R and C and along with its associated substraction, marked -, internal multiplication, marked ×, and division marked ÷, ___, etc., and the second by the "Njiki's fundamental additive operation or operation of addition" marked $+_{1,1}$ and newly defined in $\mathbb{Q}_{nve}$, $\mathbb{R}_{nve}$ and $\mathbb{C}_{nve}$ on one hand and in N,Z,Q, R and C, on the other hand, and along with its external multiplication $\bullet$ with scalars in Z and Q, and not to forget about (i) its generalizations, (including the modular forms $+_{z,1}$ or $+_{1,z}$ with z $\in\mathbb{C}$), $+_{\alpha,1}$, $+_{\alpha,\beta}$, $+_{1,\beta}$, making them be combined with the external multiplication $\bullet$ into a single one item each time, and (ii) its declinations $=,=_{1,1},=_{\alpha,1},=_{1,\beta},=_{\alpha,\beta}$ and with $=_{\alpha,\beta}==_{1,1},==_{\alpha,1}==_{1,\beta}==$ within the "Njiki's sequences or suites $\left(\frac{b}{a}\right)_{\lambda\in\mathbb{N}}$" in $\mathbb{Q}_{nve}$. And the above both additive operations, + and $+_{1,1}$, being to be considered as internal laws. As the classical or traditional + is suitable in numbers matters' treatment moving up to the fields' algebraic structures so does the new "Njiki's fundamental additive operation or operation of addition" $+_{1,1}$ be it in linear algebras studying vector spaces (i) of dimension 2 and by starting with the "Njiki's Z-module or Q-vector plane $(\mathbb{Q}_{jcn},+_{1,1},\bullet)$ or $(\mathbb{Q}_{nve},+_{1,1},\bullet)$" for

various types of calculations and among which those of averages or means' values in statistics and probability, (ii) of dimensions n $\in$N above 2 (n≥2), for much more mathematical problems' solving and (iii) of n=1 as it exceptionally follows below in the subpart 10.2.1. And other areas of mathematics including geometry, calculus, etc., being able to use either one, either another or both of them.

10.2.1. About the exceptional case of n=1 as announced earlier

A two horizontal numbers' addition, for counting, a+b, and those 2 horizontal numbers meaning non-fractions, a and b, and whatever they may be or wherever they may come from, N, Z, D, Q, R and C, is a vector translation, for measuring, into two, $t_a(b) = b + a$ or $t_b(a) = a + b$, and in the name of the commutativity property of the additive operation + in the above mathematical sets making b+a=a+b, and what also means the vector addition (a)+(b)=(a+b) of the two vectors (a) and (b) corresponding to the a and b of the b+a=a+b. And it is on that pattern or model that callipers are physically made of built up.

Reversely

10.2.2. In the vector spaces of dimension 1 and mainly in R, numbers and vectors, expressed in or as non-fractions or horizontal fractions of Q, are confused into each another and by using only the classical or traditional addition +.10.2.3. In the vector spaces of dimension 2 and specially in $\mathbb{R}_{nve} = \mathbb{R} \cup \left\{\frac{b}{0} / b \in \mathbb{R}\right\}$, numbers and vectors, expressed in or as proper fractions or vertical fractions, are confused and by using only the "Njiki's fundamental additive operation or operation of addition" $+_{1,1}$.

10.2.4. In the vector spaces of dimension n over 2 (n≥2) and mostly in R^n with n≥2, vectors take the "Njiki's fundamental additive operation or operation of addition" $+_{1,1}$ in extension. And to conclude this, all those above last 3 items being to be taken as number-vector or vector-number mathematical properties, according to a taxonomy due to Njiki, making fractions either horizontal, synonym to non-fractions or Euclidian fractions, known by the expression $\frac{hb}{ha} = \frac{b}{a} = q + \frac{r}{a}$ such that $E\left(\frac{hb}{ha}\right) = q$ and with $0 \le r < a) \wedge (a,k,q) \in \mathbb{Z}^{*3} \wedge (b,r) \in \mathbb{Z}^2$, either vertical, synonym of proper or neo-Euclidian, in honour to Euclid by Njiki, or of Njiki recognized by $\frac{hb}{ha} = k \bullet \frac{b}{a}$ with $(a,b,k) \in \mathbb{Z}^3$. And all these leading to the Njiki's fundamental logical proposition (into 2) and about his "Njiki's number-vector or vector-number approach" of the "Njiki's relative fractions". And the which fundamental logical proposition into 2 standing or holding as follows:

$$\forall \left(\frac{b}{a}, \frac{d}{c}\right) \in \mathbb{Q}^2_{nve}, \quad \overbrace{\frac{b}{a} +_{1,1} \frac{d}{c}}^{\substack{\text{vector member of} \\ \text{the equality expression}}} = \underbrace{\frac{b+d}{a+c}}_{\substack{\text{number member of} \\ \text{the equality expression}}}$$

$$\underbrace{\qquad\qquad}_{\substack{\text{number-vector or} \\ \text{vector-number} \\ \text{equality expression}}}$$

$$\forall \frac{b}{a} \in \mathbb{Q}_{nve} \wedge \forall \lambda \in \mathbb{Q}, \quad \overbrace{\lambda \bullet \frac{b}{a} = \frac{\lambda b}{\lambda a}}^{\substack{\text{vector theory equality} \\ \text{expression of the double} \\ \text{equality expression}}} = \underbrace{\frac{\lambda b}{\lambda a} = \frac{b}{a}}_{\substack{\text{number theory equality} \\ \text{expression of the double} \\ \text{equality expression}}}$$

$$\underbrace{\qquad\qquad}_{\substack{\text{number-vector or} \\ \text{vector-number double} \\ \text{equality expression leading to the} \\ \text{trilogic concept of number-vector-set}}}$$

And the common double equality expression member $\frac{\lambda b}{\lambda a}$ therein being to be considered as a mean or interfacing term representing a part into 2 of a set characteristic property. N.B.: in the above a) and b), we shift or move from left to right, from a vector writing to a number one, and though partially, and reversely, from right to left, from a number writing, and although partially, to a vector one. So, meaning that vector and number in $\mathbb{Q}_{nve}$ interpenetrate into each another and bear a set and just as that of the above Part 6 of cardinal 16 and which can be made different in nature and according to the different and numerous mathematical circumstances, and of cardinal greater than 16=4x4, like in the cases of the Q equivalence classes or classes of equivalence, or, of the $\mathbb{Q}_{nve}$ "Njiki's sequences or suites", or, of cardinal smaller than 16, like in the cases of the 3x3 or 2x2 square matrices derived from that of the Part 6. And then, the justification of the term of the "Njiki's number-vector-set approach" or of the "Njiki's triple approach" of the "Njiki's relative fractions".

That "Njiki's number-vector-set approach" or "Njiki's triple approach" of the "Njiki's relative fractions", in the "Njiki's Z-module or Q-vector plane $((\mathbb{Q}_{nve}, +_{1,1}, \bullet)$", being so then defined as the part of algebra studying, besides their relativity and rather than their rationality, according to the end of the Njiki's previous paper [1] (i) the vectoriality or linearity of the Q "Njiki's relative fractions" extended to those of the form $\frac{b}{0} = \binom{0}{b}$ tangentially $= \binom{b}{0}$ co-tangentially in $\mathbb{Q}_{jcn} = \mathbb{Z} \times \mathbb{Z}$, with $b \in \mathbb{Z}$, and (ii) their interpenetration to those of the field (Q,+,×). It will then hence be the name of a new course to be introduced and developed in algebra from all those above mathematical stuffs or materials due to Njiki and by starting by the statement and the proof of his fundamental Theorem-Definition in [1]. And its link to the arithmetic geometry being able to be studied. Now, getting back to the relativity of the Q fractions and rather than their rationality, that last one, rationality as an universal set's characteristic property, is transferred to the integers of Z, by permutation and as it is explained at the end of [1]. The "Njiki's number-vector-set approach" or the "Njiki's triple approach" of the "Njiki's relative fractions" in $((\mathbb{Q}_{nve}, +_{1,1}, \bullet)$ can then split into two successive parts namely (i) the very well-known and classical or traditional set of the relative fractions and rather than the rational in the field (Q,+,×) and (ii) the newly Njiki's one own set of vector fractions in his additive group $((\mathbb{Q}_{nve}, +_{1,1})$ leading to the Z-module or Q-vector plane $((\mathbb{Q}_{nve}, +_{1,1}, \bullet)$ of his or in short $((\mathbb{Q}_{nve}, +_{\alpha,\beta}), +_(\alpha,\beta))$, with $(\alpha,\beta) \in \mathbb{Z}^2$, and the sign $+_{\alpha,\beta}$ hereby bearing the external multiplication $\bullet$ in itself through the coefficients α and β.

From the above development, is it then mathematically logically exact or correct to write $((\mathbb{Q}_{nve}, +_{1,1}, \bullet) = (\mathbb{Q}_{nve}, +_{\alpha,\beta})$ with $(\alpha,\beta) \in Z^2$? And that is for the last Tutorial. The above being so done, the following exercises can make learn much more about the "Njiki's relative fractions"

10.3 May you please look at the way modular forms are algebraically managed or treated through and from symplectic matrixes and that is from $\binom{b\ d}{a\ c} . z$ to $\frac{bz+d}{az+c}$ and as compared to the 2 over 4 generalized $+_{1,1}$ of Njiki, $+_{z,1}$ or $+_{1,z}$, $\frac{b}{a} +_{z,1} \frac{d}{c} = \frac{bz+d}{az+c}$, $\frac{b}{a} +_{1,z} \frac{d}{c} = \frac{b+dz}{a+cz}$

10.4 What is it the most accurate to say between the 2 following expressions (i) calculate the sum total of $\frac{b}{a} + \frac{d}{c}$ or (ii) study the sum total of $\frac{b}{a} + \frac{d}{c}$? For the tip about it, make use of the notion of the "Njiki's number-vector-set approach" or of the "Njiki's triple approach" of the "Njiki's relative fractions", and then do

comments on it. In the same flight of thought, what can you say about $\frac{b}{a}+_{1,1}\frac{d}{c}$, either of calculating or of studying? More of facts, are $\frac{b}{a}+\frac{d}{c}$ or $\frac{b}{a}+_{1,1}\frac{d}{c}$ belonging respectively to the same Q equivalence class or class of equivalence as $\frac{b}{a}$ or $\frac{d}{c}$, or, to the same $\mathbb{Q}_{nve}$ "Njiki's sequence or suite" as $\frac{b}{a}$ or $\frac{d}{c}$? Why?

10.5 As there is a suite of Peano [7] $s_n=n+1$, in N, related to a sum total and then attached to the classical or traditional addition +, so does it exist a sequence or suite of Njiki $S_n = n+_{1,1}1=\frac{n}{1}+_{1,1}\frac{1}{1}=\frac{n+1}{1+1}=\frac{n+1}{2}$, , in $\mathbb{Q}_{nve}$, related to his sum mean or average and thus linked to his fundamental additive operation or operation of addition $+_{1,1}$. 1/ Represent S_n and S_n graphically and draw some observations about that. 2/Show that S_n is another case of the "Njiki's property's application-definition". And case where $a=c$ in $k=\frac{b}{a}\wedge h=\frac{d}{c}$ making so $k+_{1,1}h=\frac{b}{a}+_{1,1}\frac{d}{c}=\frac{b+d}{a+c}=\frac{b+d}{2a}=\frac{b+d}{2c}$ and specifically where $a=c=1$ in $n=\frac{n}{1}$ and $1=\frac{1}{1}$. And: $n=\frac{n}{1}\wedge 1=\frac{1}{1}\Rightarrow n+_{1,1}1=\frac{n+1}{2}$

10.6 Make your Comments about the Following Item

The modular forms and the 2 (over 4) generalized types, $+_{z,1}$ and $+_{1,z}$, of the "Njiki's fundamental addition" $+_{1,1}$, in $\mathbb{Q}_{nve}$ acting on the "Njiki's modular group" $\mathbb{Q}_{nve}$ and in venue and place of the very known one $SL_2(Z)$ of Sophus Lie followed by Roger Godement 16, 17], Rached Mneimné and not to talk about those two of (i) Galois absolute Q and mainly developed by Jean Pierre Escofier, Patrice Tauvel and Ivan Gozard and (ii) Fourrier published by Gaston Darboux and Eugene Fabry and Gaston [18-30]. Darboux, through his coefficients of modular forms; with z belonging to the half plane of Henri Poincare $H=\{z\in\mathbb{C}/\mathrm{Im}(z)>0\}$ $+_{z,1}$; with $z\in\mathbb{C}$, and $+_{1,z}$; with $z\in\mathbb{C}$, are defined as follows:

$$+_{z,1};z\in\mathbb{C};:\mathbb{Q}_{nve}\times\mathbb{Q}_{nve}\longrightarrow\mathbb{C}$$

$$\left(\frac{b}{a},\frac{d}{c}\right)\mapsto+_{z,1}\left(\frac{b}{a},\frac{d}{c}\right)=\frac{b}{a}+_{z,1}\frac{d}{c}=\frac{bz+d}{az+c} \quad (5)$$

Or

$$+_{1,z};z\in\mathbb{C}:\mathbb{Q}_{nve}\times\mathbb{Q}_{nve}\longrightarrow\mathbb{C}$$

$$\left(\frac{b}{a},\frac{d}{c}\right)\mapsto+_{1,z}\left(\frac{b}{a},\frac{d}{c}\right)=\frac{b}{a}+_{1,z}\frac{d}{c}=\frac{b+dz}{a+cz} \quad (6)$$

And they are said to be the Njiki's version of the modular forms Now, on the above patterns and as tutorials find many other mathematical situations where the "Njiki's fundamental additive operation or operation of addition" $+_{1,1}$ can get involved.

10.7 The "Njiki's relative or vector fractions" and the geometry in a 2D plane referred to an orthogonal basis A "Njiki's relative or vector fraction", $\frac{b}{a}\in\mathbb{Q}_{nve}$ with $(a,b)\in\mathbb{Z}^2$, brought into a 2D plane referred to an orthogonal basis, besides being a number expressing the slope of a vector straight line of equation $y=\frac{b}{a}x$ or of parametric equations system $\begin{cases}x=at\\y=bt\end{cases}$ with $t\in\mathbb{Q}$, according to the Euclidian geometry, does describe therein a rectangle triangle formed by itself as a vector and by its 2 coordinates a and b, tangentially, signifying $\frac{b}{a}=\binom{a}{b}$ or co-tangentially, meaning $\frac{b}{a}=\binom{a}{b}$ in accordance to the Njiki's first neo-Euclidian geometry.

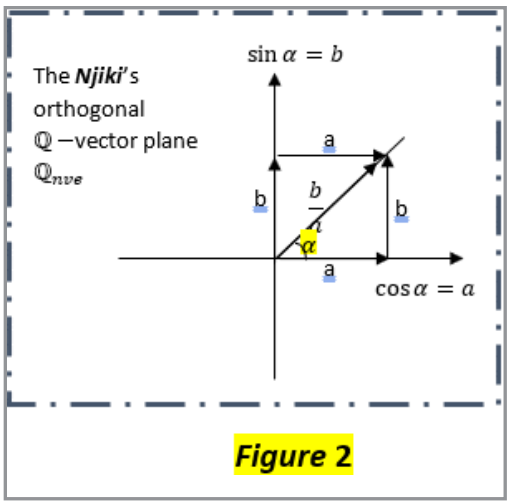

Figure 2

Figure 2

$$\frac{b}{a}=\binom{a}{b}_{tangentially}=\binom{b}{a}_{co-tangentially}$$

or

$$(-1)\bullet\frac{b}{a}=-\binom{a}{b}_{tangentially}=-\binom{b}{a}_{co-tangentially}$$

in the Njiki's first neo-Euclidian geometry. And in the Euclidian geometry: $\frac{b}{a}=\frac{\sin\alpha}{\cos\alpha}=\tan\alpha$

Now, let us consider the following two exercises.

Exercise 1: what about the norm or distance $\left\|\frac{b}{a}\right\|=\sqrt{a^2+b^2}$

Exercise 2 : may you please go through some sorts of comparison between the Njiki's orthogonal Q-vector plan $\mathbb{Q}_{nve}$ and the C's R-vector plane supporting the H. Poincare's half plane H=$\{z\in\mathbb{C}/z\geq 0\}$?

10.8 The calculation of the limits at zero (0) and at infinity (∞) of the functions resulting from the differences between the 5 values $\frac{b+d}{a+c},\frac{1}{2}\left(\frac{b}{a}+\frac{d}{c}\right),\frac{\lambda b+\mu d}{\lambda a+\mu c},\frac{\lambda b+d}{\lambda a+c},\frac{b+\mu d}{a+\mu c}$ of the expression $\frac{b}{a}+_{1,1}\frac{d}{c}$ d/c in the 3 following examples:

The analytical study of 3 differences of values issued from the expression of $\frac{b}{a}+_{1,1}\frac{d}{c}$ (1) $\frac{b+d}{a+c}-\frac{1}{2}\left(\frac{b}{a}+\frac{d}{c}\right)=y_1$, (2) $\frac{b+d}{a+c}-\frac{\lambda b+\mu d}{\lambda a+\mu c}=y_2$ and (3) $\frac{\lambda b+\mu d}{\lambda a+\mu c}-\frac{1}{2}\left(\frac{b}{a}+\frac{d}{c}\right)=y_3$ in fonction of the gap or margin $\frac{b}{a}-\frac{d}{c}$ between the elements of the two suites of Njiki $\left(\frac{b}{a}\right)_\lambda$ and $\left(\frac{d}{c}\right)_\mu$, for much more about the matter of the compatibility of the equivalence binary relation $\frac{b}{a}=\frac{d}{c}$, in $\mathbb{Q}_{nve}$, and the "Njiki's fundamental addition" $\frac{b}{a}+_{1,1}\frac{d}{c}$, in $\mathbb{Q}_{nve}$

Results of the calculation of the differences (1), (2) and (3)

(1) $y_1=\frac{a-c}{2(a+c)}\left(\frac{b}{a}-\frac{d}{c}\right)$; (2) $y_2=\frac{ac(\mu-\lambda)}{(\lambda a+\mu c)(a+c)}\left(\frac{b}{a}-\frac{d}{c}\right)$; (3) $y_3=\frac{\lambda a-\mu c}{2(\lambda a-\mu c)}\left(\frac{b}{a}-\frac{d}{c}\right)$

Conclusion: the differences of the expressions (1), (2) and (3) issued from the calculation of $\frac{b}{a}+_{1,1}\frac{d}{c}$ are all vector straight lines in $\mathbb{Q}_{nve}$ of the above respective equations with $\lim_{\frac{b}{a}\to\frac{d}{c}} y_1 = 0$, $\lim_{\frac{b}{a}\to\frac{d}{c}} y_2 = 0$, $\lim_{\frac{b}{a}\to\frac{d}{c}} y_3 = 0$, $\lim_{\frac{b}{a}\to\frac{d}{c}} y_1 = \infty$, $\lim_{\frac{b}{a}\to\frac{d}{c}} y_2 = \infty$, $\lim_{\frac{b}{a}\to\frac{d}{c}} y_3 = \infty$ in the many appropriate cases.

10.9 About the Njiki's mathematical item $\frac{b}{0}$

The Njiki's innovative set $\mathbb{Q}_{nve} = \mathbb{Q} \cup \left\{\frac{b}{o}/b \in \mathbb{Z}\right\}$ is well defined because in it $\frac{b}{0}$ is not to be taken in the sense of an Euclidian division but as a $\mathbb{Q}_{nve}$ vector generating a $\mathbb{Q}_{nve}$ vector straight line, and tangentially and co-tangentially vertical when $\mathbb{Q}_{nve}$ is referred to an orthogonal basis. It has no meaning in the Euclidian geometry but it has one in the first Neo-Euclidian geometry of Njiki through the $\mathbb{Q}_{nve}$ and within the Njiki's vector plane $(\mathbb{Q}_{nve}, +_{1,1}, \bullet)$ or $(\mathbb{Q}_{nve}, +_{\alpha,\beta})$ with $(\alpha,\beta) \in \mathbb{Q}^2$.

10.10 About the meaningfulness of the mathematical terms $\frac{b}{a}+\frac{d}{c}, \frac{1}{2}\left(\frac{b}{a}+\frac{d}{c}\right), \frac{b+d}{a+c}$ and $\frac{\alpha b+\beta d}{\alpha a+\beta c}$

The concern here is beyond the mathematical domain which is fundamentally formal. It is not philosophical either. It is intuitive.

The above being so said, the sum-total $\frac{b}{a}+\frac{d}{c}$ and the arithmetic mean or average $\frac{1}{2}\left(\frac{b}{a}+\frac{d}{c}\right)$ make sense or have some mathematical meaning only when their 2 parts $\frac{b}{a}$ and $\frac{d}{c}$ are extracted from the same physical item or assumed as such. The natural or granulometric mean or average $\frac{b+d}{a+c}$ and the (α,β)-natural or (α,β)-granulometric mean or average $\frac{\alpha b+\beta d}{\alpha a+\beta c}$ make sense or have some mathematical meaning only when the parts $\frac{b}{a}$ and $\frac{d}{c}$ are extracted from 2 different items of the same species. When mistaken, there are errors whose margin can be calculated as follows and between means or averages for example: $\frac{b+d}{a+c}-\frac{1}{2}\left(\frac{b}{a}+\frac{d}{c}\right)$, or $\frac{\alpha b+\beta d}{\alpha a+\beta c}-\frac{1}{2}\left(\frac{b}{a}+\frac{d}{c}\right)$, in one sense or in another, positive or negative.

"And when mathematics is grappling with itself, but not entirely, how can such a problem find a solution?" And that is a very big research question.

10.11 Much more about why a "Njiki's relative fraction" can be said to be both a number, in arithmetic, and a vector, in vector geometry or linear algebra

When whosoever or I makes or make, $\frac{b}{a}+_{1,1}\frac{d}{c}=\frac{b+d}{a+c}$, whosoever or I does or do both arithmetic and geometry or algebra into one same mathematical operation. I do arithmetic by the calculation of a natural or granulometric mean or average value, and, geometry or algebra by that of the coordinates $\binom{a+c}{b+d}$, tangentially, or $\binom{b+d}{a+c}$, co-tangentially, of the sum vector $\frac{b+d}{a+c}$ obtained graphically by Chasles[10]'s relation, in the Njiki's orthogonal $\mathbb{Q}$-vector plane $\mathbb{Q}_{nve}$.

Now, dear readers, if I name such an above natural approach of things "geometry-arithmetic" or "arithmetic-geometry" with a dash in between arithmetic and geometry, and for they equally are both substantives or names, what can you say about that and as compared to the current "arithmetic geometry" and not and why "geometric arithmetic", for the debate or exercise?

Besides, the traditional or classical additive operation marked + is mainly arithmetic and that is to say numerical and exceptionally geometric in the R-vector straight lines like R, whereas the Njiki's fundamental operation of addition denoted $+_{1,1}$ is mostly geometric and that is to say related to vectors and exceptionally arithmetic in the R-vector planes or Q-vector planes like $\mathbb{R}_{nve}$ or $\mathbb{Q}_{nve}$.

10.12 A Njiki's diagonally crossed table showing or bringing out the nuances between the classical or traditional addition, +, and the new modern Njiki's fundamental addition, $+_{1,1}$

The 2 majors mathematical parts / The 2 majors operations related to them	**Arithmetic**	**Geometry**
+	Much more and both with whole numbers and with fractions (for sum-totals' calculations) The first language or operation	Much less and except in vector spaces of dimension 1 like R (through vector translations of given vectors on vector straight lines and for lengths' calculations) The second language or operation
$+_{1,1}$	Much less and except in $(\mathbb{Q}_{nve}, +_{1,1}, \bullet)$ for mean or average numbers' calculations The second language or operation	Much more and in vector spaces like R^n with n>1 The first language or operation

Table or Matrix 2

Observation: Is the new modern Njiki's fundamental addition one addition too many in mathematics? Not at all. It doesn't simply give a general meaning to the addition of the dunce or of the fool, which historically already found meaning in Farey series, but which are limited to particular or restricted cases. Without it, mathematics remains incomplete, or rather, crippled, especially when it comes to moving from theory to practical applications. The classical or traditional addition, +, and the Njiki's new modern fundamental addition, $+_{1,1}$, are complementary and are respectively suited to calculating a single total and a multiple average, which must be characterized. An intuitive difference between a single total sum and a multiple average sum, whatever its nature, is that the former adds, strings together, chains, piles up, stacks, gathers, accumulates, while the latter levels, measures, or brings to the same size.

16. A suggestive multiple-choice exercise in number theory, designed to open up new avenues of investigation in number education, exploring their meanings beyond their purely formal aspects

Exercise description: A plate contains 5 alike doughnuts. A student at a school eats 3 of them one morning.

1/ What is the average quantity of doughnuts eaten? Justify your answer.

Possible answers: $_{3,}\frac{3}{1},\frac{3}{3},\frac{3}{4},\frac{3}{5}$ etc. (And all these making widen the theory of representations of fractions in to any other order than that of the Q-equivalence classes or classes of equivalence)

2/ What is the average quantity of doughnuts left to eat?

10.13 Some two much more exercises

1/ A civil engineering company has two road projects, A and B, of a specific length to build. On a given date d, it completes the $\frac{2}{5}$ lengths of A and the $\frac{1}{3}$ of B. What is the total or average length(s) of the road, A and B, that it has completed by this date d? Justify your answer regarding the adjectives "total" or "average." Repeat the exercise assuming the two fractions $\frac{2}{5}$ and $\frac{1}{3}$ are attached to only one or the other of A and B, and justify your answer accordingly.

2/A practical exercise using the Njiki's Fundamental addition$^{+}{}_{1.1}$, in Q

Given the following matrix, as an exercise, and which makes get out the PRODUCTION values of 2 COCOA SCARRED symbolised SC1 and SC2 and in terms (1) of the NUMBER of COCOA beans, good (G) or bad (B), produced by EACH of THEM and (2) of their PARTIAL MEANS, respectively, justify or explain the 2 results, $\frac{192}{197}$ and $\frac{5}{197}$, in the blocks situated at the crossing of the before last line, named PARTIAL MEAN (relative), and the column SC_1 and SC_2 of the very MATRIX, by using only the fractions $\frac{97}{99}$, $\frac{2}{99}$, $\frac{95}{98}$ and $\frac{3}{98}$ of the very line in consideration, for the WRITTEN PART of the exercise and assuming the ORAL part of it at the occasion

	SC_1		SC_2		SC_1 and SC_2	
	G	**B**	**G**	**B**	**G**	**B**
ABSOLUTE Total	97	2	95	3	192	5
PARTIAL Mean (relative)	$\frac{97}{99}$	$\frac{2}{99}$	$\frac{95}{98}$	$\frac{3}{98}$	$\frac{192}{197}$	$\frac{5}{197}$
TOTAL Mean	$^{99}/_{99}$		$^{98}/_{98}$		$^{197}/_{197}$	

Table or Matrix 3

Note: The COCOA SCARRED in the above matrix can be smartly adjusted and then be replaced by many various earthly stuffs and other worldly items, for the purposes of the cause.

10.14 Miscellaneous exercises

1/ Given $0 < \frac{b}{a} < \frac{d}{c} \wedge \Delta 1_a = \frac{b+d}{a+c} - \frac{b}{a} \wedge \Delta 1_c = \frac{d}{c} - \frac{b+d}{a+c}$ show that (1) $\frac{b}{a} < \frac{b+d}{a+c} < \frac{d}{c}$ and (2) $a\Delta 1_a = c\Delta 1_c$

2/ Given $0 < \frac{b}{a} < \frac{d}{c} \wedge (\alpha, \beta) \in \mathbb{Z}^{+*2} \wedge \Delta 2_a = \frac{ab+\beta c}{a\alpha+\beta c} - \frac{b}{a} \wedge \Delta 2_c = \frac{d}{c} - \frac{ab+\beta c}{a\alpha+\beta c}$, show that (1) $\frac{b}{a} < \frac{ab+\beta d}{a\alpha+\beta c} < \frac{d}{c}$ and (2) $\alpha a \Delta 1_a = \beta c \Delta 1_c$

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