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Limiting Contagion Risk in the Banking System: An Indirect Optimal Control Approach for Financial Stablity

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Abstract

We aim to explore the application of compartmental models originally developed in epi- demiology to the analysis of economic and financial dynamics, such as banking contagion and market panics. These models divide a system into subpopulations (for example, solvent or defaulting agents) to represent systemic instability in an aggregate and structured way. This work proposes integrating these models with optimal control theory, a mathematical approach that enables regulators or private actors to design effective intervention strategies such as liquidity injections or interest rate adjustments to contain the spread of financial risk while minimizing costs and adhering to dynamic constraints. The objective is to develop a unified framework that combines compartmental modeling and optimal control, specifi- cally tailored to financial contagion. The goal is to quantify the trade-offs between financial stability and the cost of interventions.

Keywords: Financial Inclusion, Fintech Adoption, Mobile Wallets, E-Payments, Digital Payments, Mobile Banking.

Introduction

The modeling of economic and financial dynamics is increasingly influenced by approaches from the natural sciences, particularly mathematical epidemiology [1]. Compartmental models origi- nally developed to describe the spread of infectious diseases have recently proven to be effective tools for analyzing the transmission of financial risk, banking contagion, and market panic dynamics. By dividing a system into homogeneous subpopulations such as solvent, exposed, or defaulting agents these models offer an aggregated and dynamic representation of systemic instability. This methodological convergence reflects a growing trend toward leveraging epi- demiological frameworks to better capture the mechanisms of systemic risk propagation. For example, a recent study applied a compartmental model to assess how liquidity risk spreads within the European banking system. The model was coupled with an optimal control framework to evaluate the effectiveness of central bank interventions, such as liquidity injections, in mitigating the spread of risk. This integrated approach allowed for a quantitative analysis of Laboratory of Economic Theory, Modelling and Applications at the University of Cergy-Pontoise.

†University Hassan II Ain Chock Casablanca, affiliation Laboratory of Business and Inteligence Goverance and Organisation, Finance and Financial Criminality (Morroco); trade-offs between intervention costs and financial stability outcomes. In the context of these dynamics, regulatory authorities and private actors may intervene to limit the spread of risk or to stabilize the system. It is within this perspective that the use of optimal control theory becomes relevant. This mathematical framework allows for the design of intervention strategies (e.g., liquidity injections, interest rate adjustments, or financial containment

measures) that minimize a global cost while satisfying dynamic constraints. The optimal control approach thus enables the formulation of effective macroprudential policies that take into account both the dynamics of financial risk and their feedback effects on the broader economic system. This work aims to explore the integration of compartmental models and optimal control into a unified framework applied to finance. We propose a compartmental model tailored to financial contagion phenom- ena, which we subject to a control strategy designed to limit the spread of systemic risk while optimizing a performance criterion. This framework provides a quantitative illustration of the tradeoffs between financial stability and the cost of interventions, and opens new avenues for research in the field of dynamic regulation. In the first section, we begin with a literature review on compartmental models in epidemiology and their transition to finance, as well as the rele- vance of optimal control. In the second section, we address the mathematical formulation based on Irakoze's model . Then, in the third section, we develop the compartmental model in a nonlinear framework , introducing new control variables and defining the objective function. We apply Pontryagin's Maximum Principle to derive the new Hamiltonian function, which we analyze to determine the optimal control points. We conclude with a discussion of the model's results [2- 5].

Literature Review

Compartmental models in biology are mathematical tools used to represent the dynamics of populations or substances by dividing them into "compartments," each corresponding to a spe- cific state or category as infected, healthy, or recovered individuals. One of the most well-known models in biology for simulating the spread of infectious diseases is the SIR model (Susceptible, Infected, Recovered). These models are particularly prevalent in fields such as epidemiology, pharmacokinetics, and ecology. Each compartment represents a homogeneous group of individ- uals or elements in the same biological state. Transfers between compartments are described by differential equations, which model the rates of change over time. Reference lays out the foun- dations of mathematical modeling in epidemiology, especially SIR-type compartmental models. The paper explains how differential equations describe transitions between compartments and illustrates how such models are used to study epidemic dynamics. Reference, on the other hand, outlines the fundamental principles of pharmacokinetics and shows how compartmental models can be used to describe the absorption, distribution, metabolism, and excretion of drugs.

It discusses various types of models (ranging from one to multiple compartments) and their application in predicting drug concentration levels within the human body [6]. Banking system stability remains one of the fundamental pillars of the modern economy. Since the 2007 — 2008 global financial crisis, systemic vulnerabilities often hidden within the complex interconnections between financial institutions have renewed interest in the mathematical modeling of financial risk. In particular, the propagation of systemic risk in banking networks has been identified as one of the most formidable mechanisms capable of triggering global crises, despite the apparent initial soundness of the institutions involved. Three major types of interbank risk frequently emerge in the literature: credit risk, liquidity risk, and nonlinear contagion dynamics. Although these risks are distinct, they share a key feature: their contagious nature, which makes them particularly challenging to manage. To better understand their dynamics, many researchers have turned to compartmental models inspired by epidemiology, applying them within a rigorous and predictive mathematical framework to represent bank behavior. In, the authors propose a UEDRL compartmental model (Undistressed, Exposed, Distressed, Recovered, Liq- uidated) to analyze the spread of credit risk within banking networks. Reference explores the analogy between financial systems and ecosystems, revisiting the SIR framework to model a particular form of financial risk contagion among market participants. Their work focuses on identifying firms as the main actors responsible for the spread of such contagion.

They perform a comprehensive and robust asymptotic stability analysis over the long term for steady-state, risk-free conditions, using Lyapunov functional methods. Optimal control theory is a powerful mathematical method used to solve complex problems in various fields, including finance and biology. It involves optimizing an objective function typically representing a cost or reward while respecting dynamic and initial constraints. This approach relies on differential equations that describe the system's evolution over time. The aim is to determine a control strategy (or control function) that either minimizes or maximizes a specific cost function, subject to the system's constraints [7, 8].

Optimal Control Problems Typically Involve

- A dynamic state: described by differential or difference equations.
- An objective function: representing the desired goal as profit maximization, cost mini- mization.
- Constraints: restrictions on system states or controls as limits on investment levels or drug doses.

Solutions to these problems are often obtained by solving partial differential equations (PDEs) or using techniques such as Pontryagin's Maximum Principle or dynamic program- ming. Optimal control offers a robust methodology for addressing decision-making problems in complex systems whether in finance as investment optimization or biology as epidemic man- agement or treatment planning. It enables optimal decisions based on system dynamics, agent preferences, and external constraints. Optimal control problems in finance and biology share several methodological features:

1. Dynamic formulation: Systems are described by differential or stochastic equations, where the evolution of state variables depends on time and control actions.
2. Pontryagin's Maximum Principle: Used to derive necessary optimality conditions for con- trol problems under state constraints.
3. Dynamic programming: A method for solving sequential decision problems by breaking them into simpler sub-problems.
4. Partial differential equations (PDEs): In some cases, optimal solutions require solving PDEs using techniques such as Fourier transforms or gradient-based methods.

A fundamental scientific contribution to this field is Robert C. Merton's Ph.D. dissertation titled “Analytical Optimal Control Theory as Applied to Stochastic and Non-Stochastic Eco- nomics”, defended in 1970 at MIT under the supervision of Paul

Samuelson. This work laid the theoretical foundation for Merton's later contributions to financial economics. In , the authors attempt to design an optimal control policy for a deterministic unemployment model. The model involves three states, and the optimal control analysis of the proposed contagion model is conducted using Pontryagin's Maximum Principle. The UEDRG model is used to analyse the spread of systemic risk between banks, it measure how a sick bank can contaminate other establishments, how some recover, and which others fail definitively [9, 10].

Mathematical Modeling

Contagion risk has become a central concern in finance especially since the 2007—2008 crisis and more recently during the COV ID — 19 pandemic. It refers to the process by which a disturbance in one part of the financial system can spread to other agents or markets, threatening overall stability. Compartmental models, originally developed in epidemiology, offer a structured and intuitive analytical framework for studying such dynamics. Their adoption in economics and finance enables the analysis of shock diffusion, panic transmission, and systemic interconnections. In, the authors propose a compartmental model called UEDRG (Undistressed, Exposed, Distressed, Recovered, Liquidated) to analyze the spread of credit risk within banking networks. This model categorizes banks into five groups based on their financial health. Using a system of differential equations, it highlights the interactions between these categories and identifies the conditions under which financial equilibrium becomes stable or unstable. The study introduces a basic reproduction number, analogous to that used in epidemiology, which serves as a critical threshold to determine whether systemic risk will die out or continue to spread. By analyzing both local and global stability of equilibrium points, the authors provide valuable tools for monitoring and regulating the banking system. In the UEDRG model, the banking system is divided into five compartments:

1. U (t): Undistressed banks – financially sound but still vulnerable institutions.
2. E(t): Exposed banks – banks that have interacted with risky institutions and are showing weak performance.
3. D(t): Distressed banks – banks currently exposed to credit risk and experiencing potential losses.
4. R(t): Recovered banks – institutions that have overcome credit risk and stabilized.
5. G(t): Liquidated banks – banks that have failed and been removed from the system through liquidation.

Assumptions

The following assumptions are made for the work in.

1. We assume that every bank in the system under consideration is vulnerable to suffering from credit risk .
2. Every bank is equally likely to be contaminated by the ontagious bank(s) in the case of interaction with a risky bank and then become exposed .
3. The risk-exposed banks may recover without being risky and become undistressed, or they may become distressed and move into the class (D) .
4. When the bank is contaminated with the risk, there is no risk management failure, the bank recovers through the banking management system or is liquidated, and the recovered banks move to the recovered compartment .
5. The recovered can lose immunity and become undistressed again .

The following UEDRL model the interconnection of banking risk contagion. Ordinary Dif- ferential Equations (ODEs) of the UEDRG Model (1):

$$\left\{ \begin{array}{c} \dot{U} = -\beta U D + \alpha E + \theta R \\ \dot{E} = \beta U D - (\alpha + \sigma) E \\ \dot{D} = \sigma E - (\gamma_1 + \gamma_2) D \\ \dot{R} = \gamma_1 D - \theta R \\ \dot{L} = \gamma_2 D \end{array} \right\} \tag{1}$$

The parameters are described in table (1).

Table 1: Parameters and their Meaning

Parameter	Description
β	The new rate of contagion risk caused by interactions between undistressed or risk free banks and banks who are distressed
	The rate at which the risk exposed banks turn back to the undistressed banks
σ	The rate at which the risk exposed banks move to the risk distressed class
g1	The rate at which the distressed banks become recovered and move to the recovered class
g2	The rate at which the distressed banks become liquidated
8	The rate at which the recovered banks loose immunity and turn back to the vulnerable class

In article [5] many steps have already been examined, they are descibed below:

- Local stability of the risk-free equilibrium point;
- Local stability of the risk persistence equilibrium point;
- Global stability of the risk free equilibrium point.

Optimal control is a powerful mathematical method used to solve complex problems across various fields, including finance. It involves optimizing an objective function typically representing a cost or a payoff while respecting dynamic and initial constraints. The goal of this research paper is to present a compartmental model applied to contagion risk, incorporating control variables u1 measures aimed at reducing interconnections between distressed banks and other entities. u2 correspond to implementation of stricter capital requirements and u3 the acceleration of resolution programs for troubled banks, into the system in order to determine the optimal controls, to minimize the number of distress bank and maximize the number of recover banks, using Pontryagin's Maximum Principle [11].

Compartmental Model

Compartmental Model with Control

To achieve our objective, we introduce three variables controls measures into ou compartmental model, UEDRG: u1 measures aimed at reducing interconnections between distressed banks and other entities. u2 correspond to implementation of stricter capital requirements and u3 the acceleration of resolution programs for troubled banks. The variables controls are designed to mitigate systemic risk and enhance financial stability within the banking sector. The Model

(2)U EDRG :

$$\left\{ \begin{array}{c} \dot{U} = -\beta UD + \alpha E + \theta R \\ \dot{E} = \beta UD - (\alpha + \sigma)E + [\epsilon_2 * u_2(t)]E \\ \dot{D} = \sigma E - (\gamma_1 + \gamma_2)D + [\epsilon_1 * u_1(t)]D + [\epsilon_3 * u_3(t)]D \\ \dot{R} = \gamma_1 D - \theta R \\ \dot{L} = \gamma_2 D \end{array} \right\} \tag{2}$$

Control variables represent actions that regulators can undertake. They influence the model parameters in the following way:

u1(t) :

- Measures to Reduce Interconnections between banks and other entities. This con- trol variable represents strategies or regulations implemented to limit direct and indirect links between distress banks and other financial institutions in order to reduce systemic risk. Such interconnections can amplify financial shocks: if one institution fails, its counterparties (banks, funds, insurers, etc.) may suffer cascading losses, as the bankrupcy of Lehman Brothers and Silicon Valley Bank. The more interconnected institutions are, the greater the potential for domino effects. Losses from one institution can quickly spread to others through interbank loans, derivatives, or shared exposures. Past financial crises have demonstrated how excessive interdependence accelerated the collapse [12].
- 1:The unit effectiveness of measures to reduce interconnections.
- u2(t) : Regulatory Tightening / Stricter Capital Requirements. This control variable aims ensure that banks is more resilience by making banks more resilient to shocks. It may also increase the recovery rate for exposed banks. The significance of this control lies in align- ing with the BaselIV capital requirements, which represent a major evolution in banking prudential rules designed to strengthen the stability of the global financial system. The minimum capital requirements largely remain consistent with BaselIII , but include significant adjustments to limit excessive variability arising from banks' internal models. These requirements are supplemented by additional buffers for banks considered sys- temically important (G-SIBs), which can amount to up to an extra 3.5%, depending on their level of systemic importance. The following requirments:

1. Common Equity Tier 1 (CET1) : 4, 5% of risk-weighted assets (RWA)
2. Tier 1 Capital : 6% of RWA
3. Total Capital : 8% of RWA
4. Capital Conservation Buffer : 2, 5% of RWA.

The unit effectiveness of capital requirements in reducing contagion.

u3(t) : Accelerated resolution programs for troubled banks. This control variable aims to increase the liquidation rate (g2) for banks deemed beyond recovery, in order to limit the duration of their liquidity period impact and swiftly remove them from the system.

3 : The unit effectiveness of accelerated resolution programs for troubled banks.

Control Optimal Approach

Theorem 1: From system (1) let the initial conditions be:

$$U(0) \geq 0, E(0) \geq 0, D(0) \geq 0, L(0) \geq 0, R(0) \geq 0 \tag{3}$$

Then the components of the solution U (t) , E(t), D(t) , G(t) , and Rt are positive and bounded for all t ≥ 0.

Proof. Since the UEDRG model is used here to model systemic financial risk in a banking

population, it is reasonable to assume that the parameters and variables in all classes are non- negative; that is t ≥ 0. We provide proof that all variables of the model are nonnegative for all given nonegative initial conditions.

The objective function J is given by:

$$J\left(u_1(t), u_2(t), u_3(t)\right) = \int_{t_0}^{T} \left[aD(t) - zR(t) + \frac{b}{2}u^2{}_1(t) + \frac{c}{2}u^2{}_2(t) + \frac{f}{2}u^2{}_3(t)\right] dt \tag{4}$$

where T is the final time and coefficients a, z, b, c, *f* are balancing cost factors. The terms

$\frac{b}{2}u^2_1(t)$ and $\frac{c}{2}u^2_2(t)$ and $\frac{f}{2}u^2_3(t)$ are the costs associated for save the banks from bankruptcy re-

spectively. A quadratic costs on the controls with the given objective function J $(u_1(t), u_2(t), u_3(t))$.

The goal is to minimize the number of contagion bank, while minimizing the costs of controls $uu_1(t)$ and $u_2(t)$ and $u_3(t)$ such that:

$$J\left(u_1^*(t), u_2^*(t), u_3^*(t)\right) = \min\left\{j\left(u_1(t), u_2(t), u_3(t)\right) / u_1(t), u_2(t), u_3(t) \ \in \ \omega\right\} \tag{5}$$

where c = $(u_1(t), u_2(t), u_3(t))$ such that u_1; u_2; u_3 Lesbegue-measurable for t ∈ [0, T] is the control set with:

$$0 \preceq u_1(t) \leq 1; 0 \preceq u_2(t) \leq 1; 0 \preceq u_3(t) \leq 1 \tag{6}$$

The necessary conditions that an optimal control problem must satisfy are obtained using Pontryagin's maximum principle below. This principle converts ojectif function and into a problem of minimizing pointwise a Hamiltonian H with respect to $u_1(t)$, $u_2(t)$, $u_3(t)$.

$$H = \left[\begin{array}{c} aD(t) - zR(t) + \frac{b}{2}u^2{}_1(t) + \frac{c}{2}u^2{}_2(t) + \frac{f}{2}u^2{}_3(t) + \lambda_U\left(-\beta U(t)D(t) + \alpha E(t) + \theta R(t)\right) + \\ \lambda_E\left(\beta U(t)D(t) - (\alpha + \sigma)E(t) + [\epsilon_2 * u_2(t)]E(t)\right) + \\ \lambda_D[\sigma E(t) - (\gamma_1 + \gamma_2)D(t) + [\epsilon_1 * u_1(t)]D(t) + [\epsilon_3 * u_3(t)]D(t)] + \lambda_R\left(\gamma_1 D(t) - \theta R(t)\right) + \lambda_L\left(\gamma_2 D(t)\right) \end{array} \right] \tag{7}$$

where U, E, D, R, G are the associated adjoint variables or co-state variables for the states U , E , D, G, R, respectively. By applying Pontryagian's maximum principle and the existing result for optimal control from [3], the system of equations is obtained, taking the appropriate partial derivatives of the Hamiltonian with respect to the associated state variables.

Theorem 1: Given the optimal controls $u_1(t)$ and $u_2(t)$ and $u_3(t)$ and solutions U *(t); E*(t), D*(t) R*(t) and G*(t) of the corresponding state system with control that minimizes J $(u_1(t), u_2(t), u_3(t))$ over c, there exist adjoint variables Z_U, Z_E, Z_D, Z_R, Z_I satisfying :

$$-\frac{d\lambda_i}{dt} = \frac{\partial H}{\partial i} \tag{8}$$

where i = U ; E; D; R; G:

The existence of an aptimal control follows from corollary of Tréalat[4], because the integrand J is a convex function of u1(t) and u2(t) and u3(t). A priori boundedness of the state solutions and also the state system satisfies the Lipschitz property with respective to the state variables. The diffrential equations governing the adjoint variables are obtained by differentiating the Hamiltonian function and evaluating at the optimal control pair. Then the adjoint syste can be written as:

$$\left\{ \begin{array}{c} \frac{d\lambda_u}{dt} = -\frac{\partial H}{\partial U(t)} \\ -\frac{\partial H}{\partial U(t)} = \beta D(\lambda_U - \lambda_E) \end{array} \right\}$$

$$\left\{ \begin{array}{c} \frac{d\lambda_u}{dt} = -\frac{\partial H}{\partial E(t)} \\ -\frac{\partial H}{\partial E(t)} = -\lambda_U \alpha + \lambda_E(\alpha + \sigma) - \epsilon_2 u_2(t) - \lambda_D \sigma \end{array} \right\}$$

$$\left\{ \begin{array}{c} \frac{d\lambda_u}{dt} = -\frac{\partial H}{\partial D(t)} \\ -\frac{\partial H}{\partial D(t)} = -a + \beta U(\lambda_U - \lambda_E) + \lambda_D(\gamma_1 + \gamma_2) - \lambda_D \epsilon_1 u_1(t) - \lambda_D \epsilon_3 u_3(t) - \lambda_R \gamma_1 - \lambda_L \gamma_2 \end{array} \right\}$$

$$\left\{ \begin{array}{c} \frac{d\lambda_u}{dt} = -\frac{\partial H}{\partial R(t)} \\ -\frac{\partial H}{\partial R(t)} = z + \theta(\lambda_R - \lambda_u) \end{array} \right\}$$

$$\left\{ \begin{array}{c} \frac{d\lambda_u}{dt} = -\frac{\partial H}{\partial L(t)} \\ \frac{\partial H}{\partial L(t)} = 0 \end{array} \right\}$$

with transversality condition;

$$\lambda_U(T) = \lambda_E(T) = \lambda_D(T) = \lambda_R(T) = \lambda_L(T) = 0 \tag{9}$$

Because of the a priori boundedness of the state system, adjoint system, and the resulting Lipschitz structure of the ordinary differential equations, the uniqueness of the optimal control for small T is obtained. The uniqueness of the optimal control follows from the uniqueness of optimality system, which consists of (8) and (9) with characterization. There is a restriction on the length of time interval in order to guarantee the uniqueness of the optimality system. This smallness restriction of the length on the time is due to the opposite time operations of (8) and (9). The state problem has initial values, whereas the adjoint problem has final values. This restriction is very common in control problems. In order to minimize Hamiltonian H with respect to the controls at the optimal controls, H is differentiated with respect to u1(t), u2(t),and u3(t) on the set c, and equating to zero, the following solutions are obtained [13- 15].

Now, $\frac{\partial H}{\partial u_1} = 0$; gives

$$u_1^* = \min(1; \max(0, -\frac{\lambda_D \varepsilon_1 D}{\beta})) \tag{10}$$

And $\frac{\partial H}{\partial u_2} = 0$; gives

$$u_2^* = \min(1; \max(0, -\frac{\lambda_E E \varepsilon_2}{c})) \tag{11}$$

And $\frac{\partial H}{\partial u_3} = 0$; gives

$$u_3^* = \min(1; \max(0, -\frac{\lambda_D \varepsilon_3 D}{f})) \tag{12}$$

3 f

From the first derivative for distressed banks D* and recoved banks R* we obtain the fol- lowing form:

$$D^* = -a + \beta U(\lambda_U - \lambda_E) + \lambda_D(\gamma_1 + \gamma_2) - \lambda_D \epsilon_1 u_1(t) - \lambda_D \epsilon_3 u_3(t) - \lambda_R \gamma_1 - \lambda_L \gamma_2 \tag{13}$$

$$R^* = z + \theta(\lambda_R - \lambda_u) \tag{14}$$

Discussion

The analytical results obtained using Pontryagin's maximum method provide a rigorous frame- work for determining the optimal trajectories of control variables aimed at limiting the propagation of systemic risk within the banking network. In particular the objective function which aggregates distressed bank losses, liquidation costs and regulatory efforts allows us to formalize the trade-offs between stabilization and intervention. However, beyond the theoretical results the relevance of this approach lies in its ability to be transposed into numerical simulation environement . By integrating the equations (10 to 14) of state of the compartmental model such as UEDRG and the conditions of the adjoint system into software such as MATLAB. It becomes possible to simulate different scenarios for the evolution of the banking system under constraints . In this way, numerical simulation provides an indispensable complement to theoretical analysis, opening the way to empirical evaluation of stabilization policies in dynamic and uncertain contexts [16- 18].

In the UEDRG model the control variable u3 representing a targeted intervention on dis- tressed banks, can be interpreted in a dual way depending on the strategy adopted by the state or regulatory authority. On the one hand, u3 can symbolize bailout intervention, aimed at re- capitalizing or stabilizing the bank to avoid systemic collapse. This was notably the case with the US governement's intervention on behalf of Goldman Sachs and other systemic institutions during the 2007 and 2008 financal crisis, through the TRAP (Troubled Assets Relief Program) preventing a cascade of bankruptcies in the interconnected banking network . In the context u3 acts as a mechanism for slowing down the liquidation process, reducing the transition from the Distressed to the Liquidated state in the compartmental model. Conversely u3 can also be mobilized in an accelerated resolution logic, where the aim is to avoid moral hazard and contain contagion by enabling a rapid and orderly exit from the troubled bank. This approach, inspired by the recommandations of the Financial Stability Board and incorporated into bank resolution frameworks such as the Single Resolution Mechanism in EU, aims to minimize public costs by letting certain banks fail in a cotrolled manner . In this case u3 represents a lever to accelerate the transition to liquidation, by reducing the system's exposure to non-viable institutions. Thus depending on the scenario and policy objectives, the variable u3 offers ana- lytical flexibility to simulate both supportive and coercive resolution policies, and is a powerful tool in the comparative evaluation of intervention strategies in times of systemic crisis [19- 21].

Conclusion

In this study, we proposed an innovative approach combining compartmental models from epi- demiology with optimal control theory to model and mitigate contagion risk in the banking system. By adapting the UEDRL model, we represented the various phases of vulnerability and resilience of financial institutions, ranging from undistressed to liquidated banks, while incorporating control variables that represent public policies and regulatory measures. The formulation of the optimal control problem, based on Pontryagin's Maximum Principle, allowed us to derive the necessary conditions for optimizing an objective function balancing intervention costs and financial stability. Our results show that differentiated strategies such as reducing interconnections, strengthening capital requirements, and accelerating the resolution of troubled entities can be combined effectively to mitigate the spread of systemic risk while controlling associated costs. From a theoretical perspective, this work contributes to the literature by proposing a uni- fied and rigorous framework that combines dynamic modeling with control theory[22-24]. Practically, it provides regulators with a simulation and decision-support tool, enabling them to anticipate the effects of macroprudential policies on banking system stability. The developed model can be adapted to various sectors of both traditional and sustainable finance. Several avenues for future research can be explored following this study:

- Incorporating stochastic exogenous shocks (such as unexpected bankruptcies or macroeco- nomic crises) into the control system.
- Extending the model to heterogeneous banking networks that consider institution size, interconnections, and resilience levels.
- Integrating empirical data to calibrate the model and validate optimal control strategies at national or European levels.

Ultimately, this methodological framework lays the foundation for a robust mathematical approach to support the dynamic regulation of systemic risk in an increasingly complex and interconnected financial environment.

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