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The Proof of the Riemann Conjecture

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Abstract

In order to strictly prove the conjecture in Riemann's 1859 paper on the Number of prime Numbers Not Greater than x from a purely mathematical point of view, and strictly prove the correctness of Riemann's conjecture, this paper uses Euler's formula to prove that if the independent variables of ζ(s) function are conjugate, then the values of ζ(s) function are also conjugate, thus obtaining that the independent variables of ζ(s) function are also conjugate at zero. And using the conjugation of the zeros of the Riemann ζ(s) function and the zeros of ζ(s)=0 and the zeros of ζ(1-s)=0, s and 1-s must also be conjugated, The nontrivial zero of Riemann function ζ(s) must meet s= 1/(2)+ti(t ∈R and t≠0) and s= 1/(2)-ti(t ∈ R and t≠0). And the symmetry of the zeros of Riemann ζ(s) function is the necessary condition that the nontrivial zeros of Riemann ζ(s) function are located on the critical boundary. According to the symmetry property of the zeros of Riemann ζ (s) function s and the zeros of Riemann ζ(s) function 1-s, combined with the conjugated property of the zeros of Riemann ζ(s) function s and Riemann ζ(s) function 1-s, It is shown that the real part of the nontrivial zero of the ζ(s) function must only be equal to 1/(2). And by Riemann set s=1/2+ti(t ∈C and t≠0) and auxiliary function ξ(s)=1/2s(s-1) ∈Γ(s/2)π ∈ (- s/2) ζ(s)(s ∈C and s≠1), Get ∏ s/2(s-1)π^(-s/2)ζ(s)= ξ(t) =0, combining the nontrivial zeros of Riemann function ζ(s) must meet s =1/(2)+ti (t ∈ R and t≠0) and s =1/(2)-ti (t ∈ R and t≠0), Thus it is proved equivalently that the zeros of the Riemann ξ(t) function must all be non-zero real numbers, and the Riemannian conjecture is completely correct.

Keywords: Euler's Formula, Riemann ζ(s) Function, Riemann Function ξ(t), Riemann Conjecture, Symmetric Zeros, Conjugate Zeros, Uniquenes

Introduction

The Riemann hypothesis and the Riemann conjecture is an important and famous mathematical problem left by Riemann in his 1859 paper "On the Number of primes not greater than x ", which is of great significance to the study of the distribution of prime numbers and is known as the greatest unsolved mystery in mathematics. After years of hard work, I solved this problem and rigorously proved that both the Riemann conjecture and the generalized Riemann conjecture are completely correct. The Polignac conjecture, the twin prime conjecture, and Goldbach's conjecture are also completely correct. It would be nice if you understood Riemann's conjecture thoroughly from the outset of his paper "On Prime Numbers not Greater than x" and were completely convinced of the logical reasoning behind it. You need to do this before you read my paper. The following is about the first half of Riemann's paper "On the Number of primes not Greater than x ", which I have explained and derived, which is the premise and basis for your understanding of Riemann's conjecture.In 1859, Riemann was admitted to the Berlin Academy of Sciences as a corresponding member, and in order to express his gratitude for the honor, he thought it would be best to use the permission he received immediately to inform the Berlin Academy of a study on the density of the distribution of prime numbers, a subject in which Gauss and Dirichlet had long been interested. It does not seem entirely unworthy of a report of this nature. Riemann used Euler's discovery of the following equation as his starting point:

$$\prod_{p=1}^{\infty}\left(\frac{1}{1-p^{-s}}\right)=\sum_{n=1}^{\infty}\frac{1}{n^s}$$

Where p on the left side of the equation takes all prime numbers, n on the right side takes all natural numbers, and the function of the complex variable s represented by the two series above (when they converge) is denoted by $\zeta(s)$. That is, to define a function of complex variables:

$$\zeta(s)=\sum_{n=1}^{\infty}\frac{1}{n^s}=\prod_{p=1}^{\infty}\left(\frac{1}{1-p^{-s}}\right)。$$

The two series above converge only if the real part of s is greater than 1,is also say when Re(s)>1,then $\sum_{n=1}^{\infty}\frac{1}{n^s}$ and $\prod_{p=1}^{\infty}\left(\frac{1}{1-p^{-s}}\right)$ converge only.if s=1,then $\sum_{n=1}^{\infty}\frac{1}{n^s}=\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots$, It's called a harmonic series, and it diverges.If Re(s)<1, $\sum_{n=1}^{\infty}\frac{1}{n^s}=\frac{1}{1^s}+\frac{1}{2^s}+\frac{1}{3^s}+\frac{1}{4^s}+\ldots$, it's more divergent.Because if Re(s)<1,then $\frac{1}{1^s}=\frac{1}{1},\frac{1}{2^s}>\frac{1}{2},\frac{1}{3^s}>\frac{1}{3},\frac{1}{4^s}>\frac{1}{4},\ \ldots$, .But if s is a negative number, for example s= -1,then it does not satisfy the condition that Re(s)>1. So you need to find an expression for $\zeta(s)$ function a that is always valid for any s. In modern mathematical language, that is, to carry out an analytical extension of a complex function $\zeta(s)$, and the best way to analyze the extension is to find a more extensive and effective representation of the function such as an integral representation or an appropriate function representation.Therefore, we want to define a new function, this new function also $\zeta(s)$ to represent, this new function of the independent variable s is not only full Re(s)>1, but also satisfy Re(s)≤1(s≠1), and the function image is smooth, every point on the function image can find its tangent slope, that is, the function everywhere can find the derivative. However, it is no longer called the Euler zeta function, but the Riemann ζ function. Riemann used the integral to express the function $\zeta(s)$. In this paper, I have added another complex variable to express the Riemann function $\zeta(s)$.

Because $\Pi(s)=\Gamma(s+1)=s\Gamma(s)$, where $\Pi(s)$ is the factorial function , $\Gamma(s)$ is the Euler gamma function,$\Gamma(s)=\int_0^{\infty}x^{s-1}e^{-x}dx$, Let the variable $x\rightarrow nx$ ($n\in Z^{+}$) in the integral symbol,then

$\int_0^{\infty}(nx)^{s-1}e^{-nx}d(nx)=n\int_0^{\infty}e^{-nx}\ n^{s-1}x^{s-1}=n^s\int_0^{\infty}e^{-nx}\ x^{s-1}=\Gamma(s)=\Pi(s-1)$,so

$$\int_0^{\infty}e^{-nx}\,x^{s-1}=\frac{\Pi(s-1)}{n^s}$$

That's exactly what Riemann says in his paper, he says he's going to use

$$\int_0^{\infty}e^{-nx}\ \ x^{s-1}=\frac{\Pi(s-1)}{n^s}$$

Since n is all positive integers, we need to assign $\sum$ to e^{-nx} and $\frac{1}{n^s}$ on both sides of the equation,

so $\sum_{n=1}^{\infty}e^{-nx}=1+\sum_{n=1}^{\infty}e^{-nx}-1=(1+e^{-x}+e^{-2x}+e^{-3x}+\ldots)-1=\frac{1}{1-e^{-x}}-1=\frac{e^{-x}}{1-e^{-x}}=\frac{1}{e^{x}-1}$,

The common ratio q satisfies $0<q=|e^{-x}|<=1(0<=x\rightarrow+\infty)$, $\frac{\Pi(s-1)}{n^s}=\frac{\Pi(s-1)}{1^s+2^s+3^s+4^s+5^s+\cdots}$,

and $\sum_{n=1}^{\infty}\frac{1}{n^s}=\frac{1}{1^s+2^s+3^s+4^s+5^s+\cdots}=\zeta(s)$, so according

$$\int_0^{\infty}e^{-nx}\,x^{s-1}=\frac{\Pi(s-1)}{n^s}$$

,can get $\Pi(s-1)\zeta(s)=\int_0^{\infty}\frac{x^{s-1}dx}{e^{x}-1}$, this is exactly what Riemann found in his paper.

Now consider the following integral

$$\int \frac{(-x)^{s-1}dx}{e^x - 1}$$

According to modern mathematical notation, the integral should be denoted as $\int_C \frac{(-x)^{s-1}dx}{e^x-1}$, or considering that the complex number is generally represented by Z, the integral should be denoted as $\int_C \frac{(-Z)^{s-1}dZ}{e^Z-1}$, Its integral path proceeds from +∞ to +∞ on the forward boundary of a region containing the value 0 but not any other singularities of the integrable function, where the integral path C is shown in Figure 1 below.

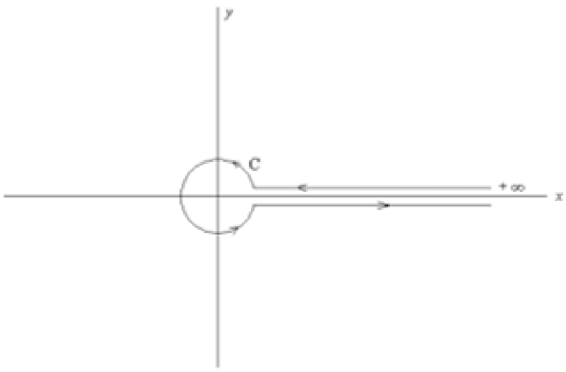


Figure 1

Riemann zeta function ζ(s) is a series expression of $\sum_{n=1}^{\infty}\frac{1}{n^s}(n \in Z^+)(\mathrm{Re}(s)>1)$on the complex plane analytical continuation. The reason for the analytical extension of the above series expression is that this expression only applies to the region of the complex plane where the real part of s Re(s)>1 (otherwise the series does not converge). Riemann found an analytical continuation of this expression (of course Riemann did not use the modern term "analytic continuation" in complex function theory). Using the circumchannel integral, the analytically extended Riemann zeta function can be expressed as:

$$\zeta(s) = \frac{\Gamma(1-s)}{2\pi i}\int_{\infty}^{\infty}\frac{(-z)^s}{e^Z-1}\frac{dz}{z}$$

The integral in the above formula is actually a circumchannel integral around the positive real axis (that is, starting from +∞, integrating above the real axis to near the origin, integrating around the origin to below the real axis, and then integrating below the real axis to +∞ - the distance from the real axis and the radius around the origin are all approaching 0); The Γ function Γ(s) in the equation is an analytical extension of the factorial function in the complex plane, for positive integers s > 1:Γ(s)=(s-1)! . It can be shown that the integral expression for ζ(s) above resolves everywhere over the entire complex plane except for a simple pole at s=1. Such an expression is an example of a so-called meromorphic function - that is, a function that resolves everywhere over the entire complex plane except for the existence of poles on an isolated set of points. This is the complete definition of the Riemann ζ function.

To obtain the value of this integral, we assume that there is a complex number of arbitrarily small moduli δ, and that the moduli $|\delta|$ of δ,$|\delta|\to 0$,Because $(-Z)^s = e^{s\ln(-Z)}$,and $\ln(-Z)=\ln(Z)+\pi i$ or $\ln(-Z) = \ln(Z) - \pi i$, so

$$\int_C \frac{(-Z)^{s-1}dZ}{e^Z-1} = \int_\infty^\delta \frac{(-Z)^{s-1}dZ}{e^Z-1} + \int_\delta^\infty \frac{(-Z)^{s-1}dZ}{e^Z-1} + k\int_{|\delta|\to 0}^{\infty} \frac{(-Z)^{s-1}dZ}{e^Z-1} = \int_{+\infty}^{\delta} \frac{(-Z)^s dZ}{(e^Z-1)Z} + \int_\delta^{+\infty} \frac{(-Z)^s dZ}{(e^Z-1)Z}$$

$$+k\int_{|\delta|\to 0} \frac{(-Z)^s dZ}{(e^Z-1)Z} = (e^{\pi si}-e^{-\pi si})\int_\delta^\infty \frac{e^{s\ln(Z)}dZ}{(e^Z-1)Z} + k\int_{|\delta|\to 0} \frac{(-Z)^s dZ}{(e^Z-1)Z}$$, k is a constant.

The definition of trigonometric functions of complex variables is given by Euler's formula

$\sin(z)=\frac{e^{iz}-e^{-iz}}{2i}$, if $z=\pi s$, then $\sin(\pi s)=\frac{e^{\pi si}-e^{-\pi si}}{2i}$.so $e^{\pi si}-e^{-\pi si}=2i\sin(\pi s)$, $i=\frac{e^{\pi si}-e^{-\pi si}}{2\sin(\pi s)}$ .so

$\int_C \frac{(-Z)^{s-1}dZ}{e^Z-1} = (e^{\pi si}-e^{-\pi si})\int_\delta^\infty \frac{e^{s\ln(Z)}dZ}{e^Z-1} + k\int_{|\delta|\to 0} \frac{(-Z)^s dZ}{(e^Z-1)Z}$, if δ is a real number and the absolute value $|\delta|$ of δ, $|\delta| \to 0$,

then $\int_{|\delta|\to 0} \frac{(-Z)^s dZ}{(e^Z-1)Z} = 0$ then $\int_C \frac{(-Z)^{s-1}dZ}{e^Z-1} = 2i\sin(\pi s)\int_0^\infty \frac{x^{s-1}dx}{e^x-1}$ $(x \in R)$.then

$\frac{1}{2i\sin(\pi s)}\int_C \frac{(-Z)^{s-1}dZ}{e^Z-1} = \int_0^\infty \frac{x^{s-1}dx}{e^x-1}$ $(x \in R)$.We got

$\Pi(s-1)\zeta(s) = \int_0^\infty \frac{x^{s-1}dx}{e^x-1} (x \in R)$ before, so $2\sin(\pi s)\,\Pi(s-1)\zeta(s) = i\int_0^\infty \frac{x^{s-1}dx}{e^x-1}$. Where we agree that in the many-valued function $(-x)^{s-1}$, the value of $\ln(-x)$ is real for negative x, thus obtaining Where we agree that in the many-valued function , the value of ln (–) is real for negative , thus obtaining 2sin ().This equation now gives the value of the function ζs for any complex variable s, and shows that it is single-valued analytic, and takes a finite value for all finite s except 1, and zero when s is equal to a negative even number.The right side of the above equation is an integral function, so the left side is also an integral function ,$\Pi(s-1) = \Gamma(s)$,and the first-order poles of $\Gamma(s)$ at $s = 0,-1,-2,-3,...$ cancels out $\sin(\pi s)$'s zero. When the real part of s is negative, the above integral can be performed not along the region positively surrounding the given value, but along the region negatively containing all the remaining complex values.See Figure 2 below, where the radius of the great circle C 'approaches infinity and thus contains all poles of the integrand, i.e. , all zeros of the denominator $e^x - 1$, $n\pi i$ (n is an integer), and the following calculation applies Cauchy's residue theorem.

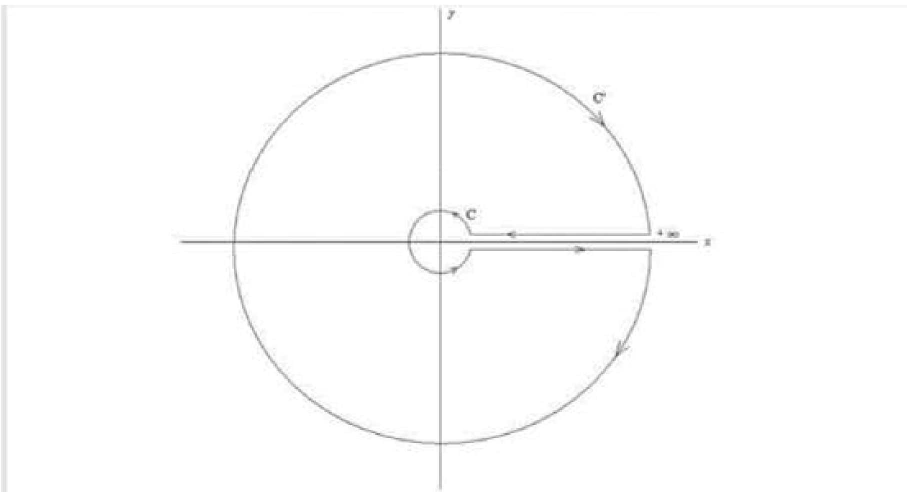


Figure 2

Since the value of the integral is infinitesimal for modular infinite complex numbers, and in this region the integrand has a singularity only if x is equal to an integral multiple of $2\pi i$, the integral is equal to the sum of the integrals negatively around these values, but the integral around the value $n2\pi i(n \in R^{+})$ is equal to $(-n2\pi i)^{s-1}(-2\pi i)(n \in R^{+})$.The residue of the integrand at $n2\pi i(n \neq 0)$ is equal to

$[\frac{(-x)^{s-1}}{(e^{x}-1)'}]_{x=n2\pi i}=[\frac{(-x)^{s-1}}{e^{x}}]_{x=n2\pi i}=(n2\pi i)^{s-1}(n \neq 0)$.

So we get

$2\sin(\pi s)\prod(s-1)\zeta(s)=(2\pi)^{s}\sum n^{s-1}((-i)^{s-1}+i^{s-1})$ [1] (Formula 3),

It reveals a relationship between ζ(s) and ζ(1-s), using known properties of the function $\Pi(s)$, that is, using the coelements formula of the gamma function $\Gamma(s)$ and Legendre's formula. It can also be expressed as: $\Gamma(\frac{s}{2})\pi^{-\frac{s}{2}}\zeta(s)$ is invariant under the transformation s⟶1-s.

based on euler's $e^{ix}=\cos(x)+i\sin(x)\ (x \in R)$, can get

$e^{i(-\frac{\pi}{2})}=\cos(\frac{-\pi}{2})+i\sin(\frac{-\pi}{2})=0-i=-i$,

$e^{i(\frac{\pi}{2})}=\cos(\frac{\pi}{2})+i\sin(\frac{\pi}{2})=0+i=i$,

then

$(-i)^{s-1}+i^{s-1}=(-i)^{-1}(-i)^{s}+(i)^{-1}(i)^{s}=(-i)^{-1}e^{i(-\frac{\pi}{2})s}+i^{(-1)}e^{i(\frac{\pi}{2})s}=$

$ie^{i(-\frac{\pi}{2})s}-ie^{i(\frac{\pi}{2})s}=i(\cos\frac{-\pi s}{2}+i\sin\frac{-\pi s}{2})-i(\cos\frac{\pi s}{2}+i\sin\frac{\pi s}{2})=i\cos(\frac{\pi s}{2})-i\cos(\frac{\pi s}{2})+\sin(\frac{\pi s}{2})+\sin(\frac{\pi s}{2})$

$=2\sin(\frac{\pi s}{2})$ (Formula 4).

According to the property of $\Pi(s-1)=\Gamma(s)$ of the gamma function,and

$\sum_{n=1}^{\infty} n^{s-1}=\zeta(1-s)$($n \in Z^{+}$ and n traves all positive integer, $s \in C$, and $s \neq 1$),

Substitute the above (Formula 4) into the above (Formula 3), will get

$2\sin(\pi s)\Gamma(s)\zeta(s)=(2\pi)^{s}\zeta(1-s)2\sin\frac{\pi s}{2}$ (Formula 5),

according to the double Angle formula $\sin(\pi s)=2\sin(\frac{\pi s}{2})\cos(\frac{\pi s}{2})$, we Will get

$\zeta(1-s)=2^{1-s}\pi^{-s}\cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)$ ($s \in C$ and $s \neq 1$) (Formula 6),

Substituting s→1-s, that is taking s as 1-s into Formula 6, we will get

$\zeta(s)=2^{s}\pi^{s-1}\sin(\frac{\pi s}{2})\Gamma(1-s)\zeta(1-s)$ ($s \in C$ and $s \neq 1$) (Formula 7),

This is the functional equation for ζ(s) ($s \in C$ and $s \neq 1$). To rewrite it in a symmetric form, use the residual formula of the gamma function

$\Gamma(Z)\Gamma(1-Z)=\frac{\pi}{\sin(\pi Z)}$ (Formula 8)

and Legendre's formula $\Gamma(\frac{Z}{2})\Gamma(\frac{Z}{2}+\frac{1}{2})=2^{1-Z}\pi^{\frac{1}{2}}\Gamma(Z)$ (Formula 9) ,

Take $z=\frac{s}{2}$ in (Formula 8) and substitute it to get

$\sin(\frac{\pi s}{2})=\frac{\pi}{\Gamma(\frac{s}{2})\Gamma(1-\frac{s}{2})}$ (Formula 10) ,

In (Formula 9), let z=1-s and substitute it in to get

$\Gamma(1-s)=2^{-s}\pi^{-\frac{1}{2}}\Gamma(\frac{1-s}{2})\Gamma(1-\frac{s}{2})$ (Formula 11)

By substituting $\sin(\frac{\pi s}{2})=\frac{\pi}{\Gamma(\frac{s}{2})\Gamma(1-\frac{s}{2})}$ (Formula 10) and $\Gamma(1-s)=2^{-s}\pi^{-\frac{1}{2}}\Gamma(\frac{1-s}{2})\Gamma(1-\frac{s}{2})$ (Formula 11) into $\zeta(s)=2^s\pi^{s-1}\sin(\frac{\pi s}{2})\Gamma(1-s)\zeta(1-s)(s\in C$ and $s\neq 1)$ (Formula 7), can get

$\pi^{-\frac{s}{2}}\Gamma(\frac{s}{2})\zeta(s)=\pi^{-\frac{1-s}{2}}\Gamma(\frac{1-s}{2})\zeta(1-s)(s\in C$ and $s\neq 1)$ (Formula 12),

Also

$\Gamma(\frac{s}{2})\pi^{-\frac{s}{2}}\zeta(s)$ is invariant under the transformation s→1-s,

And that's exactly what Riemann said in his paper.That is to say:

$\Gamma(\frac{s}{2})\pi^{-\frac{s}{2}}\zeta(s)$ is invariant under the transformation s→1-s ,

Also

$\prod(\frac{s}{2}-1)\pi^{-\frac{s}{2}}\zeta(s)=\prod(\frac{1-s}{2}-1)\pi^{-\frac{1-s}{2}}\zeta(1-s)(s\in C$ and $s\neq 1)$,

or

$\pi^{-\frac{s}{2}}\Gamma(\frac{s}{2})\zeta(s)=\pi^{-\frac{1-s}{2}}\Gamma(\frac{1-s}{2})\zeta(1-s)(s\in C$ and $s\neq 1)$(Formula 2),

Then $\zeta(s)=2^s\pi^{s-1}\sin(\frac{\pi s}{2})\Gamma(1-s)\zeta(1-s)(s\in C$ and $s\neq 1)$(Formula 7) .

This property of the function induces me to introduce $\Pi(\frac{s}{2}-1)$ instead of $\Pi(s-1)$ into the general term of the series $\sum_{n=1}^{\infty}\frac{1}{n^s}$, from which we obtain the function a very convenient expression for$\zeta(s)$, which we actually have

$$\frac{1}{n^s}\Pi\left(\frac{s}{2}-1\right)\pi^{-\frac{s}{2}}=\int_0^{\infty}e^{-n^2\pi x}\ x^{-\frac{s}{2}}dx\ .$$

To derive the above equation, let's look at $\Pi(\frac{s}{2}-1)=\Gamma(\frac{s}{2})=\int_0^{\infty}x^{\frac{s}{2}-1}e^{-x}dx$, in

$\Pi(\frac{s}{2}-1)=\Gamma(s)=\int_0^{\infty}x^{\frac{s}{2}-1}e^{-x}dx$, replace $x\rightarrow n^2\pi x$ as follows, then

$\Pi(\frac{s}{2}-1)=\Gamma(s)=\int_0^{\infty}(n^2\pi x)^{\frac{s}{2}-1}e^{-n^2\pi x}dx=n^s.n^{-2}.\pi^{\frac{s}{2}}.\pi^{-1}\int_0^{\infty}e^{-n^2\pi x}\ x^{-\frac{s}{2}}d(n^2\pi x)=$

$n^s.n^{-2}.\pi^{\frac{s}{2}}.\pi^{-1}.n^2.\pi\int_0^{\infty}e^{-n^2\pi x}\ x^{-\frac{s}{2}}dx=n^s.\pi^{\frac{s}{2}}\int_0^{\infty}e^{-n^2\pi x}\ x^{-\frac{s}{2}}dx$, so

$\frac{1}{n^s}\Pi\left(\frac{s}{2}-1\right)\pi^{-\frac{s}{2}}=\int_0^{\infty}e^{-n^2\pi x}\ x^{-\frac{s}{2}}dx$.

So, if we call $\sum_{n=1}^{\infty}e^{-n^2\pi x}=\psi(x)$, get immediately

$\frac{1}{n^s}\Pi\left(\frac{s}{2}-1\right)\pi^{-\frac{s}{2}}=\int_0^{\infty}e^{-n^2\pi x}\ x^{-\frac{s}{2}}dx=\int_0^{\infty}(\sum_{n=1}^{\infty}e^{-n^2\pi x})x^{-\frac{s}{2}}dx=\int_0^{\infty}\psi(x)x^{-\frac{s}{2}}dx.$

According to the Jacobi theta function

$\theta(x)=\sum_{n=-\infty}^{\infty}e^{-n^2\pi x}=e^{-0^2\pi x}+2\sum_{n=1}^{\infty}e^{-n^2\pi x}=1+2(e^{-\pi x}+e^{-4\pi x}+e^{-9\pi x}+e^{-16\pi x}+\cdots)$,

Easy to see $\psi(x)=\sum_{n=1}^{\infty}e^{-n^2\pi x}=\frac{\theta(x)-1}{2}$.

The transformation formula of theta function is derived as follows:$\theta(\frac{1}{x})=\sqrt{x}\ \theta(x)$.

Let the first class of complete elliptic integrals k,k' is called modulus and complement of Jacobi elliptic functions or elliptic integrals, respectively.

$k=k(k)=\int_0^{\frac{\pi}{2}}\frac{d\theta}{\sqrt{(1-k^2\sin^2\theta)}}$,

$k'=k(k')=\int_0^{\frac{\pi}{2}}\frac{d\theta}{\sqrt{(1-k'^2\sin^2\theta)}}$, let $\tau = k'/k$,then get

$\sqrt{\frac{2k}{\pi}}=\theta(\tau)=1+2(e^{-\pi\tau}+e^{-4\pi\tau}+e^{-9\pi\tau}+e^{-16\pi\tau}+\cdots)$,

The modulo k and the complement k' are interchangeable

$\sqrt{\frac{2k'}{\pi}}=\theta\left(\frac{1}{\tau}\right)=1+2(e^{-\pi/\tau}+e^{-4\pi/\tau}+e^{-9\pi/\tau}+e^{-16\pi/\tau}+\cdots)$,

Compare the two formulas to obtain $\theta(\frac{1}{\tau})=\sqrt{\tau}\theta(\tau)$. It was first obtained by Cauchy using Fourier analysis, and later proved by Jacobi using elliptic functions.

Apply the integral expression above

$$\zeta(s)=\frac{\Gamma(1-s)}{2\pi i}\int_{\infty}^{\infty}\frac{(-z)^s}{e^z-1}\frac{dz}{z}$$

can also prove Riemann zeta function satisfy the above algebraic equation - also called zeta function equation $\zeta(s)=2^s\pi^{s-1}\sin(\frac{\pi s}{2})\Gamma(1-s)\zeta(1-s)(s\in C \text{ and } s\neq 1)$ (Formula 7), is not hard to find in this relation, The Riemann ζ function takes zero at s=-2n (n is a positive integer) , because $\sin(\frac{\pi s}{2})$ is zero. The point on the complex plane where the value of the Riemann ζ function is zero is called the zero of the Riemann ζ function.So s=-2n (n is a positive integer) is the zero of the Riemann zeta function. These zeros have a simple and orderly distribution and are called trivial zeros of the Riemann ζ function. In addition to these trivial zeros, the Riemann ζ function has many other zeros whose properties are far more complex than those trivial zeros, and are rightly called nontrivial zeros. The study of the non-trivial zeros of the Riemann ζ function constitutes one of the most difficult subjects in modern mathematics. The Riemann conjecture that we are going to discuss is a conjecture about these nontrivial zeros. Here we first describe its content, and then describe its context. Riemann conjecture: All nontrivial zeros of the Riemann ζ function lie on the line $Re(s)=\frac{1}{2}$. In the study of the Riemann conjecture, mathematicians call the line $Re(s)=\frac{1}{2}$ in the complex plane a critical boundary. Using this term, the Riemann conjecture can also be expressed as: all non-trivial zeros of the Riemann ζ function lie on the critical boundary. This is the content of the Riemann conjecture, which Riemann proposed in 1859 in his paper "On the Number of Prime Numbers Not Greater than x." In its formulation, the Riemann conjecture appears to be a purely complex function proposition, but as we shall soon see, it is in fact a mysterious piece of music about the distribution of prime numbers.

How can the distribution of nontrivial zeros of a function over a complex number field, the Riemann zeta function, which we sometimes refer to simply as zeros if there is no ambiguity, be related to the distribution of prime numbers in the seemingly unrelated natural numbers (which in this book refer to positive integers)? It starts with what's called the Euler product formula. We know that as early as the ancient Greeks, Euclid proved with a wonderful proof by contradiction that there are infinitely many prime numbers. With the deepening of the study of number theory,

people are naturally more and more interested in the distribution of prime numbers on the set of natural numbers. In 1737, the mathematician Euler published a very important formula at the St. Petersburg Academy of Sciences in Russia, which laid the foundation for mathematicians to study the law of the distribution of prime numbers. This formula is the Euler product formula, which is $\sum_n n^{-s}=\prod_p(1-p^{-s})^{-1}$,The sum on the left of this formula is performed on all natural numbers, and the continued product on the right is performed on all prime numbers. It can be shown that this formula holds for all complex numbers s with Re(s)>1. The left side of this formula is the series expression of the Riemann ζ function for Re(s)>1, which we have described above, and the right side is an expression purely concerning prime numbers (and containing all prime numbers), which is a sign of the relationship between the Riemannζ function and the distribution of prime numbers. So what does this formula tell us about the distribution of prime numbers? How does the zero of the Riemann zeta function appear in this relation?

Euler himself was the first to study the information contained in this formula. He noticed that at s=1, the left-hand side of the formula

$$\sum_{n} n^{-1}$$

is a divergent series (this is a famous divergent series, called a harmonic series), which diverges logarithmically. None of this was new to Euler. To deal with the continued product on the right side of the formula, he took the logarithm of both sides of the formula, so that the continued product became a sum, from which he obtained:

$\ln(\sum_n n^{-1}) = \sum_p (p^{-1} + \frac{p^{-2}}{2} + \frac{p^{-3}}{3} + \cdots)$,

Or, rather,

$\sum_{p<N} p^{-1} \sim \ln\ln(N)$,

This result, which diverges in the form of lnln (N), is another important research result on prime numbers since Euclid proved that there are infinitely many primes. It is also a novel proof of the proposition that there are infinitely many prime numbers (because if there are only finite numbers of prime numbers, then the sum has only a finite number and cannot diverge). But this new proof by Euler contains much more than Euclid's proof, because it shows that prime numbers are not only infinitely many, but that their distribution is much denser than that of many sequences that also contain infinitely many elements, such as { n } sequences (because the sum of the reciprocal convergences of the latter).

Moreover, if we further note that the right end of $\sum_{p<N} p^{-1} \sim \ln\ln(N)$ can be rewritten as an integral expression:

$\ln\ln(N) \sim \int^{N} \frac{x^{-1}}{\ln(x)} dx$,

By introducing a density function $\rho(x)$ for the distribution of prime numbers, which gives the probability of finding prime numbers in the unit interval near x, the left end of $\sum_{p<N} p^{-1} \sim \ln\ln(N)$ can also be rewritten as an integral expression:

$\sum_{p<N} p^{-1} \sim \int^{N} x^{-1} \rho(x) dx$,

Comparing these two integral expressions, it is not difficult to guess that the distribution density of the prime numbers is $\rho(x) \sim 1/\ln x$, so that the number of prime numbers within x, usually represented by $\pi(x)$, is

$\pi(x) \sim Li(x)$,

among

$Li(x) = \int \frac{1}{\ln x} dx$,

It's a logarithmic integral function.This result is the famous prime number theorem - although this crude reasoning does not constitute a proof of the prime number theorem. So this result that Euler discovered is a secret door to the prime number theorem. Unfortunately, Euler himself did not follow this line of thinking and missed this secret door, and the time for mathematicians to develop the prime number theorem was delayed by several decades.

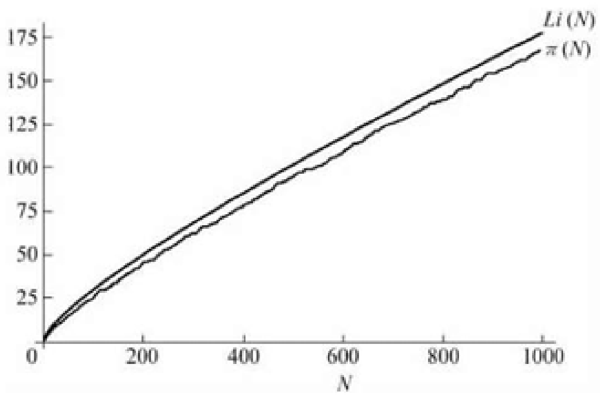
The credit for developing the prime number theorem eventually fell to two other mathematicians: the German Friedrich Gauss (1777-1855) and the French Adrien-Marie Legendre (1752-1833).Gauss's work on the distribution of prime numbers began between 1792 and 1793, when he was only fifteen years old. During that time, whenever he was "doing nothing," the precocious genius mathematician would pick a few natural number intervals of length 1,000, count the number of primes in these intervals, and compare them. After doing a lot of calculations and comparisons, Gauss discovered that the density of the prime distribution can be approximately described by the reciprocal of the logarithmic function, $\rho(x) \sim 1/\ln x$, which is the main content of the prime number theorem mentioned above. But Gauss did not publish the results. Gauss was a mathematician who pursued perfection, and he rarely published results that he thought were not perfect, and his mathematical ideas and inspiration were like a vast and surging river, which often made him start a new research topic before he had time to beautify a research result. As a result, Gauss did far more mathematical research in his lifetime than he officially published. On the other hand, Gauss often revealed some of his unpublished work through other means, such as letters, which caused considerable embarrassment to some of his contemporaries. One of the hardest hit was Legendre. The French mathematician was the first to publish the least square method for linear fitting in 1806, but Gauss mentioned in a work published in 1809 that he had discovered the same method in 1794 (that is, 12 years before Legendre), much to Legendre's dismay.

As the saying goes, friends don't get together. In the formulation of the prime number theorem, poor Legendre once again had the misfortune to collide with the mathematical giant Gauss. Legendre published his research on the distribution of prime numbers in 1798, which is the earliest document on the theorem of prime numbers in the history of mathematics. Since Gauss did not publish his results, Legendre was the rightful author of the prime number theorem. Legendre maintained this priority for a total of 51 years. But in 1849, Gauss, in a letter to the German astronomer Johann Encke (1791-1865), mentioned his work on the distribution of prime numbers in 1792-93, thus taking the half-century-old priority out of Legendre's pocket. On top of his already bulging pockets.

Fortunately, by the time Gauss wrote to Enke, Legendre had been dead for 16 years, and he had avoided another cruel blow in the most helpless way.

Both Gauss's and Legendre's studies of the distribution of prime numbers were presented in the form of guesses (Legendre's study had a certain element of inference, but it was still far from proving). Therefore, to be sure, the prime number theorem was at that time only a conjecture, that is, the prime number conjecture, and what we mean by the formulation of the prime number theorem is only the formulation of the prime number conjecture. The mathematical proof of the prime number theorem was not given until a century later, in 1896, by the French mathematician Jacques Hadamard (1865-1963) and the Belgian mathematician Charlesde la Vallee-Poussin (1866-1962), independently of each other. Their proof has a deep connection with the Riemann conjecture, and the timing and occasion of Hadamard's proof are dramatic, as we shall describe later.

The prime number theorem is concise and elegant, but its description of the distribution of prime numbers is still relatively rough, it gives only an asymptotic form of the distribution of prime numbers - the distribution of primes less than N as N approaches infinity. From the distribution of prime numbers and the prime number theorem, we can also see that there is a deviation between $\pi(x)$ and $\mathrm{Li}(x)$, and the absolute value of this deviation seems to continue to increase with the increase of x (fortunately, the increase of this deviation is still negligible compared to the increase of $\pi(x)$ and $\mathrm{Li}(x)$ itself - otherwise the prime number theorem would not hold).Is there a formula that describes the distribution of prime numbers more accurately than the prime number theorem? This was the question that Riemann set out to answer in 1859. That year, five years after Gauss's death, Riemann, 32, succeeded the German mathematician Johann Dirichlet (1805-1859) as Gauss's successor at the University of Gottingen. On 11 August of the same year, he was elected a corresponding member of the Academy of Sciences in Berlin. In return for this high honor, Riemann submitted a paper to the Berlin Academy of Sciences - a short eight-page paper entitled: On the Number of primes Less than a Given Value. It was this paper that deciphered the information contained in Euler's product formula, and it was this paper that linked the distribution of zeros of the Riemann zeta function to the distribution of prime numbers.



(The above diagram shows the distribution of prime numbers and the prime number theorem).
This paper pushed the study of the distribution of prime numbers to a magnificent peak, and left a great mystery for later generations of mathematicians.

According to Euler's formula $\zeta(s) \equiv \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_{p=1}^{\infty} (\frac{1}{1-p^{-s}})$, this is the basis for studying the distribution of prime numbers. Riemann's research also takes this formula as a starting point. In order to eliminate the continued product on the right side of this formula, Euler took the logarithm of both sides of the formula, and Riemann did the same (even the product is really something that one wants to divide quickly), thus obtaining $\ln\zeta(s) \equiv \sum_p \ln(1-p^{-s}) = \sum_p \sum_n \frac{p^{-s}}{n}$, but after this step, Riemann and euler parted ways: euler proved that sounded after prime Numbers have an infinite number not quit; Riemann, on the other hand, continued to walk along a thorny road and came out of a new world of prime number research.
It can be shown that the double summation to the right of the given $\ln\zeta(s) \equiv \sum_p \ln(1 - p-s)$=pnp−sn is absolutely converges in the region Re(s)>1 on the complex plane, and can be rewritten as the Stielchers integral:

$$\ln\zeta(s) = \int_0^{\infty} x^s \mathrm{d}\, J(x),$$

Where $J(x)$ is a special step function that takes a value of zero at x=0, increases by 1 for every prime passed, and 1/2 for every square passed,... Every time a prime number is raised to the NTH power, it increases by 1/n... And at $J(x)$ discontinuous points (i.e., x equals a prime number, the square of a prime number,... Prime number to the NTH power... The function value is defined by $J(x) = \frac{1}{2}[J(x^-) + J(x^+)]$. Obviously, such a step function can be expressed by the prime distribution function $\pi(x)$ as:

$$J(x) = \sum_n \frac{\pi(x^{\frac{1}{n}})}{n} .$$

The above Stilchers integral can be obtained by performing an integration by parts:

$$\ln\zeta(s) = s \int_0^{\infty} J(x)\, x^{-s-1} \mathrm{d}x.$$

The left side of this formula is the natural log of the Riemann zeta function, and the right side is the integral of $J(x)$, a function directly related to the prime distribution function $\pi(x)$, which can be regarded as the integral form of the Euler product formula. The method of this result differs from that of Riemann, who did not have Stieltjes integrals when he published his paper - Dutch mathematician Thomas Stieltjes (1856-1894) was only three years old at thetime. If the traditional Euler product formula is only a vague sign of the connection between the Riemann zeta function and the distribution of prime numbers, then the connection between the two is unmistakable and completely quantitative in the integral form of the Euler product formula described above. The first thing to do is obviously solve for $J(x)$ from the integral above, and

Riemann solved for $J(x)$:

$$J(x) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{\ln\zeta(Z)}{Z}x^{Z}dz,$$

Where a is a real number greater than 1. The above integral is a conditionally convergent integral, which is precisely defined as the integral from a-ib to a+ib (where b is a positive real number) and then taking the limit of b→∞. Riemann says this result is completely universal. The complete result, which actually matched Riemann's universal result, was not published until 40 years later by the Finnish mathematician Robert Mellin (1854-1933), now known as the Mellin transform.

Such a statement, written down by Riemann, but which took the mathematical community tens or even hundreds of years to prove, has several other points in Riemann's paper. This is one of the most striking features of Riemann's paper: it has a lofty vision that far surpasses other contemporary mathematical literature. Its highly condensed sentences contain extremely rich mathematical results behind, so that later mathematicians into a long reflection. Even more admirably, some of the calculations and proofs in Riemann's manuscripts, even when they were compiled decades later, were often far beyond the level of the mathematical community at the time. There is strong reason to believe that what Riemann says in his paper, in a declarative rather than a speculative tone, has a deep calculus and proof background, whether or not he gives evidence.

Ok, now back to the expression for $J(x)$, which gives the exact relationship between $J(x)$ and the Riemann ζ function. In other words, once ζ(s) is known, $J(x)$ can in principle be calculated from this expression. Knowing $J(x)$, the next obvious step is to compute $\pi(x)$. This is not difficult, since the relationship between $J(x)$ and $\pi(x)$ mentioned above can be inversely solved for $\pi(x)$ and $J(x)$ by a so-called Mobius inversion, which results in:

$$\pi(x)=\sum_n \frac{\mu(n)}{n}J(x^{\frac{1}{n}}),$$

Here μ(n) is called the Mobius function and takes the following values:

- μ(1)=1;
- μ(n)=0(If n is divisible by the square of any prime number);
- μ(n)=-1(If n is the product of an odd number of different prime number);
- μ(n)=1(If n is the product of an even number of different prime numbers).

So knowing $J(x)$ allows you to calculate $\pi(x)$, the distribution function for prime numbers. Connecting these steps together, we see that from ζ(s) to $J(x)$, and from $J(x)$ to $\pi(x)$, the secret of the distribution of prime numbers is fully and quantitatively contained in the Riemann zeta function. This is the basic idea of Riemann's study of the distribution of prime numbers.

There is a deep correlation between the distribution of prime numbers and the Riemann zeta function. At the heart of this relation is the expression for the integral of $J(x)$: $J(x) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{\ln\zeta(s)}{s}x^{s}$, which is also extremely complex due to the extremely complex nature of the Riemann ζ function. To investigate this integral further, Riemann introduced an auxiliary function ξ(s):

$$\xi(s)= \Gamma(\frac{s}{2}+1)(s-1)\pi^{-\frac{s}{2}}\zeta(s).$$

But it's better to define $\xi(s)$ as:

$\xi(s)=\frac{1}{2}s(s-1)\pi^{-\frac{s}{2}}\Gamma\left(\frac{s}{2}\right)\zeta(s)$.

Because the factor $(s-1)$ elimination first-order pole of $\zeta(s)$ at $s=1$, the factor s elimination pole of $\Gamma(\frac{s}{2})$ at $s=0$, and $\zeta(s)$'s trivial zeros -2, -4, -6,... elimination the remaining poles of $\Gamma(\frac{s}{2})$, so $\xi(s)$ is an integral function that is zero only at the nonnormal zero point of $\zeta(s)$.

What are the benefits of introducing such an auxiliary function? First of all, by type $\xi(s)=\Gamma(\frac{s}{2}+1)(s-1)\pi^{-\frac{s}{2}}\zeta(s)$define the auxiliary function of $\xi(s)$ can be proved to be the whole function, namely on all $s\neq\infty$ indicates in the complex plane up the point of analytic function. Such a function would be much simpler in nature than the Riemann zeta function, and much easier to process. In fact, of all non-mediocre complex functions, the integral function is the widest analytic region (the analytic region is larger than that, i.e. there is only one kind of function that includes $s=\infty$, and that is the constant function). This is one of the benefits of introducing $\xi(s)$.

Secondly, using this auxiliary function, the algebraic relation

$\zeta(s)=2^s\pi^{s-1}\sin(\frac{\pi s}{2})\Gamma(1-s)\zeta(1-s)(s\in C$ and $s\neq 1)$(Formula 7) for the Riemannian zeta function obtained above can be expressed as a simple form symmetric to s and 1-s: $\xi(s)=\xi(1-s)$.This is the second advantage of introducing $\xi(s)$.

Furthermore, it is not difficult to see from the definition $\xi(s)$ that the zero of $\xi(s)$ must be the zero of $\zeta(s)$. On the other hand, the zeros of $\zeta(s)$ are zeros of $\xi(s)$, except for the trivial zero $s=-2n$ (n is a natural number), which happens to be the pole of $\Gamma(s/2+1)$ and therefore not the zeros of $\xi(s)$, and thus the zeros of $\xi(s)$ coincide with the nontrivial zeros of the Riemann zeta function. In other words, $\xi(s)$ separates the nontrivial zeros of the Riemann zeta function from the total zeros. This is the third advantage of introducing $\xi(s)$.

Here it is necessary to mention a simple property of the Riemann zeta function, namely that $\zeta(s)$ has no zero in the region $Re(s)>1$. If there is no zero, of course, there is no nontrivial zero, and the latter coincides with the zero of $\xi(s)$, so the above property shows that $\xi(s)$ has no zero in the region of $Re(s)>1$; And since $\xi(s)=\xi(1-s)$, $\xi(s)$ also has no zero in the region $Re(s)<0$. This shows that all zeros of $\xi(s)$, and thus all non-trivial zeros of the Riemann ζ function - lie in the region $0\leq Re(s)\leq 1$. An important result about the distribution of zeros of the Riemann ζ function is that all nontrivial zeros of the Riemann ζ function are located in the region $0\leq Re(s)\leq 1$ in the complex plane.

All right, now back to Riemann's paper. After introducing ξ(s), Riemann decomposes lnξ(s) with the zero of ξ(s) :

$\ln\zeta(s)=\ln\xi(0)+\sum_{\rho}\ln(1-\frac{s}{\rho})-\ln\Gamma(s/2+1)+\frac{s}{2}\ln\pi-\ln(s-1)$,

Where ρ is the zero of ξ(s)(that is, the nontrivial zero of the Riemann ζ function). The summation in the resolution is performed on all ρ and in such a way that ρ is first paired with 1-ρ. Since ξ(s)=ξ(1-s), zeros always occur as ρ paired with 1-ρ. This is important because the series is conditionally convergent, but absolutely convergent after pairing ρ with 1-ρ. This factorization can also be written as the equivalent continued product relation:

$\xi(s)=\xi(0)\prod_{\rho}(1-\frac{s}{\rho})$.

Such a continued product relation is obvious for finite polynomials (as long as the condition ξ(0) ≠0 is satisfied), but is by no means obvious for infinite products, which depend on the fact that ξ(s) is an integral function. Its complete proof was not given until 1893 by Hadamard in his systematic study of infinite product expressions of integral functions. Hadamard's proof of this relationship was the first important advance in the field after Riemann's paper.

It is obvious that the convergence of the above series decomposition is closely related to the zero distribution of ξ(s). For this reason, Riemann studied the zero distribution of ξ(s) and proposed three important propositions:

Proposition 1: in 0 < Im(s) < T area, the number of zero of ξ(s) is about (T/2π)ln(T/2π)-(T/2π).

Proposition 2: in 0 < Im(s) < T area, factor ξ(s) is located in the Re(s)=1/2 of the number of zero point on the line is about (T/2π)ln(T/2π)-(T/2π).

Proposition 3: ξ(s) all zeros lie on the line Re(s)=1/2. (I will prove this proposition strictly later.)

Of these three statements, the first is needed to prove the convergence of the series decomposition (although Riemann's statement based on this statement is too brief to constitute a proof). Riemann's proof of this statement is that the number of zeros in ξ(s) in the region 0<Im(s)<T can be obtained by integrating dξ(s)/2πiξ(s) along the boundary of the rectangular region {0<Re(s)<1,0<Im(s)<T}. For Riemann, this small integral was not a big deal, so he simply wrote down the result (i.e., proposition 1). Riemann also gave this result a relative error of 1/T. But Riemann obviously greatly overestimated the level of his audience, because it was not until 1905, 46 years later, that the result he wrote was proved by the German mathematician Hansvon Mangoldt (1854-1925) (hence the Riemann-Mangolt formula). In addition to completing a small proof in the Riemann paper, it also established that there are infinitely many non-trivial zeros of the Riemann zeta function.

Comparing Riemann's second statement with the previous one shows that this second statement actually shows that nearly all zeros of ξ(s) - and thus almost all non-trivial zeros of the Riemann ζ function - lie on the line Re(s)=1/2. This is a surprising proposition, because it is much stronger than anything that has been achieved so far - that is, in the century and a half since Riemann's paper was published - on the Riemann conjecture! And the tone in which Riemann describes this proposition is completely certain, which seems to suggest that when he wrote it down he thought he had a proof for it. Unfortunately, he does not mention the details of the proof at all, so how on earth does he prove this proposition? Is his proof right or wrong? None of us will know. In addition to his 1859 paper, Riemann had mentioned this proposition in a letter, saying that it could be derived from a new expression of the ξ function, but that he had not yet reduced it to a point where it could be published. This is all that posterity has learned about this proposition from the fragments left by Riemann.

Riemann's three propositions are like three rising mountains, each taller than the last and each more difficult to climb. His first proposition kept mathematics waiting for 46 years; His second proposition has kept mathematics waiting for more than a century and a half; And his third proposition must have been seen by everyone, it is the famous Riemann conjecture! Today, the Riemann conjecture has been conquered by me, and it really does hold true, and I'm going to prove it rigorously later.

Riemann, who used to make theorems go up in smoke in conversation and laughter, finally changed his lighthearted style and adopted an uncertain tone like "very likely" when it came to expressing this third proposition, the Riemann conjecture. Riemann also wrote: "We would certainly like to have a rigorous proof of this, but after some quick and futile attempts I have set aside the search for such a proof, as it is not necessary for the immediate object of my study." Riemann put the proof aside, and the heart strings of the whole mathematical world were lifted. The validity of the Riemann conjecture is not necessary for Riemann's "immediate goal" of proving the convergence of the series factorization of lnξ(s) (since the first statement above is sufficient), but it is of vital importance to the mathematical community today. A rough count shows that there are more than a thousand mathematical statements or "theorems" in the mathematical literature today that presuppose the existence of the Riemann conjecture (or its generalized form). The fate of the Riemann conjecture is bound up with the "immediate goal" of all the mathematicians who developed these propositions or "theorems," and through those propositions or "theorems," it is inextricably linked to many branches of mathematics. On the other hand, Riemann's way of expressing the Riemann conjecture also shows from one side that Riemann distinguishes whether the propositions he writes are speculative or positive.

Now let's go back to the calculation for J(x). Using the definition ξ(s) and its decomposition, lnζ(s) can be expressed as:

$$\ln\zeta(s)=\ln\xi(0)+\sum_p \ln\left(1-\frac{s}{\rho}\right)-\ln\Gamma(s/2+1)+\frac{s}{2}\ln\pi-\ln(s-1);$$

The purpose of this decomposition of lnζ(s) is to calculate J(x). However, every single integral obtained by directly substituting this resolution into the integral expression of J(x) is not convergent, so Riemann first integrated J(x) by parts before substituting, thus obtaining:

$$J(x)=\frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty}\frac{\ln\zeta(Z)}{Z}x^{Z}dz,$$

By substituting the resolution of lnζ(s) into the above formula, the individual items can be multiplied separately. The following table shows the terms in the lnζ(s) decomposition and their corresponding integration results:

Decomposition of lnζ(s)	The corresponding integral result
$-\ln(s-1)$	$Li(x)$
$\sum_\rho \ln\left(1-\frac{s}{\rho}\right)$	$-\sum_{\mathrm{Im}(\rho)>0}\left[Li(x^{\rho})+Li(x^{1-\rho})\right]$
$-\ln\Gamma\left(\frac{s}{2}+1\right)$	$\int_x^{\infty}\frac{dt}{t(t^2-1)\ln t}$
$\ln\xi(0)$	$\ln\xi(0)=-\ln 2$
$\frac{s}{2}\ln\pi$	0

Among the above results, the integration of the series

$$\sum_{\rho} \ln(1-\frac{s}{\rho})$$

is the most complicated, and the result
is the result of integrating the series term by term. This result

$-\sum_{Im(\rho)>0}[\mathrm{Li}(x^{\rho})+\mathrm{Li}(x^{1-\rho})\ ]$

is conditionally convergent, Not only must ρ be paired with 1-ρ, as in the series expression for lnξ(s), but it must also sum Im(ρ) from smallest to largest. In giving this result, Riemann admitted that the validity of term-by-term integrals depended on a "more rigorous" discussion of the ξ function, but stated that it was easy to prove. This "easily provable" result was proved 36 years later by Mangolt in 1895. It is also worth pointing out that when Riemann integrates the individual items of this order, there is an implicit requirement that for all zeros ρ, 0<Re(ρ)<1, which is better than 0 ≤ Re(ρ) ≤ 1, which we mentioned earlier. This seemingly minor reinforcement (which is merely the elimination of the equal sign) is in fact an important consequence of number theory, which I shall prove later. Riemann's failure not only to prove this result, but also to imply it, should be regarded as a flaw in his paper. This flaw is also present in Mangolt's proof.

However, this loophole is only a loophole in the argument method, which can be filled, and the result of the argument itself does not depend on such a condition as 0<Re(ρ)<1.From these results Riemann obtained the explicit form of J(x):

$J(x)=\mathrm{Li}(x)-\sum_{Im(\rho)>0}[\mathrm{Li}(x^{\rho})+\mathrm{Li}(x^{1-\rho})]+\int_{x}^{+\infty}\frac{dt}{t(t^2-1)\ln t}-\ln 2,$

$\mathrm{Li}(x)=\int_0^x \frac{dt}{lnt}(x\in Z^+)$,

This result, together with the relationship between π(x) and J(x):

$\pi(x)=\sum_n \frac{\mu(n)}{n}J(x^{\frac{1}{n}}),$

This is the complete expression of the distribution of prime numbers obtained by Riemann, and is the main result of his 1859 paper. Riemann's result gives an exact expression for the distribution of prime numbers, the first term of which (given by the first term of J(x) and π(x) together) is precisely the result Li(x) predicted by the then-unproven prime number theorem. Since Riemann has given an exact expression for the distribution of prime numbers, he has not been able to directly prove a prime number theorem that is much coarser than this result. Why? The mystery lies in the Riemann zeta function of nontrivial, zero is J(x) the expression of those items related to the zero point, namely - $\sum_{\mathrm{Im}(\rho)>0}[\mathrm{Li}(x^{\rho})+\mathrm{Li}(x^{1-\rho})]$. In the expression for J(x), all the other terms are quite simple and relatively smooth, so that the careful laws of the distribution of prime numbers - those careful, dense fluctuations - are chiefly contained in this series relating to the nontrivial zeros of the Riemann ζ function. As mentioned above, the series is conditionally convergent, that is, its convergence depends on the cancellation of each other by the items participating in the summation, that is, the contributions from the different zeros. These contributions from the different zeros are like a zigzagging dance, guiding the careful distribution of prime numbers. And the exuberance of the dance-the way and degree to which these contributions cancel each other-determines how close the actual distribution of prime numbers is to the asymptotic distribution given by the prime number theorem. All of this depends quantitatively on the distribution of nontrivial zeros of the Riemann ζ function. The precise expression given by Riemann for the distribution of prime numbers did not immediately make a direct proof of the prime number theorem possible precisely because so little was known about the distribution of the non-trivial zeros of the Riemann ζ function (in fact, what was known then was 0≤Re(ρ≤1), as we have already mentioned above). Those contributions from zeros cannot be efficiently estimated, and hence the deviation from the prime number theorem to the actual distribution of prime numbers, which is the exact expression given by Riemann.

Then what effect does the distribution of nontrivial zeros of the Riemann ζ function have on the deviation between the prime number theorem and the actual distribution of prime numbers? Mathematicians have achieved a series of results on this question. The proof of the prime number theorem is itself one of them. After the proof of the prime number theorem, in 1901, the Swedish mathematician von Koch (1870-1924) further proved (this is an example of the mathematical statement that presupposes the existence of the Riemann conjecture as we mentioned earlier) that if the Riemann conjecture is true, Then the absolute deviation between the prime number theorem and the actual distribution of prime numbers is $O(x^{\frac{1}{2}}\ln x)$. The model of $Li(x^{\rho})$with the increaseof x $x^{Re(\rho)}/\ln x$ increases, so any pair of nontrivial zero ρ and 1-ρ asymptotic contributions given by $Li(x^{\rho})+Li(x^{1-\rho})$, at least, is $Li(x^{\frac{1}{2}}) \sim x^{\frac{1}{2}}/\ln x$. This result implies that the deviation between the prime number theorem and the actual distribution of prime numbers cannot be less than $Li(x^{\frac{1}{2}})$. In fact, the British mathematician John Littlewood (1885-1977) proved that the prime number theorem differs from the actual distribution of prime numbers by at least $Li(x^{\frac{1}{2}}) \approx \ln\ln\ln x$. This is very close to Koch's result (the main term is $x^{\frac{1}{2}}$). Therefore, the Riemann conjecture holds that the distribution of prime numbers is relatively ordered; Conversely, if the Riemann conjecture does not holdif a pair of nontrivial zeros ρ and 1-ρ of the Riemann ζ function deviate from the critical boundary (i.e. Re(ρ)>1/2 or Re(1-ρ)>1/2), then the principal term of their corresponding asymptotic contribution will be greater than $x^{\frac{1}{2}}$, and the deviation between the prime number theorem and the actual distribution of prime numbers will be greater.Thus, the study of the Riemann conjecture allowed mathematicians to see the strange laws and orders behind the seemingly random distribution of prime numbers.This law and order is reflected in the distribution of nontrivial zeros of the Riemann ζ function.

In 1885, a young Dutch mathematician named Thomas Stieltjes (1856-1894) publisheda brief at the Paris Academy of Sciences in which he claimed to have proved the following:

$M(N) \equiv \sum_{n<N} \mu(n) = O(N^{\frac{1}{2}})$,

Here μ(n) is the Mobius function we mentioned earlier, and the function M(N) given by its summation is called the Mertens function. The statement seems to be a good one: the Mobius function μ(n) is an integer function whose definition is trivial but not complicated, and the Meertens function M(N) is just the sum of μ(n), so proving that it grows by $O(N^{\frac{1}{2}})$ does not seem too difficult. But this humble proposition is actually a stronger result than the Riemann conjecture! In other words, proving the above statement is the same as proving the Riemann conjecture (but the reverse is not true, disproving the above statement is not the same as disproving the Riemann conjecture). So Stielches' presentation meant claiming to have proved the Riemann conjecture. Although the Riemann conjecture was not nearly as hot as it is today, and news did not spread nearly as fast as it does today, someone proved that the Riemann conjecture was still a big deal. If nothing else, proving the Riemann conjecture would mean proving the prime number theorem, which has plagued mathematicians for nearly a century since Gauss et al. proposed it, but has yet to be proved. At about the same time as his presentation at the Paris Academy of Sciences, Stilchers sent a letter repeating this statement to Charles Hermite (1822-1901), a major figure in French mathematics at the time. But Mr. Stilchers offered no proof, either in the briefing or in the letter, saying his proof was too complicated and needed to be simplified. Today, it would be difficult for a young mathematician to write such a blank check and cause any reaction in the mathematical community. But things were different in the 19th century, when it was common in academia for scientists to produce results without publishing (or publishing only one result), and Gauss and Riemann were among them. So to claim to have proved the Riemann conjecture, as Stielches did, without giving a concrete proof, was not unusual at the time. The academic response somewhat resembles the presumption of innocence in modern Western courts, which tend to believe claims until there is evidence to the contrary.

But to believe is to believe, of course, mathematics cannot be separated from proof, and a proof must be published in detail and tested in order to obtain final recognition. Concrete proof was therefore expected of Stilchers, and the most earnest of all was expected of Hermite, who received the letter from Stilchers. Hermite corresponded with Stielches from 1882 until his untimely death 12 years later. During that time, the two exchanged 432 letters. Hermite was one of the leading theorists of complex function theory at the time, and his relationship with Stielches is one of the more curious phenomena in the history of mathematics. At the time of his correspondence with Hermite, Stillches was only an assistant at the Leiden Observatory, and even this assistant position had been secured by the patronage of his father (Stillches's father was a prominent Dutch engineer and member of Parliament). Before that, he had failed three exams in college. It was not easy to "pull the strings, through the back door" into the observatory, but Stielches was doing astronomical observation work, but his heart was thinking about mathematics, and wrote a letter to Hermet. It would have been difficult, if not impossible, for Stielches, who had no degree and no reputation at the time, to attract the attention of a mathematical elder like Hermite. But Hermet was a devout Catholic who happened to have a peculiar belief in mathematics, believing that it existed as something supernatural and that ordinary mathematicians only occasionally had the opportunity to understand its mysteries. So what kind of person has a better chance of understanding the mysteries of mathematics than an "ordinary mathematician"? Hermite, with his mystic vision, found one, that is, the unknown stargazer Stielches. Hermite believed that Stielches had a God-given eye for the mysteries of mathematics, and he trusted it.

In his correspondence with Stielches, there was even such extreme approval as "you are always right and I am always wrong." Under the influence of this peculiar belief and the mathematical atmosphere of the nineteenth century, Hermitt believed Stielches's statement. But no matter how much Hermite urged him, Stilchez never published his full proof. Five years have passed, and Hermite is still "infatuated" with Stielches, and he decides to "entice" the other side. At Hermite's suggestion, the French Academy of Sciences set the theme of the 1890 Prize in Mathematics as "Determining the number of primes less than a given value." This topic must have a sense of deja vu to you, and yes, it is very similar to the title of the Riemann paper we have just introduced. In fact, the purpose of the prize was to seek proof of certain propositions mentioned in Riemann's paper but not proved (this was explicitly stated in the request). As for the statement itself, it can be either the Riemann conjecture or some other proposition, provided that its proof helps to "determine the number of primes less than a given value." With such a flexible requirement, prizes can be won not only for proving the Riemann conjecture, but also for proving results that are much weaker than the Riemann conjecture, such as the prime number theorem. In Hermite's view, the mathematical prize would inevitably go to Stilchers, because even if Stilchers' proof of the Riemann conjecture remained "too complex and needed to be simplified," he could still claim the prize by publishing partial or weaker results. Unfortunately, by the time the prize deadline expired, Stilchez was still silent.

But Hermite was not entirely disappointed, because his student Adama submitted a paper and won the grand prize - after all, the fat did not flow to outsiders. The main content of Hadamard's prize-winning paper is the proof of the continued product expression of the auxiliary function ξ(s) in Riemann's paper mentioned above. This proof, while not only failing to prove the Riemann conjecture and even falling some way short of proving the prime number theorem, is still a grand prize. A few years later, Hadamard continued his efforts and finally proved the prime number theorem in one fell fell. Hermite's long line failed to catch Stielches and Riemann conjectures as he wished, but it did catch Hadamard and the prime number theorem, and it was quite lucrative (the proof of the prime number theorem was actually more desirable than the proof of the Riemann conjecture at the time).

What about Stielches? Readers who have never heard of the name might think that he is a pompous and incompetent guy, but he is not. Stielches has made important contributions to many aspects of analysis and number theory. His research on continued fractions earned him the reputation of "Father of continued fraction analysis". The Riemann-Stieltjes integral, which bears his name, links him to Riemann (although there is no actual connection between the two -Stieltjes was only 10 years old when Riemann died). But his statement about the Riemann conjecture did not win him permanent suspense. It is now generally accepted by mathematicians that Stilchers' claim that $M(N)=O(N^{\frac{1}{2}})$ is false, if at all. Moreover, the validity of the proposition $M(N)=O(N^{\frac{1}{2}})$ itself has been increasingly questioned.

Since Gauss and Legendre put forward the prime number theorem in the form of empirical formula, many mathematicians have done research on it. One of the more important results was made by the Russian mathematician Pafnuty Chebyshev (1821-1894). As early as 1850, Chebyshev proved that for a sufficiently large x, the relative error between the prime distribution $\pi(x)$ and the distribution $\mathrm{Li}(x)$ given by the prime number theorem cannot exceed 1%. Before Riemann's work in 1859, the study of the distribution of prime numbers was mainly limited to real analysis. In this sense, even leaving aside specific results, Riemann's work on complex functions was a major breakthrough in the study of the distribution of prime numbers in terms of its method alone.This breakthrough paved the way for the final proof of the prime number theorem.

As mentioned earlier, the reason why Riemann's study of the distribution of prime numbers did not lead directly to the proof of the prime number theorem is that the distribution of the non-trivial zeros of the Riemann ζ function is still very little known.So, in order to prove the prime number theorem, how much do we need to know about the distribution of nontrivial zeros of the Riemann ζ function? The answer to this question became clear in 1895 with Mangolt's in-depth study of Riemann's papers.Mangolt, whose work we have already mentioned, proved Riemann's formula for $J(x)$.But the value of Mangolt's work goes much deeper than just proving Riemann's formula for $J(x)$.

As mentioned earlier, the reason why Riemann's study of the distribution of prime numbers did not lead directly to the proof of the prime number theorem is that so littleis known about the distribution of nontrivial zeros of the Riemann zeta function. So, in order to prove the prime number theorem, how much do we need to know about the distribution of nontrivial zeros of the Riemann zeta function? The answer to this question became clear in 1895 with Mangolt's in-depth study of Riemann's papers. Mangolt, whose work we have already mentioned, proved Riemann's formula for $J(x)$. But the value of Mangolt's work goes much deeper than just proving Riemann's formula for $J(x)$.

In his research, Mangolt used an auxiliary function $\Psi(x)$ that is simpler and more efficient than Riemann's $J(x)$, which is defined as:

$\Psi(x)=\sum_{n<x} \Lambda(n)$,

Where Λ(n) is called the von Mangoldt function, which takes the value ln(p) for n=pk(p is a prime number, k is a positive integer); For other n, the value is 0. Applying $\Psi(x)$, Mangolt proved a formula that is essentially equivalent to Riemann's formula for $J(x)$:

$\Psi(x)= x-\sum_{\rho}\frac{x^{\rho}}{\rho}-\frac{1}{2}\ln(1-x^{-2})-\ln(2\pi)$,

The sum of ρ, like the sum in Riemann's $J(x)$, pairs ρ with 1-ρ first and then Im(ρ) in the order from smallest to largest.

Obviously, Mangolt's $\Psi(x)$ expression is much simpler than Riemann's $J(x)$. Nowadays, $\Psi(x)$ has almost completely replaced Riemann's $J(x)$ in the study of analytic number theory. Another major benefit of the introduction of $\Psi(x)$ is that several years earlier, the aforementioned Chebyshev had already proved that the prime number theorem $\pi(x) \sim Li(x)$ was equivalent to $\Psi(x) \sim x$. In honor of Chebyshev's work, the Mangolt function is also known as the second Chebyshev function.

Linking this to Mangolt's formula concerning Ψ(x), which is essentially equivalent to Riemann's formula concerning $J(x)$, it is not difficult to see that the prime number theorem holds:

$\lim_{x\to\infty} \sum_{\rho}(x^{\rho-1}/\rho)=0$,

This condition suggests that we consider the case where $x^{\rho-1}$ approaches zero as $x\to\infty$. For $x^{\rho-1}$ to approach zero at $x\to\infty$, Re(ρ) must be less than 1. In other words, the Riemann zeta function must have no nontrivial zeros on the line Re(s)=1. This is what we need to know about the distribution of nontrivial zeros of the Riemann ζ function in order to prove the prime number theorem.

Since the nontrivial zeros of the Riemann function occur as ρ paired with 1-ρ, this information is equivalent to 0<Re(s)<1.

As mentioned earlier, all non-trivial zeros of the Riemann zeta function lie in the region 0≤Re(s) ≤1. Thus, in order to prove the prime number theorem, we needed to know slightly more about the distribution of nontrivial zeros of the Riemann zeta function than we knew (and was known to mathematicians at the time) (but still much less than the Riemann conjecture required). Thus, after the remarkable efforts of Chebyshev,Riemann, Hadamard, and Mangott, we are at last only one small step away from the proof of the prime number theorem: the removal of the little equal sign from the known law of the distribution of zeros. Although this small step is by no means easy, ithas been difficult to climb the Riemann Peak for more than 30 years, and mathematicians have waited for a century for the arrival of the complete proof of the prime number theorem. (Note; In 1896, the year after Mangolt's results were published, Hadamard and Posen independently gave proof of this last small step almost simultaneously, thus fulfilling one of the great ambitions of mathematics since Gauss. By then Stilchez had been dead for two years.

After the proof of the prime number theorem, the understanding of the distribution of non-trivial zeros of the Riemann ζ function is further advanced, that is, it is proved that all non-trivial zeros of the Riemann ζ function are located in the region of 0<Re(s)<1 on the complex plane. In the study of the Riemann ζ conjecture, mathematicians refer to this region as the critical strip.

The proof of the prime number theorem - especially in a way so closely related to Riemann's paper - led the mathematical community to pay more attention to the Riemann conjecture. Four years later, on a summer day in 1900, more than two hundred of the best mathematicians of the day gathered in Paris, and a thirty-eight-year-old German mathematician took the podium and gave a lecture that will go down in the annals of mathematics. The title of the lecture was Mathematical Problems, and the speaker's name was David Hilbert (1862-1943), who happened to be from the star-studded University of Gottingen, the academic home of Gauss and Riemann. He is the great successor of the mathematical spirit of Gottingen, a mathematical giant as famous as Gauss and Riemann. In his speech, Hilbert listed 23 mathematical problems that had a profound impact on later generations, and the Riemann conjecture was listed as part of the eighth problem, which has since become one of the problems that the entire mathematical community has focused on.

The curtain of mathematics in the 20th century opened slowly in the sound of Hilbert's speech, and Riemann conjecture ushered in a new journey of one hundred years.

Riemann described it in his paper as follows:

$$\prod\left(\frac{s}{2}-1\right)\pi^{-\frac{s}{2}}\zeta(s) = \int_1^{\infty}\psi(x)\, x^{\frac{s}{2}-1}dx + \int_1^{\infty}\psi\left(\frac{1}{x}\right) x^{\frac{s-3}{2}}dx + \frac{1}{2}\int_0^1 \left(x^{\frac{s-3}{2}} - x^{\frac{s}{2}-1}\right)dx$$

$$= \frac{1}{s(s-1)} + \int_1^{\infty}\psi(x)\left(x^{\frac{s}{2}-1}+x^{-\frac{1+s}{2}}\right)dx ,$$

Let's look at the last part of the equation, if s⟶1-s, then

$$\frac{1}{s(s-1)} = \frac{1}{(1-s)(1-s-1)} = \frac{1}{(1-s)(-s)}\,\frac{1}{(s-1)s} ,$$

$$x^{\frac{s}{2}-1}+x^{-\frac{1+s}{2}} = x^{\frac{1-s}{2}-1}+x^{-\frac{1+(1-s)}{2}} = x^{\frac{-1-s}{2}}+x^{-\frac{2-s}{2}} = x^{-\frac{1+s}{2}}+x^{\frac{s}{2}-1}, \text{so}$$

$\prod\left(\frac{s}{2}-1\right)x^{-\frac{s}{2}}\zeta(s)$ is invariant under the transformation s⟶1-s.

Riemann then derived the function equation for $\zeta(s)$ again, which is simpler than the previous derivation using the circum-channel integral and residue theorems.

If we introduce auxiliary function function $\Phi(s) = \prod\left(\frac{s}{2}-1\right)\pi^{-\frac{s}{2}}\zeta(s)$.

This can be succinctly written as Φ(s) =Φ(1– s), But it is more convenient to add the factor s(s –1) to Φ(s), which is what Riemann does next, i.e. (To keep with Riemann's notation, the number factor $\frac{1}{2}$ is introduced): $\zeta(s) = \frac{1}{2}s(s-1)\prod\left(\frac{s}{2}-1\right)\pi^{-\frac{s}{2}}\Gamma\left(\frac{s}{2}\right)\zeta(s)$.

Because factor (s –1) cancels out the pole of ζ(s) at s=1, factor s cancels out the pole of $\Gamma(\frac{s}{2})$ at s = 0, and ζ(s)'s trivial zeros -2, -4, -6,... cancel out the rest of the poles of $\Gamma(\frac{s}{2})$, so ζ(s) is an integral function and is zero only at the nonnormal zero points of ζ(s). Note that since sub s(s –1) obviously does not change under s →1– s, there is a function equation ξ(s) = ξ(1– s). The zeros of ζ(s) are all zeros of ξ(s) except the trivial zero s=-2n(n is a natural number), which, since it happens to be the pole of $\Gamma(\frac{s}{2}+1)$ in $\xi(s) = \Gamma(\frac{s}{2}+1)(s-1)\pi^{-\frac{s}{2}}\zeta(s)$, is not the zero of ξ(s), and thus the zeros of ξ(s) coincide with the nontrivial zeros of the Riemann ζ function. In other words, ξ(s) separates the nontrivial zeros of the Riemann ζ(s) function from the total zeros.

Now Riemann suppose $s=\frac{1}{2}+ti\ (t \in \mathbb{C} \text{ and } t \neq 0)$, $\prod\left(\frac{s}{2}\right)(s\ -1)\,\pi^{-\frac{s}{2}}\zeta(s) = \Gamma(\frac{s}{2}+1)\ (s\ -1)\pi^{-\frac{s}{2}}\zeta(s)=\xi(t)$, thus get

$$\xi(t) =\frac{1}{2}-(t^2+\frac{1}{4})\int_1^{\infty} \psi(x)\, x^{-\frac{3}{4}}\cos\left(\frac{1}{2}t\ln x\right)dx$$

Or

$$\xi(t) = 4\int_1^{\infty}\frac{d(x^{\frac{3}{2}}\psi'(x))}{dx}x^{-\frac{1}{4}}\cos(\frac{1}{2}t\ln x)dx.$$

The function $\prod\left(\frac{s}{2}\right)(s-1)\pi^{-\frac{s}{2}}\zeta(s)=\xi(t)$ defined by Riemann is essentially the same as the function $\xi(s)=\frac{1}{2}s(s-1)\prod\left(\frac{s}{2}-1\right)\pi^{-\frac{s}{2}}\Gamma(\frac{s}{2})\zeta(s)$ commonly used today. Because

$\prod\left(\frac{s}{2}\right)=\Gamma(\frac{s}{2}+1)=\frac{s}{2}\Gamma(\frac{s}{2})$, so $\prod\left(\frac{s}{2}\right)(s-1)\pi^{-\frac{s}{2}}\zeta(s)=\frac{s}{2}\Gamma(\frac{s}{2})(s-1)\pi^{-\frac{s}{2}}\zeta(s)=\frac{1}{2}s(s\ -1)\pi^{-\frac{s}{2}}\Gamma(\frac{s}{2})\zeta(s)=\xi(s)$。

The only difference is that Riemann takes t as the independent variable, while ζ(s), which is now commonly used, still takes s as the independent variable, and s and t differ by a linear transformation: $s=\frac{1}{2}+ti$, that's a 90 degree rotation plus a translation of $\frac{1}{2}$.In this way, the line $Re(s)=\frac{1}{2}$ in the complex plane of s corresponds to the real axis in the t plane, and the real part of the zeros of the Riemann ζ(s)(s∈C and s≠1) function on the critical line $Re(s)=\frac{1}{2}$ corresponds to the real root of ξ(t). Note that in Riemann's notation, the functional equation ξ(s) = ξ(1− s) becomes ξ(t) = ξ(−t), that is, ξ(t) is an even function, so its power series expansion is only an even power, and the zeros are symmetrically distributed with respect to t = 0.In addition, it is also clear from the above two integral representations that ξ(t) is an even function, since $\cos(\frac{1}{2}t\ln x)$ is an even function of t.

For all finite t, function $\xi(t)=\frac{1}{2}-(t^2+\frac{1}{4})\int_1^{\infty}\psi(x)\ x^{-\frac{3}{4}}\cos(\frac{1}{2}t\ln x)dx$ or function $\xi(t) = 4\int_1^{\infty}\frac{d(x^{\frac{3}{2}}\psi'(x))}{dx}x^{-\frac{1}{4}}\cos(\frac{1}{2}t\ln x)dx$ is finite in value,

And can be expanded to a power of t^2 as a rapidly convergent series, because for an s value with a real part greater than 1, the value of $\ln\zeta(s) = -\sum\ln(1-p^{-s})$ is also finite.It is same true for the logarithm of the other factors of ξ(t), so the function ξ(t) can take zero only if the imaginary part of t lies between $\frac{1}{2}$ and $-\frac{1}{2}$i. That is, A can take a zero value only if the real part of s lies between 0 and 1. The number of roots of the real part of the equation ξ(t) between 0 and T is approximately equal to $N(T)=\frac{T}{2\pi}\ln\frac{T}{2\pi}-\frac{T}{2\pi}+O(\ln T)$, approximately to $(\frac{T}{2\pi}\ln\frac{T}{2\pi}-\frac{T}{2\pi})$(this result of Riemann's estimate of the number of zeros was not strictly proved until 1859 by Mangoldt).This is because the value of the integral $\int d\ln\xi(t)$ (after omitting small quantities of order $\frac{1}{T}$) approximately equal to $(T\ln\frac{T}{2\pi}-T)i$. The value of this integral is equal to the number of roots of the equation in this region multiplied by 2πi(this is the application of the amplitude Angle principle).In fact, Riemann found that the number of real roots in this region is approximately equal to this number, and it is highly likely that all the roots are real. Riemann naturally hoped for a rigorous proof of this, but after some hasty and unsuccessful initial attempts, Riemann temporarily set aside the search for proof because it was not necessary for the purposes of Riemann's subsequent studies. What Riemann wrote down is the famous Riemann conjecture, the most famous conjecture in mathematics!

According to Riemann's assumption in the paper : $s=\frac{1}{2}+ti(t \in C \text{ and } t \neq 0)$, then the Riemann conjecture is equivalent to that for $\zeta(s)=0$, its complex roots s (except for negative even numbers) must all be complex numbers satisfying only $s=\frac{1}{2}+ti\ (t \in R \text{ and } t \neq 0)$ and $s=\frac{1}{2}-ti(t \in R$ and $t\neq0$, and they all lie on the critical boundary of the vertical real number axis satisfying $Re(s)=\frac{1}{2}$. These complex roots s (except negative even numbers) are called nontrivial zeros of Riemannn $\zeta(s)(\, s \in C \text{ and } s \neq 1 \text{ and } s \neq -2n, n \in R^{+}\,)$ functions.

Let's call the prime counting function $\pi(x)(x \in R^{+})$, the name of this function has nothing to do with PI. According to the prime number theorem, $\pi(x) \approx \frac{x}{\ln x}(x \in R^{+})$.The number of primes less than or equal to 1 is 1, the number of primes other than 1 is 0,so $\pi(1)=0$.The primes less than or equal to 2 are 1 and 2, the number of primes other than 1 is 1,so $\pi(2)=1$, The primes less than or equal to 3 are 1, 2, 3, and the number of primes other than 1 is 2 ,so $\pi(3)=2$.The primes less than or equal to 4 are 1, 2, 3, and the number of primes other than 1 is 2,so $\pi(4)=2$.The primes less than or equal to 5 are 1, 2, 3,5, and the number of primes other than 1 is 3,so $\pi(5)=3$.So $\pi(6)=3$, $\pi(7)=4$, $\pi(11)=5$, $\pi(13)=6$, ... , and so on. If we get a simple expression to calculate the prime number counting function, it will lead to amazing results, which will have great significance for the theory and application of mathematical distribution and the development of the mathematical discipline.

Riemann improved the prime counting function, and the prime counting function Riemann obtained was called $J(x)(x \in R^{+})$. The relationship between $J(x)(x \in Z^{+})$ and $\pi(x) \approx \frac{x}{\ln x}(x \in Z^{+})$ is as follows:

$$\pi(x)=\sum_{n=1}^{\infty}\frac{\mu(n)}{n}J\left(x^{\frac{1}{n}}\right)=J(x)-\frac{1}{2}J\left(x^{\frac{1}{2}}\right)-\frac{1}{3}J\left(x^{\frac{1}{3}}\right)-\frac{1}{5}J\left(x^{\frac{1}{5}}\right)+\frac{1}{6}J(x^{\frac{1}{6}})-\ldots$$

$(x \in Z^{+}, n \in Z^{+})$,

The relationship between $J(x)(x \in R^{+})$ and $\zeta(s)(s \in C \text{ and } s \neq 1)$ is as follows:

$$\frac{1}{s}\ln\zeta(s)=\int_{0}^{\infty}J(x)x^{-s-1}dx \ ,$$

$\mu(n)$ is called the Mobius function.

The Mobius function $\mu(n)$ has only three values, which are 0 and plus or minus 1, if n is ok Divisible by the square of any prime number, that is, an exponent of one or more prime factors other than 1 in the prime factorization of n. If the power is raised to the second or higher power, then $\mu(n)=0$. If n is not divisible by the square of any prime number, that is to say, the exponent of any prime factor other than 1 in the prime factorization of n has the degree 1, then let's count the number of prime factors. If there are an even number of prime factors, then $\mu(n)=1$. If the number of prime factors is odd, then $\mu(n)=-1$. This also includes the case of n=1, since 1 has no prime factors other than 1, then the number of prime factors of 1 other than 1 is 0, and 0 counts as an even number, so μ(1)=1. In the above expansion, as $n(n \in R^+)$ increases, $\frac{1}{n}(n \in Z^+)$ becomes smaller and smaller, $x^{\frac{1}{n}}(n \in Z^+)$ also gets smaller and smaller, The $n(n \in Z+$ and $n \to +\infty$th term is going to get smaller and smaller. It shows that the largest contribution to the value of π(x) is the first term J(x).

Now let's look at the following formula from Riemann:

$J(x) = \mathrm{Li}(x)\text{-}\sum_\rho \mathrm{Li}(x^\rho)+\int_x^{+\infty} \frac{dt}{t^2(t^2-1)\ln t} - \ln 2\ (x \in Z^+),$

among， $\mathrm{Li}(x)=\int_0^x \frac{dt}{\ln t}(x \in Z^+)$, $J(x)$ can also be described as:

$J(x) = \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} \frac{\ln\zeta(Z)}{Z} x^Z dz,$

$J(x)$ is called a step function, it equals zero where x equals zero, that is, J(0)=0, and then as the value of x increases, every time it passes through a prime number (such as 2,3,5,...). The value of $J(x)$ increases by 1. Every time it square a prime number (4,9,25), the value of $J(x)$ increases by $\frac{1}{2}$. Every time it pass through the third square of a prime number (such as 8,9,25,...) The value of $J(x)$ increases by 1/3. Every time it pass 4 squares of a prime number (say, 16,81,256,625,...) , the value of $J(x)$ increases by 1/4. And so on,every time it passes a prime number to $x^n\left(n \in Z+\ ,\ n \to +\infty, x\right.$ is a prime number, the value of $J(x)$ increases 1nn∈Z+and n→+∞ .You can think of it as that every time it passes a prime number to $x^n\left(n \in R^+\ ,\ n \to +\infty, x \text{ is a prime number}\right)$, J(x) increases $\frac{1}{n}(n \in Z^+ \text{and } n \to +\infty)$. Obviously, this function is closely related to the distribution of prime numbers. If you look at the right-hand side of the equation, the first term is called the logarithmic integral function $\mathrm{Li}(x) = \int_0^x \frac{dt}{\ln t}(x \in Z^+)$, When x is sufficiently large, $\mathrm{Li}(x) \approx \frac{x}{\ln x}(x \in Z^+), \pi(x) \approx \mathrm{Li}(x) \approx \frac{x}{\ln x}(x \in Z^+, x \text{ is sufficiently large})$.Let's look at the second item $\mathrm{Li}(x^\rho)\left(x \in Z^+\ ,\ \rho \in C\right)$, ρ is a complex number other than a negative even number, ρ is called the nontrivial zero of the $\zeta(s)(n \in Z^+ \text{ and } s \neq 1 \text{ and } s \neq -2n\ )$function by Riemann. ρ is denoted as: $\rho = \sigma + it\left(\sigma \in R, t \in R\right)$. On the real number line, the Riemann $\zeta(s)(s \in C, \text{and } s \neq 1 \text{ and } s \neq -2n, n \in Z^+\ )$ function has no zeros except for negative even numbers, So ρ is definitely not a real number other than a negative even number, so $x^\rho(\rho \in C, x \in Z^+, \text{and } \rho \neq 1 \text{ and } \rho \neq -2n, n \in Z^+\ )$ is definitely not a real number other than a negative even numberas also. So how do we compute $\mathrm{Li}(x^\rho)\left(x \in R+\ ,\ \rho \in C, \text{and } \rho \neq 1 \text{ and } \rho \neq -2n, n \in Z+\right.$? Just extend the domain resolution of $\mathrm{Li}(x)=\int_0^x \frac{dt}{\ln t}(x \in R^+)$ to all complex numbers except divided by 1.Riemann proved that the non-trivial zero ρ of the Riemann $\zeta(\rho)(\rho \in C \text{ and } s \neq 1 \text{ and } \rho \neq -2n, n \in Z^+)$function must satisfy 0≤Re(ρ)≤1. The vertical strip of width 1 on the complex plane is called the critical strip. and the line perpendicular to the real number axis satisfying Re(s)= $\frac{1}{2}$($s \in C$ and $s \neq 1$ and s≠−2n ,n∈Z+ is called the critical boundary, that is, the center line of the critical band. Riemann guessed that the non-trivial zeros of the Riemann $\zeta(s)(s \in C$ and $s \neq 1$ and $s \neq -2n$,n∈Z+ function all lie on the critical boundary, which is a very surprising conclusion. If the real part of the nontrivial zero of the Riemann $\zeta(s)(s \in C$ and $s \neq 1$ and $s \neq -2n, n \in$ Z+ function takes random values between 0 and 1, then the probability that it reaches exactly $\frac{1}{2}$ should equal 0, which Riemann thought was 100%. If the Riemann conjecture is strictly true, then the occurrence of prime numbers or the distribution of prime numbers is not random at all, but occurs in a definite way, and there must be a deep reason behind this. The proof of the prime number theorem is an intermediate product in the process of studying Riemann conjecture. In 1896, Hadamar and De la Vabsan proved that the nontrivial zero ρ of the Riemannn $\zeta(\rho)(\rho \in C$ and $\rho \neq 1$ and $\rho \neq -2n, n \in Z^+$)function has no zero when Re(ρ)=0 and Re(ρ)=1, thus easily proving the prime number theorem $\pi(x) \approx \frac{x}{\ln x}(x \in Z^+)$.

proving the prime number theorem $\pi(x) \approx \frac{x}{\ln x}(x \in Z^+)$.

The prime number theorem $\pi(x) \approx \frac{x}{\ln x}(x \in Z^+)$ holds, showing that for the prime counting function $\pi(x)$, the largest part of its value comes from the logarithmic integral function Li(x)=$\int_0^x \frac{dt}{\ln t}(x \in R^+)$ while the minor part of its value comes from Li(x^ρ) $\Big(x \in$ Z+ , ρ∈C and s≠1 and ρ≠−2n ,n∈Z+,since the calculation of $x\ln xx$∈Z+ is simple, but for the accurate calculation of the prime counting function $\pi(x)$, the calculation of the non-trivial zero ρ of the Riemann $\zeta(\rho)(\rho \in C$ and $s \neq 1$ and $\rho \neq -2n, n \in Z^+$) function is very important, and the strict proof of the Riemann conjecture is very important. In 1921, the British mathematician Hardy proved that the Riemann $\zeta(s)(s \in C$ and $s \neq 1$ and $s \neq -2n, n \in$ Z+ function has infinitely many nontrivial zeros on the critical boundary. But this conclusion is actually quite different from the Riemann conjecture, because the fact that there are infinitely many nontrivial zeros on the critical boundary does not mean that all zeros are on the critical boundary. Just as a line segment has an infinite number of points, but a line segment has an infinite number of lines, the percentage of Hardy's proof is almost zero compared to the number of all nontrivial zeros. It wasn't until 1942 that mathematicians pushed this percentage significantly higher than zero. That year, the Norwegian mathematician Selberg proved that the percentage was greater than zero, but did not give a specific value. In 1974, the American mathematician Liesen proved that at least 34% of nontrivial zeros lie on the critical boundary. In 1980, Chinese mathematicians Lou Shituo and Yao Qi proved that 35% of nontrivial zeros lie on the critical boundary. In 1989, the American mathematician Conrey proved that 40% of nontrivial zeros are located on the critical boundary. The calculation of the nontrivial zeros of the Riemann $\zeta(s)(s \in C$ and $s \neq 1$ and $s \neq -2n, n \in Z^+$) function is more complicated. Graham calculated the first 15 nontrivial zeros of the Riemann ζ(s) function, As shown below (six of them are listed, including the modern value to its right) :

1	1/2+14. 134 725i	1/2+14. 134 725 1i
2	1/2+21. 022 040i	1/2+21. 022 039 6i
3	1/2+25. 010 856i	1/2+25. 010 857 5i
4	1/2+30. 424 878i	1/2+30. 424 876 1i
5	1/2+32. 935 057i	1/2+32. 935 061 5i
6	1/2+37. 586 176i	1/2+37. 586 178 1i

and after 25 years, another 138 nontrivial zeros were calculated. Since then, the calculation of the nontrivial zeros of the Riemann ζ(s) function has stalled because of the clumsy methods and the lack of computers to assist it. After the calculation was halted for seven years, the deadlock was broken, and German mathematician Siegel found in Riemann's manuscript that Riemann was far ahead of the time 70 years of clever algorithm, so that the calculation of non-trivial zero points was suddenly bright. In honor of Siegel, this algorithm formula is also known as the Riemann-Siegel formula, and Siegel himself won the Fields Medal for it.

A mathematician's manuscript is worth far more than an antique. Since then, the non-trivial zeros of the Riemann ζ(s) function have been computed much faster. Hardy's students pushed the calculation of the non-trivial zeros of the Riemann ζ(s) function to 1041, the father of artificial intelligence Alan Turing pushed the calculation of the non-trivial zeros of the Riemann ζ(s) function to 11,041, and later with the application of computers, the calculation of the non-trivial zeros of the Riemann ζ(s) function from 3.5 million to 300 million, 1.5 billion. 850 billion, and now 10 trillion, These nontrivial zeros are located on what Riemann calls the critical boundary. But the ten trillion zeros on the critical boundary is nothing compared to an infinite number of zeros on the critical boundary, and no matter how large the number of zeros on the critical boundary is calculated, it is not enough to prove that the Riemann conjecture is correct. The correctness of the Riemann conjecture requires rigorous theoretical proof. People guess that the non-trivial zero of Riemann $\zeta(s)(s \in C \; and \; s \neq 1 \; and \; s \neq -2n, n \in R^{+})$ function is symmetric with respect to the real number axis based on the ten trillion zeros located on the critical boundary, but the guess is still a guess, which needs strict proof, otherwise such a guess has no meaning. In the following paper, I give a strict proof of this conjecture, and give a strict proof of Riemann conjecture, which is indeed true.

Equation for Euler ζ(s) function, $\zeta(s) = \prod_{p=1}^{\infty}(\frac{1}{1-p^{-s}}) = \sum_{n=1}^{\infty}\frac{1}{n^s}(s \in R \text{ and } s \neq 1)$and

$\zeta(s) = \prod_{p=1}^{\infty}(\frac{1}{1-p^{-s}}) = \sum_{n=1}^{\infty}\frac{1}{n^s}(s \in C \;,\; Re(s) > 1 \; and \; s \neq 1)$,they evolve into the Riemann ζ(s) function equations: $\zeta(s) = \prod_{p=1}^{\infty}(\frac{1}{1-p^{-s}}) = \sum_{n=1}^{\infty}\frac{1}{n^s}(s \in C \text{ and } s \neq 1)$, so I'm going to use Euler's formula,First of all, there are: $e^{ix} = \cos(x) + i\sin(x)(x \in R)$ and $e^{iZ} = \cos(Z) + i\sin(Z)(Z \in C)$, the exponents in the power operation of the trigonometric expression of complex numbers are extended from positive integers to general real numbers. Riemann conjecture is equivalent to $\zeta(s)=\zeta(\overline{s})=0(s \in C \text{ and } s \neq 1)$ and $\zeta(1-s=\zeta s=0(s \in C$ and $s \neq 1)$ were established.$\zeta 1-s=\zeta s=0$ can be given by $\zeta(s)=2^s\pi^{s-1}\sin(\frac{\pi s}{2})\Gamma(1-s)\,\zeta(1-s)(s \in C \text{ and } s \neq 1)$ when $\zeta(s)=0(s \in C \text{ and } s \neq 1)$, $\zeta(s)=\zeta(\overline{s})=0$ can be surrounded by $\zeta(s)=\overline{\zeta(\bar{s})}$ when $\zeta(s)=0$ ($s\in C$ and $s\neq 1$). $\zeta(s)=\overline{\zeta(\bar{s})}$ must be rigorously proved by using Euler's formula $e^{ix}=\cos(x)+i\sin(x)(x\in R)$ and

$e^{iZ}=\cos(Z)+i\sin(Z)(Z\in C)$, and by generalizing the exponents in the power operation in the trigonometric expressions of complex numbers from positive integers to general real numbers. If you want to solve the Riemann conjecture, its proof must follow such principles and methods, otherwise it may not be correct.The prime number theorem $\pi(x)\approx\frac{x}{\ln x}(x\in Z^{+})$ was independently proved by Hadamard and dela Valee Poussin in 1896. But one expects a prime number theorem with a precise error term. Under RH, it can be shown that $\pi(x)=\mathrm{Li}(x)+O(\sqrt{x}\ln x)$. Conversely, RH can also be derived from this formula. Therefore, this formula can be seen as the arithmetic equivalent of RH. This shows the extreme importance of RH. Riemann's paper also included several propositions that had not been rigorously proved. All except RH were proved by Hadamard and Mangoldt, leaving only what is now known as RH. Ordering N(T) to represent the number of zeros of $\zeta(s)$ in the rectangle $0\le\sigma\le1, 0<t<T$, Riemann made the following conjecture: $N(T)\sim\frac{T}{2\pi}\ln\frac{T}{2\pi}-\frac{T}{2\pi}$, this result has been proved by Mangolt. Hypothesis N(T) represents the number of zeros of $\zeta(s)$ on the line segment $\sigma=\frac{1}{2}$, $0<t<T$.Selberg proved that if there are normal numbers c and T, then $N_0(T)>cN(T)$. Theresult is quite striking. It shows that the number of zeros of $\zeta(s)$ on the line segment $\sigma=\frac{1}{2}$, $0<t<T$, has a positive density compared to its number on the rectangle $0\le\sigma\le 1$, $0<t<T$, and the two-dimensional measure of the line segment is zero. The Riemann ζ function and RH are both "prototypes", and there are many similarities and generalizations of $\zeta(s)$ and RH. These analogies and generalizations have a strong mathematical background, there are many RH generalizations of some kind, and their mathematical background is extremely important. For example, the plane algebraic curve on a finite field F corresponds to RH, that is, every algebraic curve satisfying certain conditions corresponds to an L function, and their zeros are located on the line $\sigma=\frac{1}{2}$. This proposition has been proved by Weil, who also conjecture RH of a higher dimensional algebraic variety.This conjecture was proved by Deligne. These are undoubtedly some of the greatest mathematical achievements of the 20th century. As far as I know, the results of Weil and Deligne gave a great boost to analytic number theory. For example, the RH proved by Weil can derive the best order estimate of the Kloosterman sum of the modular prime p with the complete triangular sum.
Here is the equation of the Riemann $\zeta(s)$ function:

For Euler $\zeta(s)$ function equation：

$\zeta(s)=\prod_{p=1}^{\infty}(\frac{1}{1-p^{-s}})=\sum_{n=1}^{\infty}\frac{1}{n^{s}}(s\in R \text{ and } s\neq 1)$and

$\zeta(s)=\prod_{p=1}^{\infty}(\frac{1}{1-p^{-s}})=\sum_{n=1}^{\infty}\frac{1}{n^{s}}(s\in C,\ Re(s)>1 \text{ and } s\neq 1)$ evolve into the Riemann $\zeta(s)$ function equation $\zeta(s)=\prod_{p=1}^{\infty}(\frac{1}{1-p^{-s}})=\sum_{n=1}^{\infty}\frac{1}{n^{s}}(s\in C \text{ and } s\neq 1)$, So we use Euler's formula $e^{ix}=\cos(x)+i\sin(x)(x\in R)$ and $e^{iZ}=\cos(Z)+i\sin(Z)(Z\in C)$, The exponents in the poweroperation of the trigonometric expression of complex numbers are generalized from positive integers to general real numbers, and thus the Euler series a and b are generalized. Then we extend the domain analysis of Euler series $\zeta(s) = \prod_{p=1}^{\infty}(\frac{1}{1-p^{-s}}) = \sum_{n=1}^{\infty}\frac{1}{n^s}(s\in \mathrm{R} \text{ and } s \neq 1)$ and $\zeta(s) = \prod_{p=1}^{\infty}(\frac{1}{1-p^{-s}}) = \sum_{n=1}^{\infty}\frac{1}{n^s}(s\in \mathrm{C}\ ,\ \mathrm{Re}(s) > 1 \ and\ s \neq 1)$ to the whole complex plane, so that it resolves everywhere except s=1, and the resulting ζ functionis equivalent to Riemann's ζ function.Riemann guess is equivalent to $\zeta(s) = \zeta(\overline{s}) = 0$ ($s \in \mathrm{C}$ and $s\neq1$ and $\zeta1-s=\zeta s=0(s\in C$ and $s\neq1)$ were established. $\zeta1-s=\zeta s=0$ can be made by $\zeta(s) = 2^s\pi^{s-1}\sin(\frac{\pi s}{2})\Gamma(1-s)\ \zeta(1-s)\ (s\in \mathrm{C}\ \ and\ \ s\neq 1)$export, $\zeta(s) = \zeta(\overline{s}) = 0$ can be obtained when ζ(s)=0 is given by $\zeta(s)=\overline{\zeta(\overline{s})}$, In order to get the $\zeta(s)=\overline{\zeta(\overline{s})}$, must use euler's formula $e^{ix} = \cos(x)+i\sin(x)(x \in R)$ and $e^{iZ} = \cos(Z)+i\sin(Z)(Z\in C)$,the exponent of the power operation in the trigonometric expression of complex number is extended from positive integer to general real number. If you want to solve the Riemann conjecture, its proof must follow such principles and methods, otherwise it may not be correct.

Let's see how $\prod(\frac{1}{1-p^{-s}})=\sum\frac{1}{n^s}$ is obtained. When Euler first came up with this formula, it was clear that both sides of the formula were series, and Euler discovered that there was this

series.This is a formula of Euler, in which n is a natural number and p is a prime number. Euler has already proved it, and I will repeat it below. If you are familiar with Euler's formulas and know exactly that they are correct, you can omit them.

Turn this Euler formula around and get:

$$\sum\frac{1}{n^s} = \prod(\frac{1}{1-p^{-s}}),$$

When Euler first proposed this formula, s only represented a positive integer more than 1. Obviously, both sides of this formula are series. Euler found that there is such a series:

$$\sum\frac{1}{n^s}=1+\frac{1}{2^s}+\frac{1}{3^s}+\frac{1}{4^s}+\frac{1}{5^s}+\frac{1}{6^s}+\ldots\text{(equation 1)}.$$

The above equation is multiplied by $\frac{1}{2^s}$ on both sides, $\frac{1}{2^s}$ on the left and $\frac{1}{2^s}$ on the right. we can get:

$$\frac{1}{2^s}\sum\frac{1}{n^s}=\frac{1}{2^s}+\frac{1}{4^s}+\frac{1}{6^s}+\frac{1}{8^s}+\frac{1}{10^s}+\frac{1}{12^s}+\ldots\text{(equation 2)}.$$

By subtracting the left and right sides of the two equations (equation 1) and (equation 2), the following results can be obtained:

$$\left(1-\frac{1}{2^s}\right)\sum\frac{1}{n^s} = 1+\frac{1}{3^s}+\frac{1}{5^s}+\frac{1}{7^s}+\frac{1}{9^s}+\frac{1}{11^s}+\frac{1}{13^s}+\frac{1}{15^s}+\ldots\text{ (equation 3)}$$

It can be observed that the product term on the left side increases by $\left(1-\frac{1}{2^s}\right)$ as the left term of equation 3 relative to equation 1. When the items on the right side of equation 1 are multiplied by $\frac{1}{2^s}$, the items whose denominator is even are eliminated, and the remaining items are regarded as the items on the right side of equation 3.

By multiplying the left and right sides of equation 3 by $\frac{1}{3^s}$, we can get:

$$\frac{1}{3^s}(1-\frac{1}{2^s})\sum\frac{1}{n^s} = \frac{1}{3^s} + \frac{1}{9^s} + \frac{1}{15^s} + \frac{1}{21^s} + \frac{1}{27^s} + \frac{1}{33^s} + \frac{1}{39^s} + \frac{1}{45^s} \ldots \text{(equation 4)}$$

By subtracting the left and right sides of the two equations (equation 3) and (equation 4), we can get:

$$(1-\frac{1}{3^s})(1-\frac{1}{2^s})\sum\frac{1}{n^s} = 1+\frac{1}{5^s} + \frac{1}{7^s} + \frac{1}{11^s} + \frac{1}{13^s} + \frac{1}{17^s} + \frac{1}{19^s} + \frac{1}{23^s} + \frac{1}{25^s} + \frac{1}{29^s} + \frac{1}{31^s} + \ldots \text{(equation 5)}$$

Similarly, multiply the left and right sides of equation 5 by $\frac{1}{5^s}$,we can get:

$$(\frac{1}{5^s})(1-\frac{1}{3^s})(1-\frac{1}{2^s})\sum\frac{1}{n^s} = \frac{1}{5^s} + \frac{1}{25^s} + \frac{1}{35^s} + \frac{1}{55^s} + \frac{1}{65^s} + \frac{1}{85^s} + \frac{1}{95^s} + \frac{1}{115^s} + \frac{1}{145^s} + \ldots$$

(equation 6)

By subtracting the left and right sides of the two equations (equation 5) and (equation 6), the following results can be:

$$(1-\frac{1}{5^s})(1-\frac{1}{3^s})(1-\frac{1}{2^s})\sum\frac{1}{n^s} = 1+\frac{1}{7^s} + \frac{1}{11^s} + \frac{1}{13^s} + \frac{1}{17^s} + \frac{1}{19^s} + \frac{1}{23^s} + \frac{1}{29^s} + \frac{1}{31^s} + \frac{1}{37^s} + \ldots \text{(equation 7)}$$

Referring to this method, in equation (2k-1) (k is a positive integer),we multiply the items on the left by $\frac{1}{p_1{}^s}$ and the items on the right by $\frac{1}{p_i{}^s}$(i is a positive integer).

p_i is the nearest prime number of the prime number p_{i-1} in the first item $\left(1-\frac{1}{p_{i-1}{}^s}\right)$ on the left side of equation (2k-1) . The "nearest prime" here refers to the one closest to p_{i-1}.There is no other prime between them, and $p_i > p_{i-1}$, equation (2k-1) add $\frac{1}{p_1{}^s}$ to the left. equation (2k-1) the right side becomes: item 1 is $\frac{1}{p_i{}^s}$, item 2 is $\frac{1}{p_i{}^s}\times\frac{1}{p_i{}^s}$, item 3 is $\frac{1}{p_i{}^s}\times\frac{1}{p_{i+1}{}^s}$, item 4 is $\frac{1}{p_i{}^s}\times\frac{1}{p_{i+2}{}^s}$, item 5 is $\frac{1}{p_i{}^s}\times\frac{1}{p_{i+3}{}^s}$, … , $\frac{1}{p_i{}^s}\times\frac{1}{p_{(i+k)}{}^s}$, … ,k is a positive integer. So go on and add them up, where p_1、p_2、p_3、 … , p_{i+1}、p_{i+2}、p_{i+3}、p_{i+4}、 … , p_{i+k} …,It is an infinite sequence of prime numbers arranged in the order of numerical size from small to large, and $p_3 = 5, p_2 = 3, p_1 = 2$.In this way, we get the expression on the right side of equation (2k-1) and mark the whole equation as equation (2k) . By The coefficient of $\sum\frac{1}{n^s}(n \in Z^+)$. on its left side is a continuous product of some forms such as $(1-\frac{1}{p_i{}^s})$. n is a natural number and p takes all prime numbers. In order to write conveniently, the symbol is introduced and the left side is written as:

referring to this method and doing it over and over again, we will eventually get such an equation:

On the right is 1, plus a score: $\frac{1}{p_i{}^s \times p_{i+k}}$.The values of $p_i{}^s$ and p_{i+k} are two infinite prime numbers, so the value of is zero, which can be omitted. So, the right side is 1.So you can get it:

$$\sum\frac{1}{n^s}=\frac{1}{\prod(1-\frac{1}{p^s})}=\prod\frac{1}{(1-\frac{1}{p^s})}=\prod\frac{1}{1-p^{-s}}$$

Riemann extends Euler's definition of positive integer s analytic to complex number, that is, the variable s is defined as complex number. And we use a function ζ(s) constructed by Euler himself to record the two series on both sides of the above equation：

$$\zeta(s)=\sum\frac{1}{n^s}=\prod\frac{1}{1-p^{-s}}\ .$$

Secondly, there is another Euler formula:$e^{ix}=\cos(x)+i\sin(x)$, x is a real number, representing the radian of an angle.This formula has been proved by Euler and can be used directly. Let me prove it again in my own way:

If we have a function $f_1(x)$=e^x, we derive $f_1(x)$=e^x(x ∈R), " ' " means derivative, then $(e^x)'=e^x$, the derivative of e^x is itself. So if we make the independent variable cx(c is constant)of function $f_1(x)$=e^x, we will get function $f_1(cx)=e^{cx}$, and derivative of function $[f_1(x)]'=(e^{cx})'=ce^{cx}$,then $[f_1(x)]'=(e^{cx})'=ce^{cx}$,If the function $f_1(cx)=e^{cx}$,c=i(i is also constant), then $f_1(ix)=e^{ix}$, then $[f_1(ix)]'=[e^{ix}]'=ie^{ix}$. Suppose that $f_2(x)=\cos(x)+i\sin(x)$=s, then s is a complex number. Now the derivative of function $f_2(x)$ is obtained:

$[f_2(x)]'=[\cos(x)+i\sin(x)]'=[\cos(x)]'+[i\sin(x)]'=-\sin x+i\cos x$ (equation 1),If $f_1(ix)=e^{ix}=\cos x+i\sin x$ is correct, then suppose that $e^{ix}=\cos(x)+i\sin(x)$ is correct based on the above $[f_1(x)]'=[e^{i}]'=ie^{ix}$,$[f_1(ix)]'=[e^{ix}]'=ie^{ix}$(equation2), replacing e^{ix} with $\cos x+i\sin x$, then: $[f_1(ix)]'=[e^{ix}]'=ie^{ix}=i(\cos x+i\sin x)=-\sin x+i\cos x$(equation 2),By comparing (equation 1) and (equation 2), it can be found that the derivatives of $f_1(ix)$ and $f_2(x)$ are equal, and since both $f_1(ix)$ and $f_2(x)$ have no constant terms, the expressions of $f_1(ix)$ and $f_2(x)$ should be consistent. We found $f_1(ix)=e^{ix}=\cos x+i\sin x=f_2(x)$,The expressions of $f_1(ix)$ and $f_2(x)$ are exactly the same,which shows that the Euler's formula $e^{ix}=\cos(x)+i\sin(x)(x\in R)$is correct.

prove $e^{ix}=\cos(x)+i\sin(x)(x\in R)$, a better method is the following, but more complex.Everyone First of all, look at the function y =e^x. If we find the derivative of this function, we will get y'= $(e^x)'$= e^x. That is to say, the derivative of y = e^x is itself. This is a very special exponential function.Let y'=$\frac{dy}{dx}$,when $\frac{dy}{dx}=0$, then y=e^x ,when $\frac{dy}{dx}=1$,then y=$e^x=1+x$,when $\frac{dy}{dx}=1+x$, y=$e^x=1+x+\frac{1}{2}x^2$,when $\frac{dy}{dx}=1+x+\frac{1}{2}x^2$,then y=$e^x=1+x+\frac{1}{2}x^2+\frac{1}{6}x^3$,when $\frac{dy}{dx}=e^x=1+x+\frac{1}{2}x^2+\frac{1}{6}x^3$,then y= $e^x=1+x+\frac{1}{2}x^2+\frac{1}{6}x^3+\frac{1}{24}x^4$,when $\frac{dy}{dx}=e^x=1+x+\frac{1}{2}x^2+\frac{1}{6}x^3+\frac{1}{24}x^4$,then y=$e^x=1+x+\frac{1}{2}x^2+\frac{1}{6}x^3+\frac{1}{24}x^4+\frac{1}{120}x^5$,by analogy, this is a preliminary proof : y=$e^x=1+x+\frac{1}{2}x^2+\frac{1}{6}x^3+\frac{1}{24}x^4+\frac{1}{120}x^5$,+... , But what about the series y=$x^n(n\in Z^+)$. in general? What about the series of y =e^x When x is treated as e and n as x, y =e^x is obtained, which requires the introduction of the concept of power series.

This is the introduction of the concept of power series:1+ x +x^2+x^3+x^4+x^5+...(x ∈R),Every item is a power in the form of $x^n(n\in Z^+)$. Let function f(x)= 1+ x

$+x^2+x^3+x^4+x^5+\ldots(x \in R)$,Equivalent to the sum of the items, if some numbers are used as the coefficients of the items, if these numbers are $a_0, a_1, a_2, a_3, a_4, a_5, \ldots, a_{i-1}, a_i, \ldots$, They are derivatives of order 0 $f^{(0)}(x)$ of the function f(x) =$x^n(n \in Z^+)$, the derivatives of order 1 $f^{(1)}(0)$ of the function f(x) =$x^n(n \in Z^+)$,the derivatives of order 2 $f^{(2)}(0)$ of the function f(x)=$x^n(n \in$

$Z+$, the derivatives of order 3 $f(3)(0)$ of the function f(x) =$xnn \in Z+$, ... ,the derivatives of order

n $f^{(n)}(0)$ of the function f(x) =$x^n(n \in Z^+)$.They are: $a_0 = f^{(0)}(0), a_1 = f^{(1)}(0), a_2 = f^{(2)}(0), a_3 = f^{(3)}(0), \ldots, a_{i-1} = f^{(i-1)}(0)$,

$a_i = f^{(i)}(0), \ldots$, If f(x) = $x^n(n \in Z^+)$ is taken as n times derivative, we will

get: $f^{(n)}(0)$=n(n-1)(n-2)(n-3)...2×1× 0^0,so that $f^{(n)}(0)$=n! , For a paivarticular function f (x)=e^x,

the

values of all these derivatives at x =0: $f^{(0)}(0), f^{(1)}(0), f^{(2)}(0), f^{(3)}(0), \ldots, f^{(n-1)}(0), f^{(n)}(0)$, ...,they must be 1, because the derivative of any order of e^x is itself. But the value of derivatives of order x^n $at\ x = 0$ are: $f^{(n)}(0)$=n(n-1)(n-2)(n-3)...2×1× 0^0=n!,therefore $a_0, a_1, a_2, a_3, a_4, a_5, \ldots, a_{i-1}, a_i, \ldots$,have to divide one by n!, can make:

$f^{(0)}(0)=1, f^{(1)}(0)=1, f^{(2)}(0)=1$, $f^{(3)}(0)=1$, ... , $f^{(n-1)}(0)=1$, $f^{(n)}(0)=1$, In order to satisfy the

coefficients of the series expression of function f(x)=e^x

correctly: $a_0, a_1, a_2, a_3, a_4, a_5, \ldots, a_{i-1}, a_i, \ldots$,Namely: $a_0=\frac{1}{0!}=1, a_1=\frac{1}{1!}, a_2=\frac{1}{2!}, a_3=\frac{1}{3!}, a_4=\frac{1}{4!}, \ldots, a_{n-1}$

$=\frac{1}{(n-1)!}, a_n=\frac{1}{(n)!}, \ldots,$

For a particular function f(x) =e^x, the method here is to multiply the n power of x by the values of the derivative functions of the function $x^n(n \in Z^+)$ at the independent variable x= 0, and then divide by the factorial of n.

So for a particular function f(x) =e^x, $a_0=\frac{1}{0!}=1, a_1=\frac{1}{1!}, a_2=\frac{1}{2!}, a_3=\frac{1}{3!}, a_4=\frac{1}{4!}, \ldots,$

$a_{n-1}=\frac{1}{(n-1)!}, a_n=\frac{1}{(n)!}$, ... ,So you can write the series of the function f(x)= e^x again: e^x==

$1+x+\frac{1}{2}x^2+\frac{1}{6}x^3+\frac{1}{24}x^4+\frac{1}{120}x^5+\ldots+\frac{1}{(n-1)!}x^{n-1}+\frac{1}{n!}x^{n-1}+\ldots,$

Let's assume f(x) = cos(x) to find the power series of cos(x). The 0-th derivative of function f (x) = cos (x) is $f^{(0)}(x)$=cos(x)(the 0-th of a function is itself).The 1-th derivative of function f(x)=cos(x) is $f^{(1)}$=-sin(x),the 2-th derivative of function f(x)=cos(x) is $f^{(2)}(x)$=-cos(x), the 3-th derivative of functionf(x)=cos(x) is $f^{(3)}(x)$=sin(x), the 4 − th derivative of function f(x) = cos(x) is $f^{(4)}(x) = -\sin(x)$, the n-th derivative of function f(x) =cos(x) is $f^{(n)}(x)$=..., If x=0 is substituted, the value of the derivative function of each order at 0 will be obtained. Because the series is derived by dividing the value of the derivative function at the independent variable x= 0 by the factorial of N and multiplying by the expansion of $x^n(n \in Z^+)$. Therefore, at x= 0, it is easy to get the value of each derivative function at x = 0 by assigning the independent variable of each derivative function to

zero: $f^{(0)}$ (0)(0)=cos(0)=1, $f^{(1)}$ (0)=-sin(0)=0, $f^{(2)}$ (0)=-cos(0)=-1, $f^{(3)}$ (0)=sin(0)=0 ,

$f^{(4)}(0)$=cos(0)=1,$f^{(5)}(0)=-\sin(0) = 0, f^{(6)}(0) = -\cos(0) = -1, f^{(7)}(0)$=sin(0)=0,...,according to 1, 0, - 1, 0, 1, 0, - 1, 0,... In the form of 1,0, -1,0, the cycle section goes on indefinitely. The function value of the derivative function of order f(x) = cos(x) at 0 of its independent variable can be used to construct the coefficients needed for the power series of cos(x). They are divided by the factorial of n, which is the coefficients of the powers of x. Now we can construct the power series of cos(x) by referring to the power series of e^x above, n is the order of the derivative function of order f(x)=cos(x), and is also the n-th power of x. So the power series of cos(x) expansion is:It starts with $\frac{f^{(0)}(0)}{0!}x^0=\frac{\cos(0)}{0!}x^0=\frac{0}{0!}\times 0=1$ as the zero term,the constant term.

Next is: $\frac{f^{(1)}(0)}{1!}x^1=\frac{-\sin(0)}{1!}x^1=\frac{0}{1!}\times x=0$, The result is zero, which means that there is no 1-th term, or that there is no first order term of x.

Next is: $\frac{f^{(2)}(0)}{2!}x^2=\frac{-\cos(0)}{2!}x^2=\frac{-1}{2!}\times x^2=-\frac{1}{2}x^2$, which means that there is no 2-th term.

Next is: $\frac{f^{(3)}(0)}{3!}x^3=\frac{\sin(0)}{3!}x^2=\frac{0}{3!}\times x^3=0$,The result is zero, which means that there is no 3-th term, or that there is no 3-th power term of x.

Next is: $\frac{f^{(4)}(0)}{4!}x^4=\frac{\cos(0)}{4!}x^4=\frac{1}{4!}x^4$,which means that there is no 4-th term.

… , If we go on doing this, we will find that n-order derivative of f(x)=cos(x) , n is a nonnegative positive number. Starting from zero, if n is an even number, then the value of $f^{(n)}(0)$ is either + 1 or - 1, according to 1, - 1, 1, - 1, 1, - 1,... The regular arrangement of, So for the power series expansion of cos(x), the sign of the value of the coefficients in front of the even power term of x is as follows: +, -, +, -, +, -, -, -,... regularly arranged.The coefficients are:$\frac{f^{(n)}(0)}{n!}=\frac{1}{n!}$ or $\frac{f^{(n)}(0)}{n!}=-\frac{1}{n!}$,If n is an odd number, the value of its coefficient is: $\frac{f^{(n)}(0)}{n!}=0$,So for the expansion of power series of cos(x), there is no odd term of x. So the power series of the function f(x) = cos(x) is:

$\cos(x)=\frac{1}{0!}x^0-\frac{1}{2!}x^2+\frac{1}{4!}x^4-\frac{1}{6!}x^6+\frac{1}{8!}x^8-\frac{1}{10!}x^{10}+\ldots=1-\frac{1}{2}x^2+\frac{1}{4!}x^4-\frac{1}{6!}x^6+\frac{1}{8!}x^8-\frac{1}{10!}x^{10}+\ldots$

Let's assume f(x)=sin(x) to find the power series of sin(x). The 0-th derivative of function f(x)=sin(x) is $f^{(0)}(x)$=sin(x)(the 0-th derivative of a function is itself),The 1-th derivative of function f(x)=sin(x) is $f^{(1)}(x)$=cos(x),The 2-th derivative of function f(x)=sin(x) is $f^{(2)}(x)$=-sin(x), The 3-th derivative of function f(x)=sin(x) is $f^{(3)}(x)$=-cos(x), The 4-th derivative of function f(x)=sin(x) is $f^{(4)}(x)$= sin(x)The n-th derivative of function f(x) =cos(x) *is* $f^{(n)}(x)$=..., If x= 0 is substituted, the value of the derivative function of each order at 0 will be obtained. Because the series is derived by dividing the value of the derivative function at the independent variable x=0 by the factorial of N and multiplying by the expansion of $x^n(n \in Z^+)$. Therefore, at x= 0, it is easy to get the value of each derivative function at x= 0 by assigning the independent variable of each derivative function to zero: $f^{(0)}(0)=\sin(0)=0$, $f^{(1)}(0)=\cos(0)=1$, $f^{(2)}(0)=-\sin(0)=0$, $f^{(3)}(0)=-\cos(0)=-1$, $f^{(4)}(0)=\sin(0)=0$, $f^{(5)}(0)=\cos(0)=1$,f60=−sin(0)=0,f(7)(0)=cos(0) = -1,... According to 0, 1, -0, -1, 0, 1, 0, -1,... In the form of 0,1, 0,-1, the cycle section goes on indefinitely. The function value of the derivative function of order f(x) = sin(x) at 0 of its independent variable can be used to construct the coefficients needed for the power series of sin(x). They are divided by the factorial of n, which is the coefficients of the powers of x. Now we can construct the power series of sin(x) by referring to the power series of e^x above, n is the order of the derivative function of order f(x) =sin(x), and is also the n-th power of x. So the power series of sin(x) expansion is:

It starts with $\frac{f^{(0)}(0)}{0!}x^0=\frac{\sin(0)}{0!}x^0=\frac{0}{0!}\times 1=0$ as the zero term, the constant term,

Next is: $\frac{f^{(1)}(0)}{1!}x^1=\frac{\cos(0)}{1!}x^1=\frac{1}{1!}\times x=x$, as 1-th term,

Next is: $\frac{f^{(2)}(0)}{2!}x^2=\frac{-\sin(0)}{2!}x^2=\frac{0}{2!}\times x^2=0$,which means that there is no 2-th term, Next is: $\frac{f^{(3)}(0)}{3!}x^2 = \frac{-\cos(0)}{3!}x^3 = \frac{-1}{3!}\times x^3 = -\frac{1}{3!}x^3$,as 3-th term,

Next is: $\frac{f^{(4)}(0)}{4!}x^2 = \frac{\sin(0)}{4!}x^4 = \frac{0}{4!}\times x^4 = 0$,which means that there is no 4-th term.

... , If we go on doing this, we will find that n-order derivative of f(x)=sin(x) , n is not a nonnegative positive number. Starting from zero, If n is an odd number, then the value of $f^{(n)}(0)$ is either + 1 or - 1, according to 1, 0, 1, - 1, 1, - 1, - 1, - 1,... Regular arrangement, if n is an even number, then the value of $f^{(n)}(0)$ is either + 1 or - 1, according to 0 ,1 ,0 ,-1 ,0 ,1 ,0 ,-1 ,... , the regular arrangement of, so for the power series expansion of sin(x), the sign of the value of the coefficients in front of the odd power term of f(x) is as follows: +, -, +, -, +, -, -, -,... regularly arranged.Th e coefficients are: $\frac{f^{(n)}(0)}{n!} = \frac{1}{n!}$ or $\frac{f^{(n)}(0)}{n!} = -\frac{1}{n!}$. If n is an even number, the value of its coefficient is: $\frac{f^{(n)}(0)}{n!} = 0$,So for the expansion of power series of sin(x), there is no even term of x. So the power series of the function f(x)=sin(x) is:

$\sin(x) == \frac{1}{1!}x^1 - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \frac{1}{9!}x^9 - ... = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \frac{1}{9!}x^9 - ...,$

Previously obtained

$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \frac{1}{5!}x^5 + ... + \frac{1}{n!}x^n = 1 + x + \frac{1}{3}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + ... + \frac{1}{n!}x^n (x \in R)$

If we change x to ix, We can get:

$e^{ix} = 1 + ix + \frac{1}{2!}(ix)^2 + \frac{1}{3!}(ix)^3 + \frac{1}{4!}(ix)^4 + \frac{1}{5!}(ix)^5 + ... + \frac{1}{n!}(ix)^n = (1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \frac{1}{8!}x^8 - - \frac{1}{10!}x^{10} + ...) + (x - \frac{1}{3!}x^3 + \frac{1}{5}x^5 - \frac{1}{7!}x^4 + \frac{1}{9!}x^9 - \cdots)(x \in R),$

because $\cos(x) = (1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \frac{1}{8!}x^8 - \frac{1}{10!}x^{10} + ...)$,$\sin(x) = \left(x - \frac{1}{3!}x^3 + \frac{1}{5}x^5 - \frac{1}{7!}x^7 + \frac{1}{9!}x^9 - ...\right.$, therefore ex= cos(x)+isin(x)(x∈R),So this is another Eulerian formula.

In the formula above, if x equals pi, we will get: $e^{i\pi}$=cos(π)+isin(π)=-1+0=-1 , therefore $e^{i\pi}+1=0$,

It's also called Euler's formula. It puts all the most important things in mathematics, 0, 1, e, i and pi, into one formula. It is a special case of Euler formula e^{ix}= cos(x)+isin(x)(x ∈R).when Z∈C,then e^{iZ}= cos(Z)+isin(Z)(Z∈C).

Conclusion Reasoning

$\sum_{n=1}^{\infty} n^{-s} = \prod_p (1 - p^{-s})^{-1}$(s ∈ Z^+ and s≠ 1, n∈ Z^+ and n goes through all the positive integers, p ∈ Z^+ and p takes all the prime numbers),this formula was proposed and proved by the Swiss mathematician Leonhard Euler in 1737 in a paper entitled "Some Observations on Infinite Series", Euler's product formula connects a summation expression for natural numbers with a continuative product expression for prime numbers, and contains important information about the distribution of prime numbers. This information was finally deciphered by Riemann after a long gap of 122 years, which led to Riemann's famous paper "On the number of primes less than a Given Value [1]. In honor of Riemann, the left end of the Euler product formula was named after Riemann, and the notation ζ(s)(s ∈ C and s ≠ 1) used by Riemann was adopted as the Riemann zeta function .

Because $e=\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=\sum_{n=0}^{\infty}\frac{1}{n!}\approx 2.7182818284\ldots$, e is a natural constant, I use " × " for Multiplication, then based on euler's $e^{ix}=\cos(x)+i\sin(x)(x\in R)$ and the principle of amplitude Angle,get $(e^{3i})^2=(\cos(3)+i\sin(3))^2=\cos(2\times3)+i\sin(2\times3)=\cos(6)+i\sin(6)$,

because $e^{6i}=\cos(6)+i\sin(6)$,

so

$(e^{3i})^2=e^{6i}$,

In general, $(e^{bi})^c=e^{b\times ci}(b\in R, c\in R)$ is established,the angle principle is extended to the case where the exponent is a real number.

so when $x>0(x\in R)$,suppose $e^y=(e=2.7182818284\ldots, x$ e is a natural constant,$x\in R$ and $x>0$, $y\in R$),then $y=\ln(x)(x>0)$,based on euler's $e^{ix}=\cos(x)+i\sin(x)(x\in R)$,will get $e^{yi}=e^{\ln(x)i}=\cos(\ln x)+i\sin(\ln x)(x\in R$ and $x>0)$.

Suppose $t\in R$ and $t\neq 0$, now let's figure out expression for $x^{ti}(x\in R$ and $x>0$, $t\in R$ and $t\neq 0)$ is $x^{ti}=(e^y)^{ti}=(e^{yi})^t=(\cos(\ln x)+i\sin(\ln x))^t(x>0)$.

Suppose s is any complex number, and Suppose $s=\sigma+ti(\sigma\in R, t\in R, s\in C$ and $s\neq 1)$,then let's find the expression of $x^s(x\in R$ and $x>0$, $s\in C)$,

You can put $s=\sigma+ti(\sigma\in R, t\in R, s\in C$,and $s\neq 1)$ and $x^{ti}=(e^y)^{ti}=(e^{yi})^t=(\cos(\ln x)+i\sin(\ln x))^t(x>0)$ into $x^s(x>0)$ and you will get $x^s=x^{(\sigma+ti)}=x^\sigma x^{ti}=x^\sigma(\cos(\ln x)+i\sin(\ln x))^t=x^\sigma(\cos(t\ln x)+i\sin(t\ln x))(x>0)$, if You put $s=\sigma-ti(\sigma\in R, t\in R)$ and $x^{ti}=(e^y)^{ti}=(e^{yi})^t=(\cos(\ln x)+i\sin(\ln x))^t(x>0)$ into x^s,you will get $x^{\bar{s}}=x^{(\sigma-ti)}=x^\sigma(x^{ti})^{-1}=x^\sigma(\cos(\ln x)+i\sin(\ln x))^{-t}=x(\cos(t\ln x)-i\sin(t\ln x))(x>0)$.

Then

$$\zeta(s)=\sum_{n=1}^{\infty}\frac{1}{n^s}=\sum_{n=1}^{\infty}\frac{1}{n^s}=\sum_{n=1}^{\infty}\frac{1}{n^{\sigma+ti}}=\sum_{n=1}^{\infty}\left(\frac{1}{n^\sigma}\times\frac{1}{n^{ti}}\right)=\sum_{n=1}^{\infty}(n^{-\sigma})\frac{1}{(\cos(\ln(n))+i\sin(\ln(n)))^t}$$

$$=\sum_{n=1}^{\infty}(n^{-\sigma}(\cos(\ln(n))+i\sin(\ln(n)))^{-t})$$

$$=\sum_{n=1}^{\infty}(n^{-\sigma}(\cos(t\ln(n))-i\sin(t\ln(n)))$$

($s\in C$ and $s\neq 1$, $n\in Z^+$ and n goes through all the positive integers),or

$$\zeta(s)=\prod_{p=1}^{\infty}\left(\frac{1}{1-p^{-s}}\right)=\prod_{p=1}^{\infty}(1-p^{-s})^{-1}=\prod_{p=1}^{\infty}(1-p^{-\sigma-ti})^{-1}=\prod_{p=1}^{\infty}\left(1-\frac{1}{p^{\sigma+ti}}\right)^{-1}=\prod_{p=1}^{\infty}\left[1-(p^{-\sigma})\frac{1}{(\cos(\ln p)+i\sin(\ln p))^t}\right]^{-1}=\prod_{p=1}^{\infty}[1-(p^{-\sigma})(\cos(t\ln p)-i\sin(t\ln p))]^{-1}$$

($s\in C$ and $s\neq 1$, $p\in Z^+$ and p goes through all the prime numbers).

And

$$\zeta(\bar{s})=\sum_{n=1}^{\infty}\frac{1}{n^{\bar{s}}}=\sum_{n=1}^{\infty}\frac{1}{n^{\bar{s}}}=\sum_{n=1}^{\infty}\frac{1}{n^{\sigma-ti}}=\sum_{n=1}^{\infty}\left(\frac{1}{n^\sigma}\times\frac{1}{n^{-ti}}\right)=\sum_{n=1}^{\infty}(n^{-\sigma})\frac{1}{\left(\cos\left(\ln(n)\right)+i\sin\left(\ln(n)\right)\right)^{-t}}$$

$$=\sum_{n=1}^{\infty}(n^{-\sigma}(\cos(\ln(n))+i\sin(\ln(n)))^t)$$

$$=\sum_{n=1}^{\infty}(n^{-\sigma}(\cos(t\ln(n))+i\sin(t\ln(n)))$$

(s ∈ C and s≠ 1, n∈ Z^+ and n goes through all the positive integers),

or

$\zeta(\bar{s}) = \prod_{p=1}^{\infty}(\frac{1}{1-p^{-\bar{s}}}) = \prod_{p=1}^{\infty}(1-p^{-\bar{s}})^{-1} = \prod_{p=1}^{\infty}(1-p^{-\sigma+ti})^{-1} = \prod_{p=1}^{\infty}(1-\frac{1}{p^{\sigma-ti}})^{-1} =$

$$\prod_{p=1}^{\infty}[1-(p^{-\sigma})\frac{1}{(\cos(lnp)-isin(lnp))^{t}}]^{-1} = \prod_{p=1}^{\infty}[1-(p^{-\sigma})(\cos(tlnp)+isin(tlnp))]^{-1}$$

(s ∈ C and s≠ 1, p ∈ Z^+ and p goes through all the prime numbers).

And

$\zeta(1-s) = \sum_{n=1}^{\infty}\frac{1}{n^{1-s}} = \sum_{n=1}^{\infty}\frac{1}{n^{1-\sigma-ti}} = \sum_{n=1}^{\infty}(n^{\sigma-1})\frac{1}{(\cos(\ln(n))+isin(\ln(n)))^{-t}} =$

$\sum_{n=1}^{\infty}(n^{\sigma-1})(\cos(\ln(n))+isin(\ln(n)))^{t}) = \sum_{n=1}^{\infty}(n^{\sigma-1})(\cos(tln(n))+isin(tln(n)))$

(s ∈ C and s≠ 1, n∈ Z^+ and n goes through all the positive integers),

or

if k∈ R, then

$\zeta(k-s) = \sum_{n=1}^{\infty}\frac{1}{n^{k-s}} = \sum_{n=1}^{\infty}\frac{1}{n^{k-\sigma-ti}} = \sum_{n=1}^{\infty}(n^{\sigma-k})\frac{1}{(\cos(\ln(n))+isin(\ln(n)))^{-t}} =$

$\sum_{n=1}^{\infty}(n^{\sigma-k})(\cos(\ln(n))+isin(\ln(n)))^{t}) = \sum_{n=1}^{\infty}(n^{\sigma-k})(\cos(tln(n))+isin(tln(n)))$

(s ∈ C and s≠ 1, k∈ R, n∈ Z^+ and n goes through all the positive integers),

and

$\zeta(k-s) = \prod_{p=1}^{\infty}(\frac{1}{1-p^{-k+s}})$

$= \prod_{p=1}^{\infty}(1-p^{s-k})^{-1} = \prod_{p=1}^{\infty}(1-p^{\sigma-k+ti})^{-1} = \prod_{p=1}^{\infty}[1-(p^{\sigma-k})(\cos(tlnp)+isin(tlnp))]^{-1}$

(s ∈ C and s≠ 1,k ∈ R, p ∈ Z^+ and p goes through all the prime numbers).

So

$X=n^{-\sigma}(\cos(tln(n))-isin(tln(n)))$,

$Y=n^{-\sigma}(\cos(tln(n))+isin(tln(n)))$,

$G=[1-(p^{-\sigma})(\cos(tlnp)-isin(tlnp))]^{-1}$,

$H=[1-(p^{-\sigma})(\cos(tlnp)+isin(tlnp))]^{-1}$,

X and Y are complex conjugates of each other, that is

$X=\bar{Y}$, and G and H are complex conjugates of each other, that is

$G=\bar{H}$, so $\zeta(s)=\sum_{n=1}^{\infty}\frac{1}{n^s} = \sum_{n=1}^{\infty}X = \prod_{p=1}^{\infty}G$ (s ∈ C and s ≠ 1), and $\zeta(\bar{s})=\sum_{n=1}^{\infty}\frac{1}{n^{\bar{s}}} = \sum_{n=1}^{\infty}Y =$

$\prod_{p=1}^{\infty}H$ (s ∈ C and s ≠ 1),so $\zeta(s)=\overline{\zeta(\bar{s})}$(s ∈ C and s ≠ 1),

and only when $\sigma=\frac{1}{2}$ then $\zeta(1-s)=\zeta(\bar{s})$(s ∈ C and s ≠ 1),

and only when $\sigma=\frac{k}{2}$(k ∈ R), then $\zeta(k-s)=\zeta(\bar{s})$(s ∈ C and s ≠ 1, k ∈ R),so

only k=1 then $\zeta(1-s)=\zeta(\bar{s})=\zeta(k-s)$(s ∈ C and s ≠ 1, k ∈ R),

only k=1(k ∈ R)is true , and when $\zeta(s)=0$, then

$\zeta(1-s)=\zeta(k-s)=\zeta(\bar{s})=\zeta(s)=0$(s ∈ C and s ≠ 1, k ∈ R).

Because

$$GRH(s,\chi(n)) = L(s,\chi(n)) = \sum_{n=1}^{\infty}\frac{\chi(n)}{n^s} = \chi(n)\sum_{n=1}^{\infty}\frac{1}{n^s} = \chi(n)\sum_{n=1}^{\infty}\frac{1}{n^{\sigma+ti}}$$

$$= \chi(n)\sum_{n=1}^{\infty}\left(\frac{1}{n^{\sigma}}\frac{1}{n^{ti}}\right) =$$

$\chi(n)\sum_{n=1}^{\infty}(n^{-\sigma})\frac{1}{(\cos(\ln((n))+i\sin(\ln(n)))^{t}} =$

$\chi(n)\sum_{n=1}^{\infty}(n^{-\sigma}(\cos(\ln(n)) + i\sin(\ln(n)))^{-t}) = \chi(n)\sum_{n=1}^{\infty} n^{-\sigma}(\cos(t\ln(n)) -$ $i\sin t\ln(n)$)(t∈C and t≠0,s∈C and s≠1,n∈Z+ ,and n goes through all positive integers),because ζ(s)=$2^s\pi^{s-1}\mathrm{Sin}(\frac{\pi s}{2})\Gamma(1\text{-}s)\zeta(1\text{-}s)$(s∈C and s≠ 1)(Formula 7) .s=-2n(n∈ Z^+) is the trivial zero of the ζ(s)function, so s=-2n(n∈ Z^+) is the trivial zero of the Landau-Siegel function L(β, ✗(n))(β∈R, ✗(n) ∈R and ✗(n)≠0,n∈ Z^+ and n traverses all positive integers). So when the dirichlet characteristic function ✗(n)≡1, then s = -2n(n∈ Z^+) is the zero of landau - siegel function L(β, ✗(n)) (β∈R and ✗(n) = 1) . So
if s=β(β ∈R) and β=-2n(s∈C), then L(β, ✗(n))=0 and ζ(s)=0.

So L(β, ✗(n))=

$\chi(n)\sum_{n=1}^{\infty}(n^{-\beta}(\cos(0\times\ln(n)) + i\sin(0\times\ln(n))) = \chi(n)\sum_{n=1}^{\infty}(n^{-\beta}) =$

$\left(\chi(1)1^{-\beta} - \chi(2)2^{-\beta} + \chi(3)3^{-\beta} - \chi(4)4^{-\beta} + \cdots\right)\left(\chi(n) \in R, \beta \in R \text{ and } \beta \neq -2n, n \in\right.$ Z+ ,"×" is the symbol for multiplication, because the real exponential function of the real number has a function value greater than zero, so

$n^{-\beta} > 0$($n \in Z^+$ and n traverses all positive integers) and $1^{\beta} - 2^{\beta} < 0, 3^{\beta} - 4^{\beta} < 0, 5^{\beta} - 6^{\beta} < 0, \ldots, (n-1)^{\beta} - (n)^{\beta} < 0, \ldots, or\ 1^{\beta} - 2^{\beta} > 0, 3^{\beta} - 4^{\beta} > 0, 5^{\beta} - 6^{\beta} > 0, \ldots, (n-1)^{\beta} - (n)^{\beta} > 0$, it can be known that if ✗(n) ≠0($n \in Z^+$ and n traverses all positive integers) and β∈R and $\beta \neq -2n(n \in Z^+)$, then L(β, ✗(n))≠0(β∈R and $\beta \neq -2n, n \in Z^+$, ✗(n) ∈R and n traverses all positive integers) and L(β, 1)≠0(β∈R and $\beta \neq -2n, n \in Z^+$, and n traverses all positive integers), so for Riemann ζ(s)(s∈C and s≠ 1, $s \neq -2n, n \in Z^+$) functions, its corresponding landau-siegel function L(β, ✗ (n))(β ∈R and $\beta \neq -2n, n \in Z^+$, ✗(n)∈R and n traverses all positive integers) of pure real zero does not exist, this means that the Riemann ζ(s)(s∈C and s≠ 1and $s \neq -2n, n \in Z^+$) function does not have a zero of a pure real variable s, the generalized Riemannian L(s, ✗(n))(s∈C and s≠1,and s≠-2n, n ∈ Z^+, ✗(n)∈R and n traverses all positive integers) function has no real zeros,and the generalized Riemann L(s, ✗ (n))=0(s∈C and s≠ 1, and $s \neq -2n, n \in Z^+$, ✗ (n)∈R and n traverses all positive integers) satisfies s=$\frac{1}{2}$+ti(t∈R,t≠0) and s=$\frac{1}{2}$-ti(t∈R,t≠0) is sufficient to prove that the twin primes, Polignac's conjecture and Goldbach's conjecture are almost true. And if ✗(n) = 0($n \in Z^+$ and n traverses all positive integers) or β∈R and $\beta = -2n(n \in Z^+)$, then L(β, ✗(n))=0(β∈R and $\beta \neq -2n, n \in Z^+$, ✗(n)∈R and n traverses all positive integers) and L(β, 1)=0(β∈R and $\beta \neq -2n, n \in Z^+$, and n traverses all positive integers), so for Riemann ζ(s)(s∈C and $s \neq 1$) functions, its corresponding landau-siegel function L(β, ✗ (n))(β ∈R, ✗ (n)∈R , $n \in Z^+$ and n traverses all positive integers) ,the pure real zero exist, this means that the Riemann ζ(s)(s∈C and s≠ 1) function have a zero of a pure real variable s, and nontrivial zero of the Generalized Riemann L(β, ✗ (n)) = 0(β∈R and $\beta \neq -2n$, $n \in Z^+$, ✗ (n)∈R and n traverses all positive integers) fuction meet $s=\frac{1}{2}+ti$ (t∈R, t≠0) and $s=\frac{1}{2}-ti$(t∈R, t≠0), It shows that the twin prime number conjecture, Polignac conjecture and Goldbach conjecture are completely valid. According $\zeta(1-s)=2^{1-s}\pi^{-s}\cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)$($s \in C$ and $s \neq 1$)(Formula 7) obtained by Riemann,so when ζ(s)=0 then ζ(1-s)=ζ(s)=0(s∈C and s≠ 1).Beacause only when $\sigma=\frac{1}{2}$, the next three equations ζ(σ+ti)=0, ζ(1-σ-ti)=0, and ζ(σ-ti)=0 are all true,so only $s=\frac{1}{2}+ti$ (t∈R and t≠0) is true.$\zeta(s) = \sum_{n=1}^{\infty}\frac{1}{n^s} = \sum_{n=1}^{\infty}\frac{1}{n^{\sigma+ti}} = \sum_{n=1}^{\infty}(\frac{1}{n^{\sigma}} \times \frac{1}{n^{ti}}) = \sum_{n=1}^{\infty}(n^{-\sigma})\frac{1}{(\cos(\ln(n))+i\sin(\ln(n)))^t} = \sum_{n=1}^{\infty}(n^{-\sigma}(\cos(\ln(n)) + i\sin(\ln(n)))^{-t}) = \sum_{n=1}^{\infty}(n^{-\sigma}(\cos(t\ln(n)) - i\sin(t\ln(n))) = \prod_{p=1}^{\infty}(\frac{1}{1-p^{-s}}) = \prod_{p=1}^{\infty}(1-p^{-s})^{-1} = \prod_{p=1}^{\infty}(1-p^{-\rho-ti})^{-1} = \prod_{p=1}^{\infty}(1-\frac{1}{p^{\sigma+ti}})^{-1} = \prod_{p=1}^{\infty}[1-(p^{-\sigma})\frac{1}{(\cos(\ln p)+i\sin(\ln p))^t}]^{-1} = \prod_{p=1}^{\infty}[1-(p^{-\sigma})(\cos(t\ln p) - i\sin(t\ln p))]^{-1}$ ($s \in C$ and $s \neq 1, t \in C$ and $t \neq 0$, p is prime number , and $p \neq 1$).

When σ =1, then if $1-\frac{1}{p}\cos(t\ln p) + i\frac{1}{p}\sin(t\ln p) \neq 0$ then $\zeta(s) = \sum_{n=1}^{\infty}\frac{1}{n^s} = \prod_{p=1}^{\infty}(\frac{1}{1-p^{-s}}) \neq 0$ .if $1-\frac{1}{p}\cos(t\ln p) \neq 0$ and $\frac{1}{p}\sin(t\ln p) \neq 0$, then $\sin(t\ln p) \neq 0$ and $\frac{1}{p}\cos(t\ln p) \neq 1$,then $t \neq \frac{k\pi}{\ln p}$ ($k \in Z$, p is prime number , and $p \neq 1$) and $\cos(t\ln p) \neq p$($t \in R$ and $t \neq 1$) , so if $p > 1$ (p is prime number , and $p \neq 1$) then $t \neq \frac{k\pi}{\ln p}$($k \in Z$, p is prime number , and $p \neq 1$) and $\cos(t\ln p) \neq p$(p is prime and p>1), or $p = 1$, then $|t| \neq |\frac{k\pi}{\ln 1}| \neq +\infty$($k \in Z$ and $p = 1$) and $\cos(t\ln 1) = 1$, $t \in R$ and $t \neq 1$. So if σ =Re(s)=1 and $t \neq \frac{k\pi}{\ln p}$($k \in Z$, and $p \neq 1$) and and $t \in R$ and $t \neq 0$, then $\zeta(1+ti) = \prod_{p=1}^{\infty}[1-\frac{1}{p}\cos(t\ln p) + i\frac{1}{p}\sin(t\ln p)]^{-1} \neq 0$($s \in C$ and $s \neq 1$).When s=1+ti($t \in R$ and $t \neq 0$) then

$\zeta(1+ti) = \prod_{p=1}^{\infty}[1-\frac{1}{p}\cos(t\ln p) + i\frac{1}{p}\sin(t\ln p)]^{-1} \neq 0$($t \in C$ and $t \neq 0$). And when Re(s)=1 and p=1(p is prime number), then $\zeta(1+ti) = \prod_{p=1}^{\infty}[1-\cos(t\ln p) + i\sin(t\ln p)]^{-1} = \prod_{p=1}^{\infty}\frac{1}{1-(p^{-1})(\cos(t\ln p)-i\sin(t\ln p))} = \prod_{p=1}^{\infty}\frac{1}{1-(1^{-1})(\cos(t\ln 1)-i\sin(t\ln 1))} = \frac{1}{0} \to +\infty$($t \in C$ and $t \neq 0$) ,then $\zeta(1+ti) \to +\infty$(t ∈

C and $\mathrm{t} \neq 0$), diverges ,without zero ,so $\zeta(1+\mathrm{ti}) \neq 0(\mathrm{t} \in \mathrm{C}$ and $\mathrm{t} \neq 0)$. When σ=0 ,if $1-\cos(\mathrm{tlnp}) \neq 0$ and $\sin(\mathrm{tlnp}) \neq 0$, then $\mathrm{tlnp} \neq k\pi(k \in \mathrm{Z})$ and $\cos(\mathrm{tlnp}) \neq 1$,then $\mathrm{t} \neq \frac{k\pi}{\mathrm{lnp}}(k \in \mathrm{Z}$ and $\mathrm{p} \neq 1)$ and $\cos(\mathrm{tlnp}) \neq 1$, so *if* $\mathrm{p}>1$, *then* $\mathrm{t} \neq \frac{k\pi}{\mathrm{lnp}}(k \in \mathrm{Z}$ and $\mathrm{p} \neq 1)$ and $\cos(\mathrm{tlnp}) \neq 1(\mathrm{p} \neq 1)$,or $\mathrm{p}=1$,then $|\mathrm{t}| \neq \left|\frac{k\pi}{\ln 1}\right| \neq +\infty$ $(k \in \mathrm{Z}$ and $\mathrm{p}=1)$ and $|\mathrm{t}| \neq +\infty$, $\mathrm{t} \in \mathrm{R}$ and $\mathrm{t} \neq 0$, then $\zeta(0+\mathrm{ti})=\prod_{p=1}^{\infty}[1-\cos(\mathrm{tlnp})+i\sin(\mathrm{tlnp})]^{-1} \neq 0(\mathrm{t} \in \mathrm{R}$ and $\mathrm{t} \neq 0)$. So when Re(s)=0 and p≠1,then $\zeta(0+\mathrm{ti})=\prod_{p=1}^{\infty}[1-\cos(\mathrm{tlnp})+i\sin(\mathrm{tlnp})]^{-1} \neq 0$. And when σ=Re(s)=0 and p=1, then

$$\prod_{p=1}^{\infty}[1-(p^{-0})(\cos(\mathrm{tlnp})-i\sin(\mathrm{tlnp}))]^{-1}=\prod_{p=1}^{\infty}\frac{1}{1-(p^{-0})(\cos(\mathrm{tlnp})-i\sin(\mathrm{tlnp}))}=$$

$$\prod_{p=1}^{\infty}\frac{1}{1-(1^{-0})(\cos(\mathrm{tln1})-i\sin(\mathrm{tln1}))} = \frac{1}{0} \rightarrow +\infty$$,then $\zeta(0+\mathrm{ti}) \rightarrow +\infty(\mathrm{t} \in \mathrm{R}$ and $\mathrm{t} \neq 0)$, diverges, without zero. So $\zeta(0+\mathrm{ti}) \neq 0(\mathrm{t} \in \mathrm{R}$ and $\mathrm{t} \neq 0)$. It is a fact that the non-trivial zeros of the Riemann ζ(s) function (meaning zeros other than negative even numbers) exist, Riemann proved that the real part $\mathrm{Re}(s)(s \in \mathrm{C}$ and $s \neq 1)$ of the nontrivial zero s of the Riemann $\zeta(s)(s \in \mathrm{C}$ and $s \neq 1)$ function must satisfy Re(s)∈[0,1]. It is not easy to calculate the non-trivial zeros of the $\zeta(s)(s \in \mathrm{C}$ and $s \neq 1)$ function by hand, and Riemann calculated a dozen of them, all of which have a real part Re(s) equal to $\frac{1}{2}$, so the non-trivial zeros of the Riemann $\zeta(s)(s \in \mathrm{C}$ and $s \neq 1)$ function (meaning zeros other than negative even numbers) exist.,and the real part $\mathrm{Re}(s)(s \in \mathrm{C}$ and $s \neq 1)$ of the nontrivial zero s of the Riemann $\zeta(s)(s \in \mathrm{C}$ and $s \neq 1)$ function must satisfy Re(s)∈(0,1).When s=1+ti(t ∈ R and t ≠ 0), Rs(s)= σ=1,

then $\zeta(s)=\zeta(1+\mathrm{ti})=\prod_{p=1}^{\infty}(\frac{1}{1-p^{-s}})=\prod_{p=1}^{\infty}(1-p^{-s})^{-1}=\prod_{p=1}^{\infty}(1-p^{-1-ti})^{-1}=\prod_{p=1}^{\infty}[1-(p^{-1})\frac{1}{(\cos(\ln p)+i\sin(\ln p))^{t}}]^{-1}=\prod_{p=1}^{\infty}[1-(p^{-1})(\cos(\mathrm{tlnp})-i\sin(\mathrm{tlnp}))]^{-1}=\prod_{p=1}^{\infty}[1-\frac{1}{p}\cos(\mathrm{tlnp})+i\frac{1}{p}\sin(\mathrm{tlnp})]^{-1} = \prod_{p=1}^{\infty}\frac{1}{[1-\frac{1}{p}\cos(\mathrm{tlnp})]+i\frac{1}{p}\sin(\mathrm{tlnp})} \neq 0(s \in \mathrm{C}$ and $s \neq 1, \mathrm{t} \in \mathrm{C}$ and $\mathrm{t} \neq 0, p \in \mathrm{Z}^{+})$,When the independent variable s is extended from a positive integer to a general complex number, in the Euler product formula, the numerator of every product fraction factor is 1, and the denominator of every product fraction factor is a polynomial related to the natural logarithm function. When $p \in \mathrm{Z}^{+}$ and p traves *all prime numbers*, then ζ(1+ti) ≠0(t ∈ R and t ≠ 0), indicating that the number of primes not greater than x is finite. From the analytic extended Euler product formula, we can see that for positive integers not greater than x, every increase of a prime p will increase a fraction factor related to ln(p) in the Euler product formula, indicating that the probability that there is a prime p near x (that is, x=p) is about $\frac{1}{\ln(p)}$, that is $\frac{1}{\ln(x)}$. If we use $\pi(x)$ to represent the number of primes not greater than x, then for a positive integer p not greater than x, the probability that it is prime is approximately $\frac{\pi(x)}{x}$, then $\frac{\pi(x)}{x} \sim \frac{1}{\ln(x)}$, $\pi(x) \sim \frac{x}{\ln(x)}$, $\pi(x) \sim \frac{x}{\ln(x)}$ is the expression for the prime number theorem.

As Riemann said in his paper, n takes all the positive integers, so n=1,2,3... ,Let's just plug in all the positive integers to $\sum \frac{1}{n^s}$.

Obviously,

ζ(s)=ζ(σ+ti)= $\sum \frac{1}{n^s}$ =∑ X=[$1^{-\sigma}$cos(tln1)+ $2^{-\sigma}$cos(tln2)+ $3^{-\sigma}$cos(tln3)+ $4^{-\sigma}$cos(tln4)+...]-i[$1^{-\sigma}$sin(tln 1)+ $2^{-\sigma}$sin(tln2)+ $3^{-\sigma}$sin(tln3)+ $4^{-\sigma}$sin(tln4)+...]= U-Vi($s \in C$ and $s \neq 1, t \in C$ and $t \neq 0$),

U=[$1^{-\sigma}$cos(tln1)+ $2^{-\sigma}$cos(tln2)+ $3^{-\sigma}$cos(tln3)+ $4^{-\sigma}$cos(tln4)+...] ,

V=[$1^{-\sigma}$sin(tln1)+ $2^{-\sigma}$sin(tln2)+ $3^{-\sigma}$sin(tln3)+ $4^{-\sigma}$sin(tln4)+...] ,

then

ζ($\overline{s}$)=ζ(σ-ti)= $\sum \frac{1}{n^{\overline{s}}}$ =∑ Y=[$1^{-\sigma}$cos(tln1)+ $2^{-\sigma}$cos(tln2)+ $3^{-\sigma}$cos(tln3)+ $4^{-\sigma}$cos(tln4)+...]+i[$1^{-\sigma}$sin(tln 1)+ $2^{-\sigma}$sin(tln2)+ $3^{-\sigma}$sin(tln3)+ $4^{-\sigma}$sin(tln4)+ ...]= U+Vi($s \in C$ and $s \neq 1, t \in C$ and $t \neq 0$),

U=[$1^{-\sigma}$cos(tln1)+ $2^{-\sigma}$cos(tln2)+ $3^{-\sigma}$cos(tln3)+ $4^{-\sigma}$cos(tln4)+...] ,

V=[$1^{-\sigma}$sin(tln1)+ $2^{-\sigma}$sin(tln2)+ $3^{-\sigma}$sin(tln3)+ $4^{-\sigma}$sin(tln4)+...] ,

$\zeta(1-s) = \sum (x^{\sigma-1})(\cos(t\ln x) + i\sin(t\ln x))$ =[$1^{\sigma-1}\cos(t\ln 1)$+ $2^{\sigma-1}\cos(t\ln 2)$+ $3^{\sigma-1}\cos(t\ln 3)$+ $4^{\sigma-1}\cos(t\ln 4)$+...]+i[$1^{\sigma-1}\sin(t\ln 1) + 2^{\sigma-1}\sin(t\ln 2) + 3^{\sigma-1}\sin(t\ln 3) + 4^{\sigma-1}\sin(t\ln 4)$ +...]($s \in C$ and $s \neq 1, t \in R$ and $t \neq 0$),

so ζ(s)= $\overline{\zeta(\overline{s})}$($s = \sigma + ti, \sigma \in R, t \in R$ and $s \neq 1$) , and when ζ(s)=0 ($s = \sigma + ti, \sigma \in R, t \in R$ and s≠1, then ζ(s)= ζ(s)=0, it shows that the zeros of the Riemann ζ(s) function must be conjugate, then there must be ζ(s)= ζ($\overline{s}$)=0, indicating that the zeros of the Riemannian ζ(s) function must be conjugate , and in the critical band of Re(s)∈(0,1), there are no non-conjugate zeros.According ζ(s)= ζ($\overline{s}$)=0 ,if s=$\overline{s}$, then $s \in R$, because $s = -2n$($n \in Z^+$)make he function ζ(s)($s \in C$ and $s \neq 1$) has the value zero in $2\sin(\pi s)\Pi(s-1)\zeta(s) = i\int_0^\infty \frac{x^{s-1}dx}{x-1}$ and $\zeta(s) = 2^s\pi^{s-1}\sin\left(\frac{\pi s}{2}\right)\Gamma(1-s)\zeta(1-s)$($s \in C$ and $s \neq 1$)((Formula 7), so a negative even number can be the zero of Riemann ζ(s)(s∈C and s≠1). If $s \neq \overline{s}$, then s and $\overline{s}$ are not both real numbers but both imaginary numbers, t∈R and t≠0. And according to $\zeta(s) = 2^s\pi^{s-1}\sin\left(\frac{\pi s}{2}\right)\Gamma(1-$ sζ1− ss∈C *and* s≠1(equation 7), if the ζ(s) = 0 (s ∈ C and s≠1) was established, then ζ(1-s)=ζ(s)=0 ($s \in C$ and $s \neq 1$) must be true , because ζ(s)= $\overline{\zeta(\overline{s})}$ ($s = \sigma + ti, \sigma \in R, t \in R$ and s≠1 , so when ζ(s)=0s=σ+ti, σ∈R,t∈R and s≠1,then ζ(s)= ζ(s)=0s=σ+ti, σ∈R,t∈R and s≠1, so the two zeros s and 1-s of Riemann ζ(s)(s∈C and s≠1) must also be conjugate. If either of s and 1-s are real numbers other than negative even numbers, since s and 1-s are conjugate, then s=1-s, then s=$\frac{1}{2}$. Since $\sin\left(\frac{\pi s}{2}\right) = \sin\left(\frac{\pi}{2} \times \frac{1}{2}\right) = \sin\left(\frac{\pi}{4}\right) \neq 0$, and because ζ($\frac{1}{2}$) diverge, then neither s nor 1-s are zeros of Riemann ζ(s)(s∈C and s≠1), that is, Riemann ζ(s)(s∈C and s≠1) has no real zeros other than negative even numbers. If Re(s)=1, then Re(1-s)=0, then s and 1-s are not conjugate, if Re(s)=0, then Re(1-s)=1, then s and 1-s are not conjugate either, so Riemann ζ(s)(s∈C and s≠1) has no zeros with real parts of 1 or 0. If Re(s)>1, then Re(1-s)<0, then s and 1-s are not conjugate, or Re(s)<0, then Re(1-s)>1, then s and 1-s are not conjugate, so the real part of Riemann ζ(s)(s∈C and s≠1) zero s must be 0<Re(s)<1, that is, Re(s)∈(0,1), which shows that the prime number theorem holds. If s and 1-s are both real and imaginary, then s and 1-s are not conjugated, then s and 1-s cannot both be zeros of Riemann ζ(s)(s∈C and s≠1), so 1-s and s can only be both imaginary and conjugate, and s cannot be pure imaginary, because if s is pure imaginary, then 1-s and s are not conjugated. So ζ(s)(s∈C and s≠1) has no pure imaginary zero. And if Re(s)≠$\frac{1}{2}$, then Re(s)≠Re(1-s), then 1-s and s are not conjugate, so Re(s)≠$\frac{1}{2}$ cannot be true. So only 1-s=$\overline{s}$ is true, that is, only 1-σ-ti=σ-ti is true, so only σ=$\frac{1}{2}$, t∈R and t≠0, so the real part of the non-real zeros of Riemann ζ(s)(s∈C and s≠1) can only be , that is, only Re(s)=$\frac{1}{2}$ is true, Equivalent to $\xi(s) = 0\left(s = \frac{1}{2} + ti \text{ or } s = \frac{1}{2} - ti, t \in R \text{ and } t \neq 0, s \in C \text{ and } s \neq 1\right)$ or $\xi\left(\frac{1}{2} + \right.$ ti=0t∈R and t≠0 and ξ12−ti=0t∈R and t≠0. Therefore, in the critical band of Re(s) ∈(0,1), Re(s)≠ $\frac{1}{2}$ is impossible, and there is no zero whose real part is not equal to $\frac{1}{2}$, so the Riemann conjecture holds. The symmetries of zeros s and zeros 1-s are not sufficient to prove that the nontrivial zeros of the Riemannn ζ(s)(s∈C, s≠1) function are on the critical line, and zeros s and zeros 1-s are symmetric only about the point ($\frac{1}{2}$,i) on the critical line. The conjugacy of s and 1-s is the fundamental reason why the nontrivial zeros of Riemann ζ(s) (s∈C, s≠1) are all located on the critical line. Let me summarize the above:

According to ζ(1-s)=$2^{1-s}\pi^{-s}\cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)$(s∈C and s≠1) (Formula 6),so when ζ(s)=0, then ζ(s)=ζ(1-s)=0 is true. Because $\zeta(s) = \overline{\zeta(\overline{s})}$(s ∈ C and s ≠ 1), then when ζ(s) =0 or ζ($\overline{s}$)=0, then it must be true that ζ(s)=ζ($\overline{s}$)=0. So when Riemann ζ(s)=0, then s and 1-s must also be conjugate. From this we get s=$\frac{1}{2}$+ti(t∈ R and t ≠ 0), or s=$\frac{1}{2}$-ti(t∈ R and t≠ 0).According to the Euler product formula, when Re(s)>1, since every product factor in the Euler product formula is not equal to zero, so when Re(s)>1, Euler ζ function is equivalent to the Riemann ζ function,Since each of the product factors in the Euler product formula is not equal to zero, when Re(s)>1, ζ(s) is not equal to zero, so the positive even number 2n(n ∈ Z^+) can make sin($\frac{\pi s}{2}$) =0, but it is not the zero of Riemann ζ(s). For real numbers other than negative and positive even numbers, in addition to not making sin($\frac{\pi s}{2}$)=0, it must satisfy that s=1-s, then s=$\frac{1}{2}$, and function ζ($\frac{1}{2}$)

diverge, so real numbers other than negative even numbers are not zeros of Riemann-ζ(s). It is also true that ζ(s)= ζ(1-s)=0, then when Re(s)<0, then ζ(s) is not equal to zero. And because when ζ(s)=0, if Re(s) =0 or Re(s) =1, then s and 1-s are not conjugate, so Re(s) =0 or Re(s) =1, thenζ(s) has no zero.So in addition to negative even numbers, Riemann ζ(s) has zero points if the value of Re(s) is in the interval (0,1). It holds that ζ(s)= ζ(1-s)=0, and we know that the zero of ζ(s) is symmetric with respect to the point ($\frac{1}{2}$,0i). But is it possible to determine that the nontrivial zeros of the Riemann ζ(s) function are all on the critical boundary where the real part is equal to $\frac{1}{2}$, just because the zeros of ζ(s) are symmetric with respect to the point ($\frac{1}{2}$,0i)? Obviously not, if Re(s)=0.54.... Then Re(1-s)=0.45... s and 1-s are symmetric about the point ($\frac{1}{2}$,0i), but Riemann argued that such a complex number is not the zero of Riemann ζ(s).Riemann was right, and it is clear that when Re(s) is not equal to $\frac{1}{2}$, then s and 1-s must not be conjugate, and according to the zeros of the ζ(s) function must be conjugate, then if Re(s) is not equal to $\frac{1}{2}$, then it must not be the zero of the ζ(s) function. To sum up, the non-trivial zeros of the Riemann ζ(s) function must all lie on the critical boundary where the real part of the complex plane is equal to $\frac{1}{2}$, and the Riemann conjecture must be true.So only when $\sigma=\frac{1}{2}$ and $\zeta(s)=0(s \in C \text{ and } s \neq 1)$,then it must be true that $\zeta(1\text{-}s)=\zeta(s)=0(s \in C \text{ and } s \neq 1)$.$\zeta(s)(s \in C \text{ and } s \neq 1)$ and $\zeta(\overline{s})(s \in C \text{ and } s \neq 1)$ are complex conjugates of each other,that is $\zeta(s)=\overline{\zeta(\overline{s})}$ $(s \in C \text{ and } s \neq 1)$, if $\zeta(s)=0(s \in C \text{ and } s \neq 1)$, then must $\zeta(\overline{s})=0(s \in C \text{ and } s \neq 1)$, and so if $\zeta(s)=0(s \in C \text{ and } s \neq 1)$, then it must be true that $\zeta(s)=\zeta(\overline{s})=0(s \in C \text{ and } s \neq 1)$.

According to Riemann's paper "On the Number of primes not Greater than x", we can obtain an expression $\zeta(1\text{-}s)=2^{1-s}\pi^{-s}\cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)\,(s \in C \text{ and } s \neq 1)$ in relation to the Riemann ζ(s) $(s \in C \text{ and } s \neq 1)$function, which has long been known to modern mathematicians, and which I derive later. According $\zeta(1\text{-}s)=2^{1-s}\pi^{-s}\cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)\,(s \in C \text{ and } s \neq 1)$(Formula 7) obtained by Riemann,so when ζ(s)=0 then ζ(1-s)=ζ(s)=0(s∈C and s≠1) eacause only when $\sigma=\frac{1}{2}$, the next three equations ζ(σ+ti)=0, ζ(1-σ-ti)=0, and ζ(σ-ti)=0 are all true,so only $s=\frac{1}{2}$+ti (t∈R and t≠0) and $s=\frac{1}{2}$-ti (t∈R and t≠0) is true.And when ζ(s)=0 then according ζ(1-s)=ζ(s)and $\zeta(s)=\overline{\zeta(\overline{s})}=0(s \in C \text{ and } s \neq$

1),is also say $\zeta(s)=\zeta(\overline{s})=0$ and $\zeta(1-s)=\zeta(\overline{s})$ $=0(s \in C$ and $s \neq 1)$,then only $\zeta(\sigma+ti)=\zeta(\sigma-ti)=0$ is true.Since Riemann has shown that the Riemann $\zeta(s)(s \in C$ and $s \neq 1)$ function has zero, that is, in $\zeta(1-s)=2^{1-s}\pi^{-s}\cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)$ $(s \in C$ and $s \neq 1)$, $\zeta(s)=0(s \in C$ and $s \neq 1)$(Formula 7) is true, so when $\zeta(s)=0$,In the process of the Riemann conjecture proved about $\zeta(s)=\zeta(\overline{s})=0$ and $\zeta(1-s)=\zeta(\overline{s})$ $=0(s \in C$ and $s \neq 1)$, is refers to the $\zeta(s)(s \in C$ and $s \neq 1)$ is a functional numbe. Does $\zeta(s)=\zeta(\overline{s})=0$ and $\zeta(1-s)=\zeta(\overline{s})=0(s \in C$ and $s \neq 1)$ mean the symmetry of the $\zeta(s)(s \in C$ and $s \neq 1)$ function equation? Does that mean the symmetry of the equation $s=\overline{s}=1-s$? Not really. In my analyst, $\zeta(s)(s \in C$ and $s \neq 1)$、$\zeta(1-s)(s \in C$ and $s \neq 1)$ and $\zeta(\overline{s})(s \in C$ and $s \neq 1)$ function expression are both from $\sum_{n=1}^{\infty} n^{-s} = \prod_p (1-p^{-s})^{-1}(s \in c$ and $s \neq 1$, $n \in Z^+$ and n goes through all the positive integers, $p \in Z^+$ and p goes through all the prime numbers), so according to $\sum_{n=1}^{\infty} n^{-s} = \prod_p (1-p^{-s})^{-1}(s \in C$ and $s \neq 1$, $n \in Z^+$ and n goes through all the positive integers, $p \in Z^+$ and p goes through all the prime numbers),$\zeta(s)$ $(s \in C$ and $s \neq 1)$function of the independent variable s, the relationship between $\overline{s}$ and 1-s only $C_3^2=3$ kinds, namely $s=\overline{s}$ or $s=1-s$ or $\overline{s}=1-s$. As follows:according $\zeta(s)=\zeta(1-s)=0$ $(s \in C$ and $s \neq 1)$ and $\zeta(s)=\zeta(\overline{s})=0$ and $\zeta(1-s)=\zeta(\overline{s})$ $=0(s \in C$ and $s \neq 1)$,then only $s=\overline{s}$ or $s=1-s$ or $\overline{s}=1-s$,so $s \in R$ and $s=-2n(n \in Z^+)$, or $\sigma+ti=1-ti$,or $\sigma-ti=1-\sigma-ti$, so $s \in R$ and $s=-2n(n \in Z^+)$, or $\sigma=\frac{1}{2}$ and $t=0$,or $\sigma=\frac{1}{2}$ and $t \in R$ or $\sigma+ti=1-ti$,or $\sigma-ti=1-\sigma-ti$, so $s \in R$ and $s=-2n(n \in Z^+)$, or $\sigma=\frac{1}{2}$ and $t=0$,or $\sigma=\frac{1}{2}$ and $t \in R$ and $t \neq 0$,so $s \in R$, or $s=\frac{1}{2}+0i$,or $s=\frac{1}{2}+ti$ $(t \in R$ and $t \neq 0)$ and $s=\frac{1}{2}-ti(t \in R$ and $t \neq 0)$, because $\zeta\left(\frac{1}{2}\right) \to +\infty, \zeta(1) \to +\infty$,$\zeta(1)$ is divergent, $\zeta(\frac{1}{2})$ is more divergent,so drop s=1 and $s=\frac{1}{2}$.According the equation $\xi(s)=$ $\frac{1}{2}s(s-1)\Gamma(\frac{s}{2})\pi^{-\frac{s}{2}}\zeta(s)(s \in C$ and $s \neq 1)$obtained by Riemann, so $\xi(s)=\xi(1-s)(s \in C$ and $s \neq 1, s \neq -2n$, and n traverses all positive integers),

because $\Gamma(\frac{s}{2})=\overline{\Gamma(\frac{\overline{s}}{2})}$, and $\pi^{-\frac{s}{2}}=\overline{\pi^{-\frac{\overline{s}}{2}}}$, and because $\zeta(s)=\overline{\zeta(\overline{s})}(s \in C$ and $s \neq 1)$, so $\xi(s)=\overline{\xi(\overline{s})}(s \in C$ and $s \neq 1, s \neq -2n$, and n traverses all positive integers).So when $\zeta(s)=0(s \in C$ and $s \neq 1)$,then $\zeta(s)=\zeta(1-s)=\zeta(\overline{s})=0(s \in C$ and $s \neq 1)$ and $\xi(s)=\xi(1-s)=\xi(\overline{s})=0(s \in C$ and $s \neq 1, s \neq -2n$, and n traverses all positive integers) must be true , so the nontrivial zeros of the Riemann $\zeta(s)(s \in C$ and $s \neq 1)$ function and the nontrivial zeros of the Riemann $\xi(s)(s \in C$ and $s \neq 1, s \neq -2n$, and n traverses all positive integers) function are identical, So the nontrivial zero of the Riemannian $\zeta(s)(s \in C$ and $s \neq 1)$ function is the same as the zero of the Riemannian $\xi(s)(s \in C$ and $s \neq 1)$ function, And the imaginary parts of the zeros of $\xi(s)=0$ are real roots of $\prod \frac{s}{2}(s-1)\pi^{-\frac{s}{2}}\zeta(s) = \prod \frac{(\frac{1}{2}+ti)}{2}(-\frac{1}{2}+ti)\pi^{-\frac{\frac{1}{2}+ti}{2}}\zeta(\frac{1}{2}+ti)=\xi(t)=0$, In addition, the previous proof of Riemann $\zeta(s)(s=\sigma+ti,(\sigma \in R, t \in R$, and $s \neq 1)$, based on the Landau-Siegel function has no real zeros except negative even numbers, so the complex root of Riemann $\xi(s)=0$ $(s \in C$ and $s \neq 1, s \neq -2n$, and n traverses all positive integers) satisfies $s=\frac{1}{2}+ti(t \in R$ and $t \neq 0)$ or $s=\frac{1}{2}-ti(t \in R$

and t≠0). According to the Riemann function $\prod\frac{s}{2}(s\text{-}1)\pi^{-\frac{s}{2}}\zeta(s)=\xi(t)(t\in C \text{ and } t\neq 0, s\in C \text{ and } s\neq 1, s\neq -2n, \text{and n traverses all positive integers})$ defined by Riemann and he defined $s=\frac{1}{2}+ti(t\in C \text{ and } t\neq 0)$, because s≠1, and $\prod\frac{s}{2}\neq 0$, $\pi^{-\frac{s}{2}}\neq 0$, so $\prod\frac{s}{2}(s\text{-}1)\pi^{-\frac{s}{2}}\neq 0$, and when $\zeta(s)=0(s\in C \text{ and } s\neq 1, s\neq -2n, \text{and n traverses all positive integers})$, then $\prod\frac{s}{2}(s-1)\pi^{-\frac{s}{2}}\zeta(s)=\prod\frac{(\frac{1}{2}+ti)}{2}(-\frac{1}{2}+ti)\pi^{-\frac{\frac{1}{2}+ti}{2}}\zeta(\frac{1}{2}+ti)=\xi(t)=0(t\in C \text{ and } t\neq 0, s\in C \text{ and } s\neq 1, s\neq -2n, \text{and n traverses all positive integers})$, and

$\zeta(\frac{1}{2}+ti)=\frac{\xi(t)}{\prod\frac{(\frac{1}{2}+ti)}{2}(-\frac{1}{2}+ti)\pi^{-\frac{\frac{1}{2}+ti}{2}}}=\frac{0}{\prod\frac{(\frac{1}{2}+ti)}{2}(-\frac{1}{2}+ti)\pi^{-\frac{\frac{1}{2}+ti}{2}}}=0(t\in C \text{ and } t\neq 0)$,so t∈R and $t\neq 0$.So the root t of the equations $\prod\frac{(\frac{1}{2}+ti)}{2}(-\frac{1}{2}+ti)\pi^{-\frac{\frac{1}{2}+ti}{2}}\zeta(\frac{1}{2}+ti)=\xi(t)=0\ (t\in C \text{ and } t\neq 0)$ and $4\int_1^{\infty}\frac{d(x^{\frac{3}{2}}\Psi'(x))}{dx}x^{-\frac{1}{4}}\cos(\frac{1}{2}t\ln x)dx=\xi(t)=0(t\in C \text{ and } t\neq 0)$ and $\xi(t)=\frac{1}{2}\text{-}(t^2+\frac{1}{4})\int_1^{\infty}\Psi(x)\ x^{-\frac{3}{4}}\cos(\frac{1}{2}t\ln x)=0(t\in C \text{ and } t\neq 0)$must be real and $t\neq 0$.Riemann got $\prod\frac{(\frac{1}{2}+ti)}{2}(-\frac{1}{2}+ti)\pi^{-\frac{\frac{1}{2}+ti}{2}}\zeta(\frac{1}{2}+ti)=\xi(t)\ (t\in C \text{ and } t\neq 0)$ and $\xi(t)=\frac{1}{2}-(t^2+\frac{1}{4})\int_1^{\infty}\Psi(x)\ x^{-\frac{3}{4}}\cos(\frac{1}{2}t\ln x)\,dx(t\in C \text{ and } t\neq 0, s\in C \text{ and } s\neq 1)$ in his paper, or $\prod\frac{(\frac{1}{2}+ti)}{2}(-\frac{1}{2}+ti)\pi^{-\frac{\frac{1}{2}+ti}{2}}\zeta(\frac{1}{2}+ti)=\xi(t)(t\in C \text{ and } t\neq 0)$ and $\xi(t)=4\int_1^{\infty}\frac{d(x^{\frac{3}{2}}\Psi'(x))}{dx}x^{-\frac{1}{4}}\cos(\frac{1}{2}t\ln x)\,dx(t\in C \text{ and } t\neq 0)$ because the roots of $\zeta(\frac{1}{2}+ti)=0\ (t\in C \text{ and } t\neq 0)$ are the roots of

$\prod\frac{s}{2}\ (s\text{-}1)\ \pi^{-\frac{s}{2}}\ \zeta(s)=\prod\frac{(\frac{1}{2}+ti)}{2}(-\frac{1}{2}+ti)\pi^{-\frac{\frac{1}{2}+ti}{2}}\zeta(\frac{1}{2}+ti)=\xi(t)=0\ (t\in C \text{ and } t\neq 0, s\in C \text{ and } s\neq 1, s\neq -2n, \text{and n traverses all positive integers})$, and because the roots of $\zeta(\frac{1}{2}+ti)=0\ (t\in C \text{ and } t\neq 0)$ are the roots of $4\int_1^{\infty}\frac{d(x^{\frac{3}{2}}\Psi'(x))}{dx}x^{-\frac{1}{4}}\cos(\frac{1}{2}t\ln x)\,dx=\xi(t)=0\,(t\in C \text{ and } t\neq 0)$, and because the roots of $\zeta(\ \frac{1}{2}\ +ti)=0\ \ (t\in C \text{ and } t\neq 0)$ are the roots of $\xi(t)=\frac{1}{2}\text{-}(\ t^2+\frac{1}{4})\ \int_1^{\infty}\Psi(x)\,x^{-\frac{3}{4}}\cos\left(\frac{1}{2}t\ln x\right)=0(t\in C \text{ and } t\neq 0)$, so the roots of equations $\prod\frac{(\frac{1}{2}+ti)}{2}(-\frac{1}{2}+ti)\pi^{-\frac{\frac{1}{2}+ti}{2}}\zeta(\frac{1}{2}+ti)=\xi(t)=0(t\in C \text{ and } t\neq 0)$and the roots of equations $4\int_1^{\infty}\frac{d(x^{\frac{3}{2}}\Psi'(x))}{dx}x^{-\frac{1}{4}}\cos(\frac{1}{2}t\ln x)dx=\xi(t)=0(t\in C \text{ and } t\neq 0)$ and the roots of equations $\xi(t)=\frac{1}{2}\text{-}(t^2+\frac{1}{4})\int_1^{\infty}\Psi(x)\ x^{-\frac{3}{4}}\cos(\frac{1}{2}t\ln x)=0(t\in C \text{ and } t\neq 0)$ must all be real numbers and consistent.

So when $\zeta(s)=0\ (s\in C\ \textit{and}\ s\neq 1 \text{ and } s\neq -2n, n\in Z^+$, n take all positive integer), and because the roots of $\zeta(s)=0\left(s\in C\ \textit{and}\ s\neq 1\ , \text{ and } s\neq -2n, n\in Z^+\ , \text{ n take all positive integer}\right)$ is $s=\frac{1}{2}+ti(t\in$

R and $t \neq 0$)and $s = \frac{1}{2}$-ti($t \in R$ and $t \neq 0$) , so

the roots of $\prod\frac{s}{2}$ (s-1) $\pi^{-\frac{s}{2}}\zeta(s)= \prod\frac{(\frac{1}{2}+ti)}{2}(-\frac{1}{2}+ti)\pi^{-\frac{\frac{1}{2}+ti}{2}}\zeta(\frac{1}{2}+ti)=\xi(t)=0$ ($t \in C$ and $t \neq 0, s \in$ C and $s\neq1$ and 41∞d(x32Ψ′(𝑥))d𝑥𝑥−14cos(12tln𝑥)d𝑥=ξ(t)=0(t∈C and t≠0) and

$\xi(t)=\frac{1}{2}\cdot(t^2+\frac{1}{4})\int_1^{\infty}\Psi(x)\,x^{-\frac{3}{4}}\cos\left(\frac{1}{2}t\ln x\right)=0$ $\left(t \in C \text{ 且 } t \neq 0\right)$ must all be real numbers and consistent,so when

$\zeta(s)=0\left(s \in C \text{ and } s \neq 1 \text{and } s \neq -2n, n \in Z^+ \text{ , n take all positive integer}\right)$,then

$\prod\frac{s}{2}(s\text{-}1)\,\pi^{-\frac{s}{2}}\zeta(s) = \xi(s) = 0(s \in C \text{ and } s \neq 1\text{and } s \neq -2n, n \in Z^+ \text{ , n take all positive integer})$,

and the roots of $\prod\frac{(\frac{1}{2}+ti)}{2}(-\frac{1}{2}+ti)\pi^{-\frac{\frac{1}{2}+ti}{2}}\zeta(\frac{1}{2}+ti)=\xi(t)=0$($t \in C$ and $\neq 0$)are all real numbers. The number of roots of $\xi(t)=0$($t \in C$ and $t \neq 0$)(which the real part of roots between 0 and T) is approximately equal to $\frac{T}{2\pi}\ln\frac{T}{2\pi}-\frac{T}{2\pi}$. Riemann's estimate of the number of zeros ξ (t)=0 was strictly proved by Mangoldt in 1895, all the roots of $\xi(t)=0$($t \in C$ and $t \neq$ 0 are real numbers , so the Riemann conjecture are perfectly valid.

Because the number of roots t of $\zeta\left(\frac{1}{2}+it\right) = \sum_{n=1}^{\infty}(n^{-\frac{1}{2}}(\cos(t\ln(n)) - i\sin(t\ln(n)))$ = $\sum_{n=1}^{\infty}(n^{-\frac{1}{2}}(\cos(\ln(n^t)) - i\sin(\ln(n^t)))$ =0 is the number of roots of

$\xi(t)=\frac{1}{2}\cdot(t^2+\frac{1}{4})\int_1^{\infty}\Psi(x)\;x^{-\frac{3}{4}}\cos(\frac{1}{2}t\ln x)=0$. Because when t=0, then $\zeta\left(\frac{1}{2}\right)$ is divergent, when $\ln(n^t) \in [0, 2\pi]$, the numbers of the root t of

$\zeta\left(\frac{1}{2}+it\right) = \sum_{n=1}^{\infty}(n^{-\frac{1}{2}}(\cos(t\ln(n)) - i\sin(t\ln(n)))$ =

$\sum_{n=1}^{\infty}(n^{-\frac{1}{2}}(\cos(\ln(n^t)) - i\sin(\ln(n^t)))$ =0 is $\ln\frac{T}{2\pi} - 1$, so when t∈(0, T] , the numbers of the roots of

$\zeta\left(\frac{1}{2}+it\right) = \sum_{n=1}^{\infty}(n^{-\frac{1}{2}}(\cos(t\ln(n)) - i\sin(t\ln(n)))$ = $\sum_{n=1}^{\infty}(n^{-\frac{1}{2}}(\cos(\ln(n^t)) - i\sin(\ln(n^t)))$ =0 is $N=n_1 \times n_2 = \frac{T}{2\pi} \times (\ln\frac{T}{2\pi} - 1)$.

Formula 2

Let's say I have any complex number Z= x+yi(x ∈R,y∈R), and I have any complex number s=σ+ui(σ ∈R,u∈R).We use r(r∈R,and r>0) to represent the module |Z| of complex Z= x+yi (x ∈R ,y∈R), and ϕ to represent the argument Am(Z) of complex Z= x+yi(x∈R ,y∈R).That is |Z|=r, then r= $(x^2+y^2)^{\frac{1}{2}}$, so Z=r(Cos(ϕ)+iSin(ϕ)) and ϕ=|arccos($\frac{x}{(x^2+y^2)^{\frac{1}{2}}}$)|,and ϕ∈(-π, π], then ϕ=Am(Z).

Base on $x^s = x^{\sigma+ui} = x^{\sigma}x^{ui}=x^{\sigma}(\cos(\ln x) + i\sin(\ln x))^{u}=x^{\sigma}(\cos(u\ln x) + i\sin(u\ln x))$ can get $r^s = r^{(\sigma+ui)} = r^{\sigma}r^{ui} = r^{\sigma}(\cos(\ln x) + i\sin(\ln x))^{u} = r^{\sigma}(\cos(u\ln x) + i\sin(u\ln x))$ (r>0), then $f(Z,s)=z^s=(r(\cos(\varphi) + i\sin(\varphi))^{\sigma+u\,i}=(r(\cos(\varphi) + i\sin(\varphi))^{\sigma}r(\cos(\varphi) + i\sin(\varphi))^{ui}$ = $r^{\sigma}(\cos(\sigma\varphi) + i\sin(\sigma\varphi))(r(\cos(\varphi) + i\sin(\varphi))^{ui} = r^{\sigma}(\cos(\sigma\varphi) + i\sin(\sigma\varphi))r^{ui}(\cos(\varphi) +$

$\text{isin}(\varphi))ui = r\sigma(\cos(\sigma\varphi)+i\sin(\sigma\varphi))(\cos(u\ln r)+i\sin(u\ln r))(\cos(u\varphi)+i\sin(u\varphi))i$

$=r^{\sigma}(\cos(\sigma\varphi + u\ln r) + i\sin(\sigma\varphi + u\ln r))(\cos(u\varphi) + i\sin(u\varphi))^{i}$.

Beacuse of

$Z = e^{\ln|Z|+iAm(Z)} = e^{\ln|Z|}e^{iAm(Z)} = e^{\ln|Z|}$ (cos(Am(Z))+isin(Am (Z)))=r(cos(Am(Z))+isin(Am (Z))),so

lnZ=ln|Z|+iAm(Z)($-\pi$<Am(Z)<= π).

Suppose a>0,then $a^{x}=e^{\ln(a^{x})} = e^{x\ln a}$, then $z^{s}=e^{s\ln z}$.

Suppose any complex Number Q= $\cos(u\varphi) + i\sin(u\varphi)$, and Suppose

the complex ψ= i, then lnQ= ln‖Q‖+iAm(Q) ($-\pi$<Am(Q)<= π).

Because 0<=|sin($u\varphi$)|<=1,

so

If $-\pi<u\varphi<=\pi$,then Am(Q)=$u\varphi$ and $-\pi$<Am(Q)<= π ;

If $u\varphi>\pi$, then Am(Q)=$u\varphi$-2kπ($k \in Z^{+}$) and $-\pi$<Am(Q)<= π ;

if $u\varphi < -\pi$,then Am(Q)=$u\varphi$+2kπ($k \in Z^{+}$) and $-\pi$<Am(Q)<= π. Then

If Am(Q)=$u\varphi$, then

$(\cos(u\varphi) + i\sin(u\varphi))^{i} = Q^{\psi} = e^{\psi\ln Q} = e^{\psi(\ln|Q|+iAm(Q))}=e^{i(o+iAm(Q))}=e^{-u\varphi}$.then

$f(Z,s)=z^{s}=r^{\sigma}(\cos(\sigma\varphi + u\ln r) + i\sin(\sigma\varphi + u\ln r))(\cos(u\varphi) + i\sin(u\varphi))^{i}$

$=e^{-u\varphi}r^{\sigma}(\cos(\sigma\varphi + u\ln r)+ie^{-u\varphi}r^{\sigma}\sin(\sigma\varphi + u\ln r)$。 Substituting

$r= (x^{2} + y^{2})^{\frac{1}{2}}$ into the above equation gives:

$f(Z,s)=z^{s}=e^{-u\varphi}(x^{2} + y^{2})^{\frac{\sigma}{2}}(\cos(\sigma\varphi + u\ln(x^{2} + y^{2})^{\frac{1}{2}}))$

$+ie^{-u\varphi}(x^{2} + y^{2})^{\frac{\sigma}{2}}(\sin(\sigma\varphi + u\ln(x^{2} + y^{2})^{\frac{1}{2}}))$.

If Am(Q)= $u\varphi$-2kπ($k\in Z^{+}$),then

$(\cos(u\varphi) + i\sin(u\varphi))^{i} = Q^{\psi} = e^{\psi\ln Q} = e^{\psi(\ln|Q|+iAm(Q))}=e^{i(o+i(u\varphi-2k\pi))}=e^{2k\pi-u\varphi}$, then

$f(Z,s)=z^{s}=r^{\sigma}(\cos(\sigma\varphi + u\ln r) + i\sin(\sigma\varphi + u\ln r))(\cos(u\varphi) + i\sin(u\varphi))^{i}$

$=e^{2k\pi-u\varphi}r^{\sigma}(\cos(\sigma\varphi + u\ln r)+ie^{2k\pi-u\varphi}r^{\sigma}\sin(\sigma\varphi + u\ln r)$.

Substituting $r= (x^{2} + y^{2})^{\frac{1}{2}}$ into the above equation gives:

$f(Z,s)=Z^{s}=e^{2k\pi-u\varphi}(x^{2} + y^{2})^{\frac{\sigma}{2}}(\cos(\sigma\varphi + u\ln(x^{2} + y^{2})^{\frac{1}{2}}))$

$+ie^{2k\pi-u\varphi}(x^{2} + y^{2})^{\frac{\sigma}{2}}(\sin(\sigma\varphi + u\ln(x^{2} + y^{2})^{\frac{1}{2}}))$.

If Am(Q)=$u\varphi$+2kπ($k\in Z^{+}$), then

$(\cos(u\varphi) + i\sin(u\varphi))^{i} = Q^{\psi} = e^{\psi\ln Q} = e^{\psi(\ln|Q|+iAm(Q))}=e^{i(o+i(u\varphi+2k\pi))}=e^{-2k\pi-u\varphi}$, then

$f(Z,s)=Z^{s}=r^{\sigma}(\cos(\sigma\varphi + u\ln r) + i\sin(\sigma\varphi + u\ln r))(\cos(u\varphi) + i\sin(u\varphi))^{i}$

$=e^{-2k\pi-u\varphi}r^{\sigma}(\cos(\sigma\varphi + u\ln r)+ie^{-2k\pi-u\varphi}r^{\sigma}\sin(\sigma\varphi + u\ln r)$.

Substituting $r= (x^{2} + y^{2})^{\frac{1}{2}}$ into the above equation gives:

$f(Z,s)=z^{s}=e^{-2k\pi-u\varphi}(x^{2} + y^{2})^{\frac{\sigma}{2}}(\cos(\sigma\varphi + u\ln(x^{2} + y^{2})^{\frac{1}{2}}))$

$+ie^{-2k\pi-u\varphi}(x^{2} + y^{2})^{\frac{\sigma}{2}}(\sin(\sigma\varphi + u\ln(x^{2} + y^{2})^{\frac{1}{2}}))$.

Reasoning 1:

For any complex number s, when $Rs(s) > 0$ and $(s \neq 1)$, and if $s=\sigma+ti(\sigma \in R, t \in R$ and $t \neq 0, s \in C)$, then according to Dirichlet $\eta(s)$, then the relationship between the Riemannn ζ(s)(s∈C and Rs(s)>0 and s≠1) function and the Dirichlet η(s)(s∈C and Rs(s)>0 and s≠1) function is :

because $\eta(s)=\frac{1}{1^s}-\frac{1}{2^s}+\frac{1}{3^s}-\frac{1}{4^s}+\frac{1}{5^s}-\frac{1}{6^s}+\ldots(s \in C$ and $Rs(s) > 0$ and $s \neq 1)$,

$\zeta(s)=\frac{1}{1^s}+\frac{1}{2^s}+\frac{1}{3^s}+\frac{1}{4^s}+\frac{1}{5^s}+\frac{1}{6^s}+\ldots(s \in C$ and $Rs(s) > 0$ and $s \neq 1)$, so

$\eta(s)-\zeta(s)=$

$-(\frac{2}{2^s}+\frac{2}{4^s}+\frac{2}{6^s}+\cdots)=-\frac{2}{2^s}(\frac{1}{1^s}+\frac{1}{2^s}+\frac{1}{3^s}+\frac{1}{4^s}+\frac{1}{5^s}+\frac{1}{6^s}+\ldots)=-\frac{2}{2^s}\zeta(s)(s \in C$ and $Rs(s) > 0$ and $s \neq$ 1 , then ηs=1−22sζs=(1−21−s)ζss∈C and Rss>0 and s≠1, then

$\eta(s)=\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^s}(s \in C$ and $Rs(s) > 0$ and $s \neq 1)$ and $\eta(s)=(1-2^{1-s})\zeta(s)(s \in C$ and $Rs(s) >$ 0 and s≠1, ζsis the Riemann Zeta function, η(s) is the Dirichlet η(s) function,

so Riemann $\zeta(s)=\frac{\eta(s)}{(1-2^{1-s})}=\frac{1}{(1-2^{1-s})}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^s}=\frac{(-1)^{n-1}}{(1-2^{1-s})}\prod_p(1-p^{-s})^{-1}(s \in C$ and $Rs(s) >$ 0 and $s \neq 1, n\in Z^+, p \in Z^+, s \in C$, n goes through all the positive integers, p goes through all the prime numbers). Let's prove that ζ(s) and $\zeta((\bar{s})$ are complex conjugations of each other.

$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^s}$=[$1^{-\sigma}$cos(tln1)− $2^{-\sigma}$cos(tln2)+ $3^{-\sigma}$cos(tln3)−$4^{-\sigma}$cos(tln4)-...]-i[$1^{-\sigma}$sin(tln1)− $2^{-\sigma}$sin(tln2)+ $3^{-\sigma}$sin(tln3) − $4^{-\sigma}$sin(tln4)+...]= U-Vi,

$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^{\bar{s}}}$=[$1^{-\sigma}$cos(tln1)−$2^{-\sigma}$cos(tln2)+ $3^{-\sigma}$cos(tln3)−$4^{-\sigma}$cos(tln4)-...]+i[$1^{-\sigma}$sin(tln1)− $2^{-\sigma}$ sin(tln2)+ $3^{-\sigma}$sin(tln3)−$4^{-\sigma}$sin(tln4)+...]= U+Vi,

$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^{1-s}}$=[$1^{\sigma-1}$ cos(tln1) − $2^{\sigma-1}$ cos(tln2)+ $3^{\sigma-1}$ cos(tln3) −$4^{-\sigma}$ cos(tln4)-...]+i[$1^{-\sigma}$ sin(tln1) − $2^{-\sigma}$sin(tln2)+ $3^{-\sigma}$sin(tln3)− $4^{-\sigma}$sin(tln4)+...],

$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^{k-s}}$=[$1^{\sigma-k}$ cos(tln1) − $2^{\sigma-k}$ cos(tln2)+ $3^{\sigma-k}$ cos(tln3) −$4^{\sigma-k}$ cos(tln4)-...]+i[$1^{\sigma-k}$ sin(tln − $2^{\sigma-k}$sin(tln2)+ $3^{\sigma-k}$sin(tln3) − $4^{\sigma-k}$sin(tln4)+...],

$(s \in C$ and $s \neq 1, n \in Z^+$ and n traves all positive integer, $k \in R$),

because ,

$\frac{(-1)^{n-1}}{(1-2^{1-s})}=\overline{\frac{(-1)^{n-1}}{(1-2^{1-\bar{s}})}}$,

$\prod_p(1-p^{-s})^{-1}=\overline{\prod_p(1-p^{-\bar{s}})^{-1}}$

$(s \in C$ and $s \neq 1, p \in Z^+$ and p traves all prime numbers),

so

$\frac{(-1)^{n-1}}{(1-2^{1-s})}=\overline{\frac{(-1)^{n-1}}{(1-2^{1-\bar{s}})}}$,

so

$\frac{(-1)^{n-1}}{(1-2^{1-s})}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^s}=\overline{\frac{(-1)^{n-1}}{(1-2^{1-\bar{s}})}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^{\bar{s}}}}$,

$\frac{(-1)^{n-1}}{(1-2^{1-s})}\prod_p(1-p^{-s})^{-1}=\overline{\frac{(-1)^{n-1}}{(1-2^{1-\bar{s}})}\prod_p(1-p^{-\bar{s}})^{-1}}$,

$\zeta(s)=\frac{1}{(1-2^{1-s})}\sum_1^{\infty}\frac{(-1)^{n-1}}{n^s}=\frac{(-1)^{n-1}}{(1-2^{1-s})}\prod_p(1-p^{-s})^{-1}$,

$\zeta(\bar{s})=\frac{1}{(1-2^{1-\bar{s}})}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^{\bar{s}}}=\frac{(-1)^{n-1}}{(1-2^{1-\bar{s}})}\prod_p(1-p^{-\bar{s}})^{-1}$ ($s \in C$ and $s \neq 1, n \in Z^+$ and n traves all positive integer, $p \in Z^+$ and p traves all prime numbers),

so

only $\zeta(s)=\overline{\zeta(\bar{s})}$ ($s \in C$ and $s \neq 1$), [2]so

$p^{1-s}=p^{(1-\sigma-ti)}=p^{1-\sigma}p^{-ti}=p^{1-\sigma}(\cos(\ln p)+i\sin(\ln p))^{-t}=p^{1-\sigma}(\cos(t\ln p)-i\sin(t\ln p))$,
$p^{1-\bar{s}}=p^{(1-\sigma+ti)}=p^{1-\sigma}p^{ti}=p^{1-\sigma}(p^{ti})=p^{1-\sigma}(\cos(\ln p)+i\sin(\ln p))^{t}=(p^{1-\sigma}(\cos(t\ln p)+i\sin(t\ln p))$,($s \in C$ and $s \neq 1, t \in C$ and $s \neq 0, p \in Z^+$)

then

$p^{-(1-s)}=p^{(-1+\sigma+ti)}=p^{\sigma-1}p^{ti}=p^{\sigma-1}\frac{1}{(\cos(t\ln p)-i\sin(t\ln p))}=(p^{\sigma-1}(\cos(t\ln p)+i\sin(t\ln p))$,

$p^{-(\bar{s})}=p^{-(\sigma-ti)}=p^{-\sigma}p^{ti}=(p^{-\sigma}(\cos(t\ln p)+i\sin(t\ln p))$
($s \in C$ and $s \neq 1, t \in C$ and $s \neq 0, p \in Z^+$),

so

$(1-p^{-(1-s)})=1-(p^{\sigma-1}(\cos(t\ln p)+i\sin(t\ln p))=1-p^{\sigma-1}\cos(t\ln p)-ip^{\sigma-1}\sin(t\ln p)$,

$(1-p^{-(\bar{s})})=1-(p^{-\sigma}(\cos(t\ln p)+i\sin(t\ln p))=1-p^{-\sigma}\cos(t\ln p)-ip^{-\sigma}\sin(t\ln p)$,
($s \in C$ and $s \neq 1, t \in C$ and $t \neq 0, p \in Z^+$),

$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^{1-s}}=[1^{\sigma-1}\cos(t\ln 1)-2^{\sigma-1}\cos(t\ln 2)+3^{\sigma-1}\cos(t\ln 3)-4^{\sigma-1}\cos(t\ln 4)-...]+i[1^{\sigma-1}\sin(t\ln 1)-2^{\sigma-1}\sin(t\ln 2)+3^{\sigma-1}\sin(t\ln 3)-4^{\sigma-1}\sin(t\ln 4)+...]$,

$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^{\bar{s}}}=[1^{-\sigma}\cos(t\ln 1)-2^{-\sigma}\cos(t\ln 2)+3^{-\sigma}\cos(t\ln 3)-4^{-\sigma}\cos(t\ln 4)-...]+i[1^{-\sigma}\sin(t\ln 1)-2^{-\sigma}\sin(t\ln 2)+3^{-\sigma}\sin(t\ln 3)-4^{-\sigma}\sin(t\ln 4)+...]$
($s \in C$ and $s \neq 1, s \neq -2n$, and n traverses all positive integers),

when $\sigma=\frac{1}{2}$,then

$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^{1-s}}=\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^{\bar{s}}}$ ($s \in C$ and $s \neq 1, n \in Z^+$ and n traves all positive integer, $k \in R$),

$(1-p^{-(1-s)})=(1-p^{-\bar{s}})$($s \in C$ and $s \neq 1, p \in Z^+$),

and

$(1-p^{-(1-s)})^{-1}=(1-p^{-\bar{s}})^{-1}$($s \in C$ and $s \neq 1, p \in Z^+$),
$\prod_p(1-p^{-(1-s)})^{-1}=\prod_p(1-p^{-\bar{s}})^{-1}$ ($s \in C$ and $s \neq 1$, and p traves all prime numbers, $k \in R$),
$\prod_p(1-p^{-(1-s)})^{-1}=\prod_p(1-p^{-\bar{s}})^{-1}$ ($s \in C$ and $s \neq 1, p \in Z^+$ and p traves all prime numbers, $k \in R$),

and

$\frac{(-1)^{n-1}}{(1-2^{s})}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^{1-s}}=\frac{(-1)^{n-1}}{(1-2^{1-\bar{s}})}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^{\bar{s}}}$,

$\frac{(-1)^{n-1}}{(1-2^{s})}\prod_p(1-p^{-(1-s)})^{-1}=\frac{(-1)^{n-1}}{(1-2^{1-\bar{s}})}\prod_p(1-p^{-\bar{s}})^{-1}$

($s \in C$ and $s \neq 1$and n traverses all positive integers, $p \in Z^+$ and p traves all prime numbers),

And

$\zeta(1-s)=\frac{(-1)^{n-1}}{(1-2^{s})}\prod_{p}(1-p^{-(1-s)})^{-1}$,

$\zeta(\overline{s})=\frac{(-1)^{n-1}}{(1-2^{1-\overline{s}})}\prod_{p}(1-p^{-\overline{s}})^{-1}$,

$\zeta(1-s)=\frac{(-1)^{n-1}}{(1-2^{s})}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^{1-s}}$,

$\zeta(\overline{s})=\frac{(-1)^{n-1}}{(1-2^{1-\overline{s}})}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^{\overline{s}}}$

($s \in C$ and $s \neq 1, p \in Z^{+}$ and p traves all prime numbers, $n \in Z^{+}$ and n traves all positive integer),

so when $\sigma=\frac{1}{2}$, then only $\zeta(1-s)=\zeta(\overline{s})$($s \in C$ and $s \neq 1$)must be true.

$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^{k-s}}$=[$1^{\sigma-k}$cos(tln1)$-$ $2^{\sigma-k}$cos(tln2)+ $3^{\sigma-k}$cos(tln3) $-4^{\sigma-k}$cos(tln4)-...]+i[$1^{\sigma-k}$sin(tln1) $-$ $2^{\sigma-k}$sin(tln2)+ $3^{\sigma-k}$sin(tln3) $-$ $4^{\sigma-k}$sin(tln4)+...],

$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^{s}}$=[$1^{-\sigma}$cos(tln1)$-2^{-\sigma}$cos(tln2)+ $3^{-\sigma}$cos(tln3)$-4^{-\sigma}$cos(tln4)-...]+i[$1^{-\sigma}$Sin(tln1)$-$ $2^{-\sigma}$sin(tln2)+ $3^{-\sigma}$sin(tln3)$-4^{-\sigma}$sin(tln4)+...],

$p^{k-s}=p^{(k-\sigma-ti)}=p^{k-\sigma}p^{-ti}=p^{k-\sigma}(\cos(\ln p)+i\sin(\ln p))^{-t}=p^{k-\sigma}(\cos(t\ln p)-i\sin(t\ln p))$,

$p^{1-\overline{s}} = p^{(1-\sigma+ti)} = p^{1-\sigma}p^{ti} = p^{1-\sigma}(p^{ti}) = p^{1-\sigma}(\cos(\ln p)+i\sin(\ln p))^{t} = (p^{1-\sigma}(\cos(t\ln p)+i\sin(t\ln p))$,

($s \in C$ and $s \neq 1, p \in Z^{+}$ and p traves all prime numbers, $n \in Z^{+}$ and n traves all positive integer, $k \in R$),

Then

$p^{-(k-s)}=p^{(-k+\sigma+ti)}=p^{\sigma-k}p^{ti} = p^{\sigma-k}\frac{1}{(\cos(t\ln p)-i\sin(t\ln p))}$ =$(p^{\sigma-k}(\cos(t\ln p)+i\sin(t\ln p))$,

$p^{-(\overline{s})}=p^{-(\sigma-ti)}=p^{-\sigma}p^{ti} = (p^{-\sigma}(\cos(t\ln p)+i\sin(t\ln p))$,

$p^{-(k-s)}=(p^{\sigma-k}(\cos(t\ln p)+i\sin(t\ln p))$,

($s \in C$ and $s \neq 1, p \in Z^{+}$ and and p is a prime number $k \in R$),

so

$(1-p^{-(k-s)})$=1-$(p^{\sigma-k}(\cos(t\ln p)+i\sin(t\ln p))$ =$1-p^{\sigma-k}\cos(t\ln p)-ip^{\sigma-k}\sin(t\ln p)$,

$(1-p^{-\overline{s}})$=1-$(p^{-\sigma}(\cos(t\ln p)+i\sin(t\ln p))$ =$1-p^{-\sigma}\cos(t\ln p)-ip^{-\sigma}\sin(t\ln p)$,

($s \in C$ and $s \neq 1, p \in Z^{+}$ and and p is a prime number $k \in R$),

So when $\sigma=\frac{k}{2}$($k \in R$) then

$\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^{1-k+s}}=\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^{\overline{s}}}$ ($s \in C$ and $s \neq 1, k \in R, n \in Z^{+}$ and n traves all positive integer),

$(1-p^{-(k-s)})$ =$(1-p^{-\overline{s}})$ ($s \in C$ and $s \neq 1, k \in R, p \in Z^{+}$ and p is a prime number),

and $(1-p^{-(k-s)})^{-1}$=$(1-p^{-\overline{s}})^{-1}$($s \in C$ and $s \neq 1, k \in R$, $p \in Z^{+}$ and p is a prime number),

$\prod_{p}\left(1-p^{-(k-s)}\right)^{-1}=\prod_{p}\left(1-p^{-\overline{s}}\right)^{-1}$,($s \in C$ and $s \neq 1, p \in Z^{+}$ and p traves all prime numbers, $n \in Z^{+}$ and n traves all positive integer, $k \in R$),

and

$\frac{1}{(1-2^{1-k+s})}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^{k-s}} = \frac{1}{(1-2^{1-\overline{s}})}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^{\overline{s}}}$

($s \in C$ and $s \neq 1, p \in Z^{+}$ and p traves all prime numbers, $n \in Z^{+}$ and n traves all positive integer, $k \in R$),

and

$\zeta(k-s)=\frac{(-1)^{n-1}}{(1-2^{1-k+s})}\prod_p(1-p^{-(k-s)})^{-1}$,

$\zeta(\overline{s})=\frac{(-1)^{n-1}}{(1-2^{1-\overline{s}})}\prod_p(1-p^{-\overline{s}})^{-1}$,

$\zeta(k-s)=\frac{1}{(1-2^{1-k+s})}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^{k-s}}$ (s∈C and s≠ 1, k ∈R),

$\zeta(\overline{s})=\frac{1}{(1-2^{1-\overline{s}})}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^{\overline{s}}}$ (s∈C and s≠ 1),

($s \in C$ and $s \neq 1, p \in Z^+$ and p traves all prime numbers, $n \in Z^+$ and n traves all positive integer, $k \in R$),

so when $\sigma=\frac{k}{2}$(k∈R) then only $\zeta(k-s)=\zeta(\overline{s})(s \in C$ and $s \neq 1, k \in R)$.

According the equation $\zeta(1\text{-}s)=2^{1-s}\pi^{-s}\cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)(s \in C$ and $s \neq 1)$ obtained by

Riemann,since Riemann has shown that the Riemann ζ(s) function has zero, that is, in

$\zeta(1\text{-}s)=2^{1-s}\pi^{-s}\cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)$ ($s \in C$ and $s \neq 1$)(Formula 6), $\zeta(s)=0(s \in C$ and $s \neq 1)$ is true.

When $\zeta(s)=0(s \in C$ and $s \neq 1)$, then only $\zeta(k-\overline{s})= \zeta(s)=0(s \in C$ and $s \neq 1)$, and When $\zeta(\overline{s})=0(s \in C$ and $s \neq 1)$,then $\zeta(k-s)=\zeta(\overline{s})=0(s \in C$ and $s \neq 1)$. And because when $\zeta(\overline{s})=0(s \in C$ and $s \neq 1)$, then only $\zeta(1-s)=\zeta(\overline{s})=0(s \in C$ and $s \neq 1)$, which is $\zeta(k-s) = \zeta(\overline{s})(s \in C$ and $s \neq 1, k \in R)$,so only k=1 be true. According $\zeta(s)=\zeta(1\text{-}s)$ =0 ($s \in C$ and $s \neq 1$) and $\zeta(s)=\zeta(\overline{s})=\zeta(1\text{-}\overline{s})=0(s \in C$ and $s \neq 1)$,then $s=\overline{s}$ or $s=1\text{-}s$ or $\overline{s}=1\text{-}s$,so s∈R and $s=-2n(n \in Z^+)$,

or σ+ti=1-σ-ti ,or σ-ti=1-σ-ti, so $s \in R$, or $\sigma=\frac{1}{2}$ and t=0,or $\sigma = \frac{1}{2}$ and t ∈R and t≠ 0,

so $t \in R$, or $s=\frac{1}{2}+0i$,or $s=\frac{1}{2}+ti$(t ∈R and t ≠ 0) and $s=\frac{1}{2}-ti$(t ∈R and t ≠ 0), because $\zeta\left(\frac{1}{2}\right) \to +\infty, \zeta(1) \to +\infty$, ζ(1) is divergent, $\zeta(\frac{1}{2})$ is more divergent,so drop them.Beacause only when $\sigma=\frac{1}{2}$,the next three equations, $\zeta(\sigma+ti)= 0$, $\zeta(1-\sigma-ti)=0$, and $\zeta(\sigma\text{-}ti)= 0$ are all true, because $\zeta\left(\frac{1}{2}\right) \to +\infty, \zeta(1) \to +\infty$, ζ(1) is divergent, $\zeta(\frac{1}{2})$ is more divergent, so drop s=1 and $s=\frac{1}{2}$, so only $s=\frac{1}{2}+ti$(t∈R and t ≠ 0,s∈C) is true.Since Riemann has shown that the Riemann ζ(s) ($s \in C$ and $s \neq 1$) function has zero, that is, in $\zeta(1\text{-}s)= 2^{1-s}\pi^{-s}\text{Cos}(\frac{\pi s}{2})\Gamma(s)\zeta(s)$ ($s \in C$ and $s \neq 1$)(Formula 6), $\zeta(s)=0(s \in C$ and $s \neq 1)$ is true. According the equation $\xi(s) = \frac{1}{2}s(s\text{-}1)\,\Gamma(\frac{s}{2})\pi^{-\frac{s}{2}}\zeta(s)(s \in C$ and $s \neq 1, s \neq -2n)$ obtained by Riemann , so $\xi(s) = \xi(1-s)(s \in C$ and $s \neq 1, s \neq -2n$, and n traverses all positive integers), because

$\Gamma(\frac{s}{2})=\overline{\Gamma(\frac{\overline{s}}{2})}$, and $\pi^{-\frac{s}{2}}=\overline{\pi^{-\frac{\overline{s}}{2}}}$, and because $\zeta(s)=\overline{\zeta(\overline{s})}$(s∈C and s≠ 1) , so $\xi(s)=\overline{\xi(\overline{s})}(s \in C$ and $s \neq 1, s \neq -2n$, and n traverses all positive integers). So when $\zeta(s)=0$ ($s \in C$ and $s \neq 1$) ,then $\zeta(s)=\zeta(1-s) = \zeta(\overline{s}) = 0(s\in C$ and $s\neq 1)$ and $\xi(s) = \xi(1-s) = \xi(\overline{s}) = 0(s \in C$ and $s \neq 1, s \neq -2n$, and n traverses all positive integers) must be true , The zeros of ζ(s) are all zeros of ξ(s) ($s \in C$ and $s \neq 1, s \neq -2n$, and n traverses all positive integers), except the trivial zero s=-2n (n is a natural number), which, since it happens to be the pole of $\Gamma(\frac{s}{2}+1)$ in $\xi(s)= \Gamma(\frac{s}{2}+$

1)(s-1)$\pi^{-\frac{s}{2}}\zeta(s)$, is not the zero of $\xi(s)$,therefore the zero of $\xi(s)$ coincides with the nontrivial zero of the Riemannian ζ function, so the nontrivial zero of the Riemannian ζ(s)(s∈C and s≠1) function is the same as the zero of the Riemannian $\xi(s)(s \in C \text{ and } s \neq 1, s \neq -2n, \text{and n}$ traverses all positive integers)function.So the zeros of the Riemann $\xi(s)$ function and the nontrivial zeros of the Riemann ζ(s)(s∈C and s≠ 1) function are identical. In other words, ξ(s) separates the nontrivial zeros of the Riemann ζ(s) function from all zeros. Since the nontrivial zeros of the Riemann ζ(s) function satisfy $s=\frac{1}{2}+ti$(t∈R and t≠0) and $s=\frac{1}{2}-ti$(t∈R and t≠0),

Therefore, the complex root of the function $\xi(s)=0(s \in C \text{ and } s \neq 1, s \neq -2n, \text{and n}$ traverses allpositive integers) introduced by Riemann must satisfy $s=\frac{1}{2}+ti$(t∈R and t≠0) and $s=\frac{1}{2}-ti$(t∈R and t≠0), which is consistent with the nontrivial zero of the Riemann Riemann ζ(s) function. ξ(t)=0 is also the same as the imaginary part of the nontrivial zero of the Riemann ζ(s) function and is a real number. According to the Riemann function $\prod\frac{s}{2}(s-1)\pi^{-\frac{s}{2}}\zeta(s)=\xi(t)(s\in C \text{ and } s\neq 1, t \in C \text{ and } t \neq 0)$ and Riemann defined $s=\frac{1}{2}+ti(t \in C \text{ and } t \neq 0)$, because s≠1, and $\prod\frac{s}{2} \neq o$, $\pi^{-\frac{s}{2}} \neq 0$, so $\prod\frac{s}{2}(s-1)\pi^{-\frac{s}{2}} \neq 0(s \in C \text{ and } s \neq 1)$,when $\zeta(s)=0(s \in C \text{ and } s \neq 1)$, then $\prod\frac{s}{2}(s-1)\pi^{-\frac{s}{2}}\zeta(s)=\prod\frac{(\frac{1}{2}+ti)}{2}(-\frac{1}{2}+ti)\pi^{-\frac{\frac{1}{2}+ti}{2}}\zeta(\frac{1}{2}+ti)=\xi(t)=0(s\in C \text{ and } s\neq 1, t \in C \text{ and } t \neq 0)$ and $\zeta(\frac{1}{2}+ti)=\frac{\xi(t)}{\prod\frac{(\frac{1}{2}+ti)}{2}(-\frac{1}{2}+ti)\pi^{-\frac{\frac{1}{2}+ti}{2}}} = \frac{0}{\prod\frac{(\frac{1}{2}+ti)}{2}(-\frac{1}{2}+ti)\pi^{-\frac{\frac{1}{2}+ti}{2}}} = 0(s \in C \text{ and } s \neq 1, s \neq -2n,, \text{and n}$ traverses all positive integers, $t \in C \text{ and } t \neq 0$),so $t\in R \text{ and } t \neq 0$ .So the root t of the equations $\prod\frac{(\frac{1}{2}+ti)}{2}(-\frac{1}{2}+ti)\pi^{-\frac{\frac{1}{2}+ti}{2}}\zeta(\frac{1}{2}+ti)=\xi(t)=0(t \in C \text{ and } t \neq 0)$ and

$4\int_1^{\infty}\frac{d(x^{\frac{3}{2}}\Psi'(x))}{dx}x^{-\frac{1}{4}}\cos(\frac{1}{2}t\ln x)dx=\xi(t=0(t \in C \text{ and } t \neq 0)$and

$\xi(t)=\frac{1}{2}-(t^2+\frac{1}{4})\int_1^{\infty}\Psi(x)x^{-\frac{3}{4}}\cos\left(\frac{1}{2}t\ln x\right)=0\ (t \in C \text{ and } t \neq 0)$ must be real and $t \neq 0$ .If $Re(s)=\frac{k}{2}(k \in R)$,then $\zeta(k-s)=2^{k-s}\pi^{-s}\cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)\ (s\in C \text{ and } s\neq 1, k \in R)$ and $\xi(k-s) = \frac{1}{2}\ s(s-k)\ \Gamma\left(\frac{s}{2}\right)\pi^{-\frac{s}{2}}\zeta(s)(s \in C \text{ and } s \neq 1, s \neq -2n, \text{and n traverses all positive integers}, k \in R)$are true, so when $\zeta(s)=0(s \in C \text{ and } s \neq 1)$,then $\zeta(s)=\zeta(k-s) = \zeta(\overline{s}) = 0(s\in C \text{ and } s\neq 1)$and $\xi(s)=\xi(k-s)=\xi(\overline{s})=0(s \in C \text{ and } s \neq 1, s \neq -2n, \text{and n traverses all positive integers})$ must be true , and $s=\frac{k}{2}+ti$ (k∈R , $t\in R \text{ and } t \neq 0$) must be true,then $\prod\frac{s}{2}(s-k)\pi^{-\frac{s}{2}}\zeta(s)=\prod\frac{(\frac{k}{2}+ti)}{2}(-k+\frac{1}{2}+ti)\pi^{-\frac{\frac{k}{2}+ti}{2}}\zeta\left(\frac{k}{2}+ti\right)=\xi(t)=0(s \in C \text{ and } s \neq 1, s \neq -2n, \text{and n traverses all positive integers}, k \in R)$, and $\zeta(\frac{k}{2}+ti)=\frac{\xi(t)}{\prod\frac{(\frac{k}{2}+ti)}{2}(-k+\frac{1}{2}+ti)\pi^{-\frac{\frac{k}{2}+ti}{2}}} = \frac{0}{\prod\frac{(\frac{k}{2}+ti)}{2}(-k+\frac{1}{2}+ti)\pi^{-\frac{\frac{k}{2}+ti}{2}}}=0(s\in C \text{ and } s\neq 1, t \in C \text{ and } t \neq$

$0, k \in R$),so t∈R and $t \neq 0$. So the root t of the equations $\prod \frac{\left(\frac{k}{2}+ti\right)}{2}(-k+\frac{1}{2}+ti)\pi^{-\frac{\frac{k}{2}+ti}{2}}\zeta\left(\frac{k}{2}+ti\right)$=ξ(t)=0($t \in C$ and $t \neq 0, k \in R$) must be real and $t \neq 0$. But the Riemann ζ(s)(s∈C and s≠ 1)function only satisfies $\zeta(1\text{-}s)=2^{1-s}\pi^{-s}\cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)$(s∈C and s≠ 1)(Formula 6) and $\xi(s) = \frac{1}{2}s(s\text{-}1)\Gamma(\frac{s}{2})\pi^{-\frac{s}{2}}\zeta(s)$ ($s \in C$ and $s \neq 1, s \neq -2n, n \in Z^+$) , is also say that only

$\zeta(1\text{-}s)=2^{1-s}\pi^{-s}\cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)$(s∈C and s≠ 1)(Formula 6)) is true , so only $Re(s)=\frac{k}{2}=\frac{1}{2}$ is true, so only k=1 is true.The Riemann hypothesis and the Riemann conjecture must satisfy the properties of the Riemann ζ(s)(s∈C and s≠ 1) function and the Riemann ξ(s)($s \in C$ and $s \neq 1, s \neq -2n$, and n traverses all positive integer) function, The properties of the Riemann ζ(s)(s∈C and s ≠ 1) function and the Riemann ξ(s)($s \in C$ and $s \neq 1, s \neq -2n$, and n traverses all positive integer) function are fundamental, the Riemann conjecture must be correct to reflect the properties of the Riemann ζ(s)(s∈C and s≠ 1) function and the Riemann ξ(s)($s \in C$ and $s \neq 1, s \neq -2n$, and n traverses all positive integer) function, that is, the roots of the Riemann ξ(t)(t∈C and t≠ 0)function can only be real, that is, Re(s) can only be equal to $\frac{1}{2}$, and Im Im(s) must be real, and Im(s) is not equal to zero.So the Riemann conjecture must be correct.

For any complex number s, when Rs(s) is any real number, including Rs(s)>0 and(s ≠ 1)and Rs(s) ≤ 0 and s ≠ 0), then Riemann ζ(s) function is $\zeta(s)=2^s\pi^{s-1}\sin(\frac{\pi s}{2})\Gamma(1\text{-}s)\zeta(1\text{-}s)$(s∈C and s≠ 1)(Formula 7). Suppose s= σ+ti(σ ∈R,t∈R and t ≠ 0,s∈C),let's prove that ζ(s)(s∈C and s≠ 1) and $\zeta(\bar{s})$(s∈C and s≠ 1) are complex conjugations of each other and get the equation $\zeta(s)=2^s\pi^{s-1}\sin(\frac{\pi s}{2})\Gamma(1\text{-}s)\zeta(1\text{-}s)$(s∈C and s≠ 1)(Formula 7).

Reasoning 2:

The reasoning in Riemann's paper goes like:

$2\sin(\pi s)\prod(s-1)\zeta(s)=(2\pi)^s\sum n^{s-1}((-i)^{s-1}+i^{s-1})$ [1] (Formula 3),

based on euler's $e^{ix}=\cos(x)+i\sin(x)$ $(x \in R)$ can get

$e^{i(-\frac{\pi}{2})}=\cos(\frac{-\pi}{2})+i\sin(\frac{-\pi}{2})=0\text{-}i=\text{-}i$,

$e^{i(\frac{\pi}{2})}=\cos(\frac{\pi}{2})+i\sin(\frac{\pi}{2})=0+i=i$,

then

$(-i)^{s-1}+i^{s-1}=(-i)^{-1}(-i)^s+(i)^{-1}(i)^s=(-i)^{-1}e^{i\left(-\frac{\pi}{2}\right)s}+i^{(-1)}e^{i\left(\frac{\pi}{2}\right)s}=$

$ie^{i\left(-\frac{\pi}{2}\right)s}\text{-}ie^{i\left(\frac{\pi}{2}\right)s}=i(\cos\frac{-\pi s}{2}+i\sin\frac{-\pi s}{2})\text{-}i(\cos\frac{\pi s}{2}+i\sin\frac{\pi s}{2})=i\cos(\frac{\pi s}{2})\text{-}i\cos(\frac{\pi s}{2})+\sin(\frac{\pi s}{2})+\sin(\frac{\pi s}{2})$

$=2\sin(\frac{\pi s}{2})$ (Formula 4).

According to the property of Π(s-1)=Γ(s) of the gamma function,and

$\sum_{n=1}^{\infty} n^{s-1}=\zeta(1\text{-}s)$($n \in Z^+$ and n traves all positive integer, $s \in C$, and $s \neq 1$),

Substitute the above (Formula 4) into the above (Formula 3), will get

$2\sin(\pi s)\Gamma(s)\zeta(s)=(2\pi)^s\zeta(1-s)2\sin\frac{\pi s}{2}$ (Formula 5),

If I substitute it into (Formula5), according to the double Angle formula $\sin(\pi s)=2\mathrm{Sin}(\frac{\pi s}{2})\cos(\frac{\pi s}{2})$, we Will get $\zeta(1\text{-}s)=2^{1-s}\pi^{-s}\cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)$ ($s\in C$ and $s\neq 1$) (Formula 6),because $\pi^{-\frac{1-s}{2}}\neq 0\neq 0$ and $\Gamma(\frac{1-s}{2})\neq 0$,so when $\zeta(s)=0$($s\in C$ and $s\neq 1$), then $\zeta(1\text{-}s)=0$($s\in C$ and $s\neq 1$),

Substituting s→1-s, that is taking s as 1-s into Formula 6, we will get

$\zeta(s)=2^s\pi^{s-1}\sin(\frac{\pi s}{2})\Gamma(1\text{-}s)\zeta(1\text{-}s)$($s\in C$ and $s\neq 1$)(Formula 7),

This is the functional equation for $\zeta(s)$ ($s\in C$ and $s\neq 1$). To rewrite it in a symmetric form, use the residual formula of the gamma function [3]

$\Gamma(Z)\Gamma(1\text{-}Z)=\frac{\pi}{\sin(\pi Z)}$ (Formula 8)

and Legendre's formula

$\Gamma(\frac{Z}{2})\Gamma(\frac{Z}{2}+\frac{1}{2})=2^{1-Z}\pi^{\frac{1}{2}}\Gamma(Z)$ (Formula 9) ,

Take $z=\frac{s}{2}$ in (Formula 8) and substitute it to get

$\sin(\frac{\pi s}{2})=\frac{\pi}{\Gamma(\frac{s}{2})\Gamma(1-\frac{s}{2})}$ (Formula 10) ,

In (Formula 9), let z=1-s and substitute it in to get

$\Gamma(1\text{-}s)=2^{-s}\pi^{-\frac{1}{2}}\Gamma(\frac{1-s}{2})\Gamma(1\text{-}\frac{s}{2})$ (Formula 11)

By substituting (Formula 10) and (Formula 11) into (Formula 7), we get

$\pi^{-\frac{s}{2}}\Gamma(\frac{s}{2})\zeta(s)=\pi^{-\frac{1-s}{2}}\Gamma(\frac{1-s}{2})\zeta(1\text{-}s)$($s\in C$ and $s\neq 1$), also

$\Gamma(\frac{s}{2})\pi^{-\frac{s}{2}}\zeta(s)$ is invariant under the transformation s→1-s,

And that's exactly what Riemann said in his paper.
That is to say:

$\Gamma(\frac{s}{2})\pi^{-\frac{s}{2}}\zeta(s)$ is invariant under the transformation s→1-s ,

also

$\prod(\frac{s}{2}-1)\pi^{-\frac{s}{2}}\zeta(s)=\prod(\frac{1-s}{2}-1)\pi^{-\frac{1-s}{2}}\zeta(1\text{-}s)$($s\in C$ and $s\neq 1$),

or

$\pi^{-\frac{s}{2}}\Gamma(\frac{s}{2})\zeta(s)=\pi^{-\frac{1-s}{2}}\Gamma(\frac{1-s}{2})\zeta(1\text{-}s)$($s\in C$ and $s\neq 1$)(Formula 2),

Then $\zeta(s)=2^s\pi^{s-1}\text{Sin}(\frac{\pi s}{2})\Gamma(1\text{-}s)\zeta(1\text{-}s)$($s\in C$ and $s\neq 1$) (Formula 7) ,

under the transformation s→1-s ,will get

$\zeta(1\text{-}s)=2^{1-s}\pi^{-s}\cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)$($s\in C$ and $s\neq 1$) (Formula 6). Then $\zeta(1\text{-}s)=\frac{\zeta(s)}{2^s\pi^{s-1}\sin(\frac{\pi s}{2})\Gamma(1-s)}$ ($s\in C$ and $s\neq 1$), when $\zeta(s)=0$ and $s\neq 2n(n\in Z^+)$, then if $\zeta(1\text{-}s)=\frac{\zeta(s)}{2^s\pi^{s-1}\sin(\frac{\pi s}{2})\Gamma(1-s)}$ ($s\in C$ and $s\neq 1$) is going to make sense, then the denominator $2^s\pi^{s-1}\sin(\frac{\pi s}{2})\Gamma(1\text{-}s)\neq 0$, Clearly indicates $2^s\neq 0$($s\in C$ and $s\neq 1$), $\pi^{s-1}\neq 0$($s\in C$ and $s\neq 1$), $\Gamma(1\text{-}s)\neq 0$($s\in C$ and $s\neq 1$), so $\sin(\frac{\pi s}{2})$ can not equal to zero, so $\sin(\frac{\pi s}{2})\neq 0$($s\in C$ and $s\neq 1$), so So when $\zeta(s)=0$ and $s\neq 2n(n\in Z^+)$, then $\zeta(1-s)=$

ζ(s) =0(s∈C and s≠ 1 ,and s ≠ −2n , n ∈ Z^+).

Because

L(s, ✗(n)) = ✗(n) ζ(s) (s ∈ C and s ≠ 1, n ∈ Z_+ and n goes through all the positive integer) and

L(1 − s, ✗(n))= ✗(n)ζ(1-s)(s∈C and s≠ 1, n ∈ Z^+ and n goes through all the positive integer) ,

and according to $\zeta(s)=2^s\pi^{s-1}\mathrm{Sin}(\frac{\pi s}{2})\Gamma(1-s)\zeta(1-s)$(s∈C and s≠ 1)(Formula 7), So

only $L(s, ✗(n))=2^s\pi^{s-1}\mathrm{Sin}(\frac{\pi s}{2})\Gamma(1-s)L(1-s, ✗(n))$(s ∈ C and s ≠ 1, n ∈ Z^+)(Formula 13).

According to the property that Gamma function Γ(s) and exponential function are nonzero, is also that $\Gamma(\frac{1-s}{2})\neq 0$, and $\pi^{-\frac{1-s}{2}}\neq 0$, according to $\pi^{-\frac{s}{2}}\Gamma(\frac{s}{2})\zeta(s)=\pi^{-\frac{1-s}{2}}\Gamma(\frac{1-s}{2})\zeta(1-s)$(s∈C and s≠ 1) (Formula 12),Mathematicians have shown that the real part of the complex independent variable s of the Riemann ζ(s)(s∈C and s≠ 1)function will have zero only if 0<Re(s)<1 and Im(s) ≠ 0, so we agree on Riemann $\zeta(s)=\frac{\eta(s)}{(1-2^{1-s})}=\frac{1}{(1-2^{1-s})}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^s}=\frac{(-1)^{n-1}}{(1-2^{1-s})}\prod_p(1-p^{-s})^{-1}$ (s ∈ C and 0 < Rs(s) < 1 and s ≠ 1 and Im(s) ≠ 0 , n∈ n ∈ Z^+, p ∈ n ∈ Z^+, s ∈C , n goes through all the positive integers , p goes through all the prime numbers).According the equation $\zeta(1-s)=2^{1-s}\pi^{-s}\cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)$(s∈C and s≠ 1) obtained by Riemann,since Riemann has shown that the Riemann ζ(s) (s ∈ C and s ≠ 1) function has zero, that is, in $\zeta(1-s)=2^{1-s}\pi^{-s}\cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)$ (s∈C and s≠ 1) (Formula 6), so ζ(s)=0(s∈C and s≠ 1) is true, and so we agree on $\zeta(1-s)=2^{1-s}\pi^{-s}\cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)$ (s ∈ C and 0 < Rs(s) < 1 and s ≠ 1 and Im(s) ≠ 0 ,n∈ Z^+, p ∈ Z^+, s ∈C , n goes through all the positive integers, p goes through all the prime numbers).

According to the property that Gamma function Γ(s) and exponential function are nonzero, is also that $\Gamma(\frac{1-s}{2})\neq 0$, and $\pi^{-\frac{1-s}{2}}\neq 0$,

So when ζ(s)=0(s∈C and s≠ 1), then ζ(1-s)=0(s∈C and s≠ 1), also must ζ(s)=ζ(1-s)=0(s∈C and s≠ 1).

Because $e=\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=\sum_{n=0}^{\infty}\frac{1}{n!}\approx 2.7182818284\ldots$,

and because $\sin(Z)=\frac{e^{iZ}-e^{-iZ}}{2i}$, Suppose Z=s= σ+ti (σ ∈R,t∈R and t ≠ 0), then

$\sin(s)=\frac{e^{is}-e^{-is}}{2i}=\frac{e^{i(\sigma+ti)}-e^{-i(\sigma+ti)}}{2i}$,

$\sin(\bar{s})=\frac{e^{i\bar{s}}-e^{-i\bar{s}}}{2i}=\frac{e^{i(\sigma-ti)}-e^{-i(\sigma-ti)}}{2i}$,

according $x^s=x^{(\sigma+ti)}=x^\sigma x^{ti}=x^\sigma(\cos(\ln x)+i\sin(\ln x))^t=x^\sigma(\cos(t\ln x)+i\sin(t\ln x))(x>0)$, then

$e^s=e^{(\sigma+ti)}=e^\sigma e^{ti}=e^\sigma(\cos(t)+i\sin(t))=e^\sigma(\cos(t)+i\sin(t))$,

$e^{is}=e^{i(\sigma+ti)}=e^{\sigma i}(\cos(it)+i\sin(it))=(\cos(\sigma)+i\sin(\sigma))(\cos(it)+i\sin(it))$,

$e^{i\bar{s}}=e^{i(\sigma-ti)}=e^{\sigma i}(\cos(-it)+i\sin(-it))=(\cos(\sigma)+i\sin(\sigma))(\cos(it)-i\sin(it))$,

e^{-is}=$e^{-i(\sigma+ti)}$=$e^{-\sigma i}(\cos(-it)+i\sin(-it)) = (\cos(\sigma)-i\sin(\sigma))(\cos(it)-i\sin(it))$,

$e^{-i\bar{s}} = e^{-i(\sigma-ti)}$=$e^{-\sigma i}(\cos(it)+i\sin(it)) = (\cos(\sigma)-i\sin(\sigma))(\cos(it)+i\sin(it))$,

2^s=$2^{(\sigma+ti)}$=$2^\sigma 2^{ti}$=$2^\sigma(\cos(\ln 2)+i\sin(\ln 2))^t$=$2^\sigma(\cos(t\ln 2)+i\sin(t\ln 2))$,

$2^{\bar{s}}$=$2^{(\rho-ti)}$=$2^\sigma 2^{-ti}$=$2^\sigma(\cos(\ln 2)+i\sin(\ln 2))^{-t}$ =$2^\sigma(\cos(t\ln 2)-i\sin(t\ln 2))$,

π^{s-1}=$\pi^{(\sigma-1+ti)}$=$\pi^{\sigma-1}\pi^{ti}$=$\pi^{\rho-1}(\cos(\ln\pi)+i\sin(\ln\pi))^t$=$\pi^{\sigma-1}(\cos(t\ln\pi)+i\sin(t\ln\pi))$,

$\pi^{\bar{s}-1}$=$\pi^{(\sigma-1-ti)}$=$\pi^{\sigma-1}\pi^{-ti}$=$\pi^{\sigma-1}(\cos(\ln\pi)+i\sin(\ln\pi))^{-t}$=$2^{\sigma-1}(\cos(t\ln\pi)-i\sin(t\ln\pi))$,

So

2^s=$\overline{2^{\bar{s}}}$, π^{s-1}=$\overline{\pi^{\bar{s}-1}}$,

and

$\frac{e^{is}-e^{-is}}{2i}$ =$\overline{\frac{e^{i\bar{s}}-e^{-i\bar{s}}}{2i}}$,

So

sin(s)=$\overline{\sin(\bar{s})}$,

and

sin($\frac{\pi s}{2}$)=$\overline{\sin(\frac{\pi\bar{s}}{2})}$.

And the gamma function on the complex field is defined as:

Γ(s)=$\int_0^{+\infty} t^{s-1}\, e^{-t}$dt,

Among Re(s)>0,this definition can be extended by the analytical continuation principle to the entire field of complex numbers, except for non-positive integers,

So

Γ(s)=$\overline{\Gamma(\bar{s})}$,

and

Γ(1-s)= $\overline{\Gamma(1-\bar{s})}$ .When ζ(1-$\bar{s}$)= $\overline{\zeta(1-\bar{s})}$ =0=ζ(s)=ζ(1-s)=0 (s ∈ C and $s \neq 1$), and according

ζ(s)=$2^s\pi^{s-1}$Sin($\frac{\pi s}{2}$)Γ(1-s)ζ(1-s)(s∈C and s≠ 1and $s \neq 0$), then ζ(s)=$\overline{\zeta(\bar{s})}$ $= 0$(s∈C and s≠ 1),is also say ζ(s)=ζ($\bar{s}$)=ζ(1-$\bar{s}$)=0(s∈C and s≠ 1). so only ζ(σ+ti)=ζ(σ-ti)=0 is true. According the equation

ζ(1-s)= $2^{1-s}\pi^{-s}$cos($\frac{\pi s}{2}$)Γ(s)ζ(s) (s ∈ C and s ≠ 1) obtained by Riemann,since Riemann has shown that the Riemann ζ(s)(s ∈ C and s ≠ 1) function has zero, that is, in

ζ(1-s)=$2^{1-s}\pi^{-s}$cos($\frac{\pi s}{2}$)Γ(s)ζ(s)(s ∈ C, and s ≠ 1)(Formula 7), ζ(s)=0(s ∈ C, and s ≠ 1) is true, so when ζ(s)=0(s ∈ C, and s ≠ 1), then only ζ(s)=ζ(1-s)=0(s ∈ C, and s ≠ 1) is true.in the process of the Riemann hypothesis proved about ζ(s)=ζ(1-s)= ζ($\bar{s}$)=0, is refers to the ζ(s) is a functional number? It's not. Does ζ(s)=ζ(1-s)=ζ($\bar{s}$)(s ∈ C and s ≠ 1) mean the symmetry of the ζ(s) function equation? Does that mean the symmetry of the equation s=$\bar{s}$=1-s? Not really. In my analyst, ζ(s)、ζ(1-s) and ζ($\bar{s}$) function expression are $\sum_{n=1}^{\infty} n^{-s}$(n∈ Z^+ and n traves all positive integer, s ∈ C, and s ≠ 1), so according to $\sum_{n=1}^{\infty} n^{-s}$ (n ∈ Z^+ and n traves all positive integer, s ∈ C, and s ≠ 1) ,ζ(s) (s ∈ C, and s ≠ 1) function of the independent variable s, the relationship between $\bar{s}$ and 1-s only C_3^2=3 kinds, namely s=$\bar{s}$ or s=1-s or $\bar{s}$=1-s. As follows: According ζ(s)=ζ(1-s)=0(s ∈ C, and s ≠ 1) and ζ(s)=ζ($\bar{s}$) =0(s ∈ C, and s ≠ 1), so s and 1-s are also conjugate,then only s=$\bar{s}$ or s=1-s or $\bar{s}$=1-s ,so s∈R, or σ+ti=1-σ-ti ,or σ-ti=1-σ-ti, so $s \in R$ and $s = -2n(n \in Z^+)$, or σ=$\frac{1}{2}$and t=0,or $\sigma = \frac{1}{2}$ and t ∈R and t≠ 0,so s ∈ R, or s=$\frac{1}{2}$+oi ,or s=$\frac{1}{2}$+ti(t ∈R and t ≠ 0) and s=$\frac{1}{2}$-ti(t ∈R and t ≠ 0),because $\zeta\left(\frac{1}{2}\right) \to +\infty, \zeta(1) \to +\infty$, ζ(1) is divergent, ζ($\frac{1}{2}$) is more divergent,so drop them.Beacause only when ρ=$\frac{1}{2}$,the next three equations, $\zeta(\sigma + ti)$= 0, $\zeta(1 - \sigma - ti)$= 0, and ζ(σ-ti)=0 are all true, because $\zeta\left(\frac{1}{2}\right) \to +\infty, \zeta(1) \to +\infty$, ζ (1) is divergent, ζ($\frac{1}{2}$) is more divergent,so only s=$\frac{1}{2}$+ti(t∈R and t ≠ 0) and s=$\frac{1}{2}$-ti(t∈R and t ≠ 0) are true, or say only s=$\frac{1}{2}$+ti(t∈R and t ≠ 0) and s=$\frac{1}{2}$-ti(t∈R and t ≠ 0)are true.Since Riemann has shown that the Riemann ζ(s) (s ∈ C and s ≠ 1) function has zero, that is, in ζ(1-s)= $2^{1-s}\pi^{-s}\cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)$ (s ∈ C and s ≠ 1)(Formula 7), ζ(s)=0(s ∈ C, and s ≠ 0 and s ≠ 1)is true. According the equation $\xi(s) = \frac{1}{2}$ s(s-1) $\Gamma\left(\frac{s}{2}\right)\pi^{-\frac{s}{2}}\zeta(s)$(s ∈ C , and s ≠ 1 , and $s \neq -2n, n \in Z^+$) obtained by Riemann, so ξ(s)=ξ(1 − s)(s ∈ C, and s ≠ 1and $s \neq -2n, n \in Z^+$), because $\Gamma(\frac{s}{2})=\overline{\Gamma(\frac{\bar{s}}{2})}$, and $\pi^{-\frac{s}{2}}=\overline{\pi^{-\frac{\bar{s}}{2}}}$, and because $\zeta(s)=\overline{\zeta(\bar{s})}$(s ∈ C, and s ≠ 1) , so $\xi(s)=\overline{\xi(\bar{s})}$(s ∈ C, and s ≠ 1 and $s \neq -2n, n \in Z^+$, and n traverses all positive integers), So when ζ(s)=0 (s ∈ C, and s ≠ 1 and $s \neq -2n, n \in Z^+$) ,then $\zeta(s) = \zeta(1 - s) = \zeta(\bar{s}) = 0$(s ∈ C and s ≠ 1and $s \neq -2n, n \in Z^+$) and $\xi(s)=\xi(1-s)=\xi(\bar{s})=0$(s ∈ C, and s ≠ 1and $s \neq -2n, n \in Z^+$) must be true , so the zeros of the Riemannξ(s) function and the nontrivial zeros of the Riemann ζ(s)(s ∈ C, and s ≠ 1) function are identical, so the complex root of Riemann ξ(s)=0(s ∈ C, and s ≠ 1) satisfies s=$\frac{1}{2}$+ti(t∈R and t ≠ 0) and s=$\frac{1}{2}$-ti(t∈R and t ≠ 0).According to the Riemann function $\prod\frac{s}{2}(s-1)\pi^{-\frac{s}{2}}\zeta(s)=\xi(t)$(t ∈ C and t ≠ 0, s ∈ C and s ≠ 1and s ≠ 0) and he Riemann definded s=$\frac{1}{2}$+ti(t∈C and t ≠ 0), because s≠1 and and $s \neq -2n(n \in Z^+)$, and $\prod\frac{s}{2} \neq o$, $\pi^{-\frac{s}{2}} \neq 0$, so $\prod\frac{s}{2}(s-1)\pi^{-\frac{s}{2}} \neq o$, and when ξ(t)=0, then $\prod\frac{(\frac{1}{2}+ti)}{2}(-\frac{1}{2}+ti)\pi^{-\frac{\frac{1}{2}+ti}{2}}\zeta(\frac{1}{2}+ti)$ =ξ(t)=0, and $\zeta(\frac{1}{2}+ti)=\frac{\xi(t)}{\prod\frac{s}{2}(s-1)\pi^{-\frac{s}{2}}}=\frac{0}{\prod\frac{s}{2}(s-1)\pi^{-\frac{s}{2}}}=0$, so t∈R and t ≠ 0. So the root t of the equations $\prod\frac{(\frac{1}{2}+ti)}{2}(-\frac{1}{2}+ti)\pi^{-\frac{\frac{1}{2}+ti}{2}}\zeta(\frac{1}{2}+ti)=\xi(t)=0$ and $4\int_1^{\infty}\frac{d(x^{\frac{3}{2}}\Psi'(x))}{dx}x^{-\frac{1}{4}}\cos(\frac{1}{2}t\ln x)dx=$ ξ(t)=0 (t ∈ C and t ≠ 0)and $\xi(t)=\frac{1}{2}-(t^2+\frac{1}{4})\int_1^{\infty}\Psi(x)\,x^{-\frac{3}{4}}\cos(\frac{1}{2}t\ln x)=0$(t ∈ C and t ≠ 0) must be real and t ≠ 0. If Re(s)= $\frac{k}{2}$(k ∈ R) ,then ζ(k-s)= $2^{k-s}\pi^{-s}\cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)$ (s ∈ C and s ≠ 1 and $s \neq -2n, n \in Z^+$, k ∈ R) and $\xi(k - s) = \frac{1}{2}$ s(s-k) $\Gamma(\frac{s}{2})\pi^{-\frac{s}{2}}\zeta(s)$(s ∈ C, and s ≠ 1and $s \neq -2n, n \in Z^+$, s ∈ C, and s ≠ 1, and $s \neq -2n, n \in Z^+$, n traverses all positive integers, k ∈ R)are true. So when ζ(s)=0(s ∈ C, and s ≠ 1 and $s \neq -2n, n \in Z^+$),then $\zeta(s)=\zeta(k-s) = \zeta(\bar{s}) = 0$(s ∈ C, and s ≠ 1, and $s \neq -2n, n \in Z^+$, n traverses all positive integers, k ∈ R) and

$\xi(s) = \xi(k-s) = \xi(\bar{s}) = 0$ $(s \in C$ and $s \neq 1$, and $s \neq -2n, n \in Z^+$, n traverses all positive integers, $k \in R)$ must be true, and $s=\frac{k}{2}+ti$ $(k\in R, t\in R$ and $t \neq 0)$

must be true, then $\prod\frac{s}{2}(s-k)\,\pi^{-\frac{s}{2}}\zeta(s)= \prod\frac{(\frac{k}{2}+ti)}{2}(-k+\frac{1}{2}+ti)\,\pi^{-\frac{\frac{k}{2}+ti}{2}}\zeta(\frac{k}{2}+ti)=\xi(t)=0$ $(k \in R, t \in C$ and $t \neq 0, s \in C$, and $s \neq 1$, and $s \neq -2n, n \in Z^+$, n traverses all positive integers),

$\zeta(\frac{k}{2}+ti)=\frac{\xi(t)}{\prod\frac{(\frac{k}{2}+ti)}{2}(-k+\frac{1}{2}+ti)\pi^{-\frac{\frac{k}{2}+ti}{2}}}=\frac{0}{\prod\frac{(\frac{k}{2}+ti)}{2}(-k+\frac{1}{2}+ti)\pi^{-\frac{\frac{k}{2}+ti}{2}}}=0$ $(k\in R, t\in C$ and $t \neq 0)$, so $t\in R$ and $t \neq 0$. So

the root of the equations $\prod\frac{(\frac{k}{2}+ti)}{2}(-k+\frac{1}{2}+ti)\pi^{-\frac{\frac{k}{2}+ti}{2}}\zeta(\frac{k}{2}+ti)=\xi(t)=0$ $(k\in R, t\in C$ and $t \neq 0)$ must be real and $t \neq 0$. But the Riemann $\zeta(s)$ function only satisfies $\zeta(1-s)=2^{1-s}\pi^{-s}\cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)$ $(s \in C$ and $s \neq 1)$ and $\xi(s) = \frac{1}{2}s(s-1)\Gamma\left(\frac{s}{2}\right)\pi^{-\frac{s}{2}}\zeta(s)$

$(s \in C$, and $s \neq 1$, $s \neq -2n, n \in Z^+$, n traverses all positive integers), is also say that only

$\zeta(1-s)=2^{1-s}\pi^{-s}\cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)$ $(s \in C$ and $s \neq 1)$ (Formula 7) is true, so only $Re(s)=\frac{k}{2}=\frac{1}{2}$ $(k \in R)$ is true, so only k=1 is true.The Riemann conjecture must satisfy the properties of the Riemann $\zeta(s) = 0(s \in C$, and $s \neq 1)$ function and the Riemann $\xi(s) = 0(s \in C$ and $s \neq 1$ and $s \neq -2n, n \in Z^+$, n traverses all positive integers$)$ function, The properties of the Riemann $\zeta(s)= 0(s \in C$, and $s \neq 1$, $s \neq -2n, n \in Z^+)$ function and the Riemann $\xi(s) = 0(s \in C$, and $s \neq 1$ and $s \neq -2n, n \in Z^+$, n traverses all positive integers$)$ function are fundamental, the Riemann conjecture must be correct to reflect the properties of the Riemann $\zeta(s)= 0(s \in C$, and $s \neq 1$, $s \neq -2n, n \in Z^+$, n traverses all positive integers$)$ function and the Riemann $\xi(s) = 0(s \in C$, and $s \neq 1$ and $s \neq -2n, n \in Z^+$, n traverses all positive integers$)$ function, that is, when $\zeta(s)= 0(s \in C$, and $s \neq 1$ and $s \neq -2n, n \in Z^+)$, the roots of the Riemann $\xi(t)(t \in C$ and $t \neq 0)$ function can only be real, that is, Re(s) can only be equal to $\frac{1}{2}$, and Im(s) must be real, and Im(s) is not equal to zero.So the Riemann Riemann conjecture must be correct.Riemann found in his paper that

$$\prod\left(\frac{s}{2}-1\right)\pi^{-\frac{s}{2}}\zeta(s) = \int_1^{\infty}\psi(x)\,x^{\frac{s}{2}-1}dx+\int_1^{\infty}\psi(\tfrac{1}{x})\,x^{\frac{s-3}{2}}dx + \frac{1}{2}\int_0^1(x^{\frac{s-3}{2}}-x^{\frac{s}{2}-1})dx$$

$= \frac{1}{s(s-1)} + \int_1^{\infty}\psi(x)\,(x^{\frac{s}{2}-1}+x^{-\frac{1+s}{2}})dx$ $(s \in C$ and $s \neq 1)$ $(s \in C$ and $s \neq 1)$, Because $\frac{1}{s(s-1)}$ and $\int_1^{\infty}\psi(x)(x^{\frac{s}{2}-1}+x^{-\frac{1+s}{2}})dx$ are all invariant under the transformation s→1-s If I introduce the auxiliary function $\psi(s)=\prod\left(\frac{s}{2}-1\right)\pi^{-\frac{s}{2}}\zeta(s)(s \in C$, and $s \neq 1$ and $s \neq -2n, n \in Z^+)$,So I can just write it as $\psi(s)=\psi(1-s)$. But it would be more convenient to add the factor $s(s-1)$to $\psi(s)$ and introduce the coefficient $\frac{1}{2}$, which is exactly what Riemann did, is that to take $\xi(s) = \frac{1}{2}s(s-1)\Gamma\left(\frac{s}{2}\right)\pi^{-\frac{s}{2}}\zeta(s)(s\in C$ and $s \neq -2n, n \in Z^+$, and $s \neq 1)$.Because the factor (s-1) cancels out the first pole of $\zeta(s)$ at s=1, And the factor s cancels out the pole of $\Gamma\left(\frac{s}{2}\right)$ at s=0, and s is equal to -2, -4, -6,...,the rest of the poles of $\Gamma\left(\frac{s}{2}\right)$ cancel out . So $\xi(s)$is an integral function.And the factor $s(s-1)$ obviously doesn't change under the transformation $s\rightarrow 1-s$,so we also have the function

$\xi(s) = \xi(1-s) = 0(s \in C \text{ and } s \neq 1 \text{ and } s \neq -2n, n \in Z^{+}, n \text{ traverses all positive integers })$, base on $\zeta(1\text{-}s)= 2^{1-s}\pi^{-s}\cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)(s \in C \text{ and } s \neq 1)(Formula\ 7)$. At the same time, according to $\zeta(1\text{-}s)=2^{1-s}\pi^{-s}\cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)(s \in C \text{ and } s \neq 1)$, if $\zeta(s)=0$ ($s\in C$ and $s\neq 1$),then must $\zeta(1\text{-}s)=0$ ($s\in C$ and $s\neq 1$), is that to say $\zeta(s)=\zeta(1\text{-}s)=0(s\in C$ and $s\neq 1)$. According to Riemann definded $s=\frac{1}{2}+ti(t\in C \text{and } t \neq 0)$, s and t differ by a linear transformation . It's a 90 degree rotation plus a translation of $\frac{1}{2}$. So line Re(s)=$\frac{1}{2}$ in the s plane corresponds to the real number line in the t plane,the zero of Riemann $\zeta(s)(s \in C \text{ and } s \neq 1 \text{ and } s \neq -2n, n \in Z^{+}, n \text{ traverses all positive integers})$ on the critical line Re(s)=$\frac{1}{2}$ corresponds to the real root of $\xi(t)(t\in C \text{ and } t \neq 0)$. In Riemann function $\xi(t)(t \in C \text{ and } t \neq 0)$, the function equation $\xi(s)=\xi(1-s)\left(s \in C \text{ and } s \neq 1 \text{ and } s \neq -2n, n \in Z^{+}\right)$becomes equation $\xi(t)=\xi(-t)(t \in C \text{ and } t \neq 0)$ is an even function, an even function is a symmetric function, it's zeros are distributed symmetrically with respect to t=0 .The function $\xi(t)(t \in C \text{and } t \neq 0)$ designed by Riemann and $s = \frac{1}{2} + ti(t \in C \text{ and } t \neq 0)$ definded by Riemann and $\xi(s) = \xi(1-s)(s \in C \text{ and } s \neq 1 \text{and } s \neq -2n, n \in Z^{+}, n \text{ traverses all positive integers})$are equivalent to $\xi(t)=\xi(-t)(t \in C \text{ and } t \neq 0)$.So the function $\xi(s)(s \in C \text{ and } s \neq 1 \text{ and } s \neq -2n, n \in Z^{+}, n \text{ traverses all positive integers})$ is also an even function.The zero points on the graph of an even function $\xi(s)\left(s \in C \text{ and } s \neq 1 \text{and } s \neq -2n, n \in Z^{+}, n \text{ traverses all positive integers}\right)$ with respect to the coordinates of its argument on the real number line equal to some value are symmetrically distributed on the line perpendicular to the real number line of the complex plane. When $\xi(t)=0(t \in C \text{ and } t \neq 0)$, is also that $\xi(t)=\xi(-t)=0(t \in C \text{ and } t \neq 0)$,the zeros of $\xi(t)(t \in C \text{ and } t \neq 0)$are symmetrically distributed with respect to t equals 0 . When $\xi(s) = 0\left(s \in C \text{ and } s \neq 1 \text{and } s \neq -2n, n \in Z^{+}, , n \text{ traverses all positive integers}\right)$,is also that $\xi(s) = \xi(1-s) = 0\left(s \in C \text{ and } s \neq 1 \text{ and } s \neq -2n, n \in Z^{+}\right)$,the zeros of $\xi(s)\left(s \in C \text{ and } s \neq 1 \text{and } s \neq -2n, n \in Z^{+}, , n \text{ traverses all positive integers}\right)$are symmetrically distributed with respect to point ($\frac{1}{2}$,0i) on a line perpendicular to the real number line of the complex plane.So when $\xi(s)=\xi(1-s)=0\left(s \in C \text{ and } s \neq 1 \text{ and } s \neq -2n, n \in Z^{+}, , n \text{ traverses all positive integers}\right)$, s and 1-s are pair of zeros of the function $\xi(s)\left(s \in C \text{ and } s \neq 1 \text{ and } s \neq -2n, n \in Z^{+}\right)$ symmetrically distributed in the complex plane with respect to point ($\frac{1}{2}$, 0i) on a line perpendicular to the real number line of the complex plane.When $\zeta(s)=0(s\in C$ and $s\neq 1$ and $s \neq -2n, n \in Z^{+}, n \text{ traverses all positive integers})$, then $\zeta(1\text{-}s)=0(s\in C$ and $s\neq 1)$, is aslo that

$\zeta(s)=\zeta(1\text{-}s)=0\left(s \in C \text{ and } s \neq 1 \text{ and } s \neq -2n, n \in Z^{+}, n \text{ traverses all positive integers}\right)$. We find $\zeta(s)=\zeta(1\text{-}s)=0\left(s \in C \text{ and } s \neq 1 \text{and } s \neq -2n, n \in Z^{+}, n \text{ traverses all positive integers}\right)$ and $\xi(s) = \xi(1-s) = 0\left(s \in C \text{ and } s \neq 1 \text{ and } s \neq -2n, n \in Z^{+}, n \text{ traverses all positive integers}\right)$ are just the name of the function is idifferent,the independent variable s is equal to $\frac{1}{2}+ti(t\in C$ and

$t \neq 0$),that means that the zero arguments of function $\zeta(s)(s \in C$ and $s \neq 1$ and $s \neq -2n, n \in Z+$ and function ξssϵC and s≠1 and s≠−2n,nϵZ+,n traverses all positive integers are exactly the same,so the zeros of the

$\zeta(s)$ $(s \in C$ and $s \neq 1$, and $s \neq -2n, n \in Z^+$, n traverses all positive integers$)$ function in the complex plane also correspond to the symmetric distribution of point $(\frac{1}{2}, 0i)$ on a line perpendicular to the real number line in the complex plane, so When $\zeta(s) = \zeta(1-s) = 0(s \in C$, and $s \neq 1$and $s \neq -2n, n \in Z^+$, n traverses all positive integers),s and 1-s are pair of zeros of the function

$\zeta(s)(s \in C$ and $s \neq 1$and $s \neq -2n, n \in Z^+$, n traverses all positive integers$)$ symmetrically distributed in the complex plane with respect to point $(\frac{1}{2}, 0i)$ on a line perpendicular to the real number line of the complex plane.We got $\overline{\zeta(s)}=\zeta(\overline{s})(s=\sigma+ti, \sigma \in R,\ t \in R$ and $t \neq 0$, and $s \neq 1$and $s \neq -2n, n \in Z^+$, n traverses all positive integers) before,When t in Riemann definded $s=\frac{1}{2}+ti(t\in C$ and $t \neq 0)$ is a complex number, then s in $\overline{\zeta(s)}=\zeta(\overline{s})(s=\sigma+ti, \sigma \in R,\ t \in R$ and $t \neq 0, s \neq 1$and $s \neq -2n, n \in Z^+$, n traverses all positive integers) are consistent with s in Riemann's appoint $s=\frac{1}{2}+ti(t\in C$ and $t \neq 0)$ and $s=\frac{1}{2}-ti(t\in C$ and $t \neq 0)$.If $\zeta(s)=\zeta(\overline{s})=0(s \in C$, and $s \neq 1$and $s \neq -2n, n \in Z^+$),Since s and $\overline{s}$ are a pair of conjugate complex numbers,So s and $\overline{s}$ must be a pair of zeros of the function $\zeta(s)(s \in C$, and $s \neq 1$and $s \neq -2n, n \in Z^+)$ in the complex plane with respect to point $(\sigma,0i)$ on a line perpendicular to the real number line.s is a symmetric zero of 1-s, and a symmetric zero of $\overline{s}$. By the definition of complex numbers, how can a symmetric zero of the same function $\zeta(s)(s \in C$, and $s \neq 1$and $s \neq -2n, n \in Z^+)$ of the same zero independent variable s on a line perpendicular to the real number axis of the complex plane be both a symmetric zero of 1-s on a line perpendicular to the real number axis of the complex plane with respect to point $(\frac{1}{2}, 0i)$ and a symmetric zero of $\overline{s}$ on a line perpendicular to the real number axis of the complex plane with respect to point $(\sigma, 0i)$? Unless σ and $\frac{1}{2}$ are the same value, is also that $\sigma = \frac{1}{2}$, and only 1-s=$\overline{s}$ is true, and 1-s=s is wrong.Otherwise it's impossible,this is determined by the uniqueness of the zero of the function $\zeta(s)(s \in C$, and $s \neq 1$and $s \neq -2n, n \in Z^+)$ on the line passing through that point perpendicular to the real number axis of the complex plane with respect to the vertical foot symmetric distribution of the zero of the line and the real number axis of the complex plane,only one line can be drawn perpendicular from the zero independent variable s of the function $\zeta(s)(s \in C$, and $s \neq 1$and $s \neq -2n, n \in Z^+)$ to the real number line of the complex plane, the vertical line has only one point of intersection with the real number axis of the complex plane. In the same complex plane, the same zero point of the function

$\zeta(s)(s \in C$, and $s \neq 1$and $s \neq -2n, n \in Z^+)$ on the line passing through that point perpendicular to the real number line of the complex plane there will be only one zero point about the vertical foot symmetric distribution of the line and the real number line of the complex plane.Because $\overline{\zeta(s)} = \zeta(\overline{s})$($s \in C$, and $s \neq 1$and $s \neq -2n, n \in Z^+$), then if $\zeta(\sigma+ti)$=0, then $\zeta(\sigma-ti)$=0, and because $\zeta(s)=\zeta(1-s)=0(s \in C$, and $s \neq 1$and $s \neq -2n, n \in Z^+$), then ζ(1-σ-ti)=0, and because $\zeta(s)=\zeta(1-s)=0(s \in C$, and $s \neq 1$and $s \neq -2n, n \in Z^+)$, then ζ(1-σ-ti)=0. The next three equations, $\zeta(\sigma+ti)$=0, $\zeta(\sigma-ti)$=0, and ζ(1-σ-ti)=0, are all true, so only 1-σ=σ is true,only $s=\frac{1}{2}+ti(t\in R$ and $t \neq 0)$ and $s=\frac{1}{2}-ti(t\in R$ and $t \neq 0)$are true.Since the harmonic series ζ(1) diverges, it has been proved by the late medieval French scholar Orem (1323-1382).The Riemann hypothesis and the Riemann conjecture must satisfy the properties of the Riemann ζ(s) $(s \in C, \text{and } s \neq 1 \text{and } s \neq -2n, n \in Z^+)$ function and the Riemann ξ(s)$(s \in C, \text{and } s \neq 1 \text{and } s \neq -2n, n \in Z^+)$ function, The properties of the Riemann ζ(s)$= 0(s \in C \text{ and } s \neq 1 \text{and } s \neq -2n, n \in Z^+)$ function and the Riemann $\xi(s) = 0(s\in C$ and $s\neq 1)$ function are fundamental, the Riemann conjecture must be correct to reflect the properties of the Riemann ζ(s)$(s \in C, \text{and } s \neq 1 \text{and } s \neq -2n, n \in Z^+)$ function and the Riemann ξ(s)$(s \in C \text{ and } s \neq 1 \text{and } s \neq -2n, n \in Z^+)$ function, that is, the roots of the Riemann ξ(t)(t∈C and t$\neq 0$) function must only be real, that is, Re(s) can only be equal to $\frac{1}{2}$, and Im(s) must be real, and Im(s) is not equal to zero.So the Riemann hypothesis and the Riemann conjecture must be correct. Riemann

got $\prod \frac{s}{2}(s-1)\pi^{-\frac{s}{2}}\zeta(s)=\xi(t)(t\in R \text{ and } t \neq 0,\ s \in C, \text{and } s \neq 1, s \neq -2n, n \in Z^+)$,and $\xi(t)=\frac{1}{2} -(t^2 + \frac{1}{4})\int_1^{\infty} \Psi(x)\, x^{-\frac{3}{4}} \cos\left(\frac{1}{2}t\ln x\right) dx(t\in R \text{ and } t \neq 0)$ in his paper, or

$\prod \frac{s}{2}(s-1)\pi^{-\frac{s}{2}}\zeta(s) = \prod \frac{(\frac{1}{2}+ti)}{2}(-\frac{1}{2}+ti)\pi^{-\frac{\frac{1}{2}+ti}{2}}\zeta(\frac{1}{2}+ti)=\xi(t)(t\in R \text{ and } t \neq 0,\ s \in C, \text{and } s \neq 1 \text{and } s \neq -2n, n \in Z^+)$ and $\xi(t)=4\int_1^{\infty} \frac{d\left(x^{\frac{3}{2}}\Psi'(x)\right)}{dx} x^{-\frac{1}{4}}\cos(\frac{1}{2}t\ln x)\ dx(t \in C \text{ and } t \neq 0, s \in C \text{ and } s \neq 1 \text{ and } s \neq -2n, n \in Z^+)$. Because $\zeta(\frac{1}{2}+ti)=0(t\in C \text{ and } t \neq 0)$, so the roots of

$\prod \frac{(\frac{1}{2}+ti)}{2}(-\frac{1}{2}+ti)\pi^{-\frac{\frac{1}{2}+ti}{2}}\zeta(\frac{1}{2}+ti)=\xi(t)= 0(t\in C \text{ and } t \neq 0)$ and $4\int_1^{\infty} \frac{d(x^{\frac{3}{2}}\Psi'(x))}{dx} x^{-\frac{1}{4}}\cos(\frac{1}{2}t\ln x)dx=\xi(t)= 0$ $(t\in C \text{ and } t \neq 0)$ and $\xi(t)=\frac{1}{2} -(t^2 + \frac{1}{4})\int_1^{\infty} \Psi(x)\ x^{-\frac{3}{4}}\cos(\frac{1}{2}t\ln x)=0(t\in C \text{ and } t \neq 0)$ must all be real numbers. According to the $2\sin(\pi s)\prod(s-1)\zeta(s) = \int_{\infty}^{\infty} \frac{x^{s-1}dx}{e^x-1}$ Riemann got in his paper and the $\zeta(1-s)=2^{1-s}\pi^{-s}\cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)(s \in C \text{ and } s \neq 1)$, We know that the Riemann ζ(s)(s∈C and s≠1) function is a two-to-one mapping, or even a many-to-one mapping deterministic universal function, or a one-to-two mapping, or even a one-to- many mapping deterministic universal function. If we consider the Riemann ζ(s)(s∈C and $s \neq 1$) function as a general complex number whose domain includes real numbers, then s=-2n(n is a positive integer) is the only class of real zeros of the Riemann ζ(s)(s∈C and s≠1) function at the root, If we consider the Riemann ζ(s)(s∈C and s≠1) function as a general complex number whose domain does not include real numbers, then $s=\frac{1}{2}+ti$(t∈R and t≠0) is the only class of complex zeros of the Riemann ζ(s)(s∈C and s≠1) function at the root, so the zero real root of the Landau-Siegel function $L(\beta, \chi(n))(\beta \in R, \chi(n)=1)$ does not exist.

When $\zeta(s)= 0(s \in C, \text{and } s \neq 1 \text{ and } s \neq -2n, n \in Z^+)$ and $\xi(t)=0(t \in C \text{ and } t \neq 0)$, the real part of the equation ξ(t)=0(t∈C and $t \neq 0$) must be real between 0 and T. Because the real part of the equation ξ(t)=0 has the number of complex roots between 0 and T approximately equal to $\frac{T}{2\pi}\ln\frac{T}{2\pi} - \frac{T}{2\pi}$ [1] ,This result of Riemann's estimate of the number of zeros was rigorously proved by Mangoldt in 1895. Then,when $\zeta(s)=0(s\in C$ and $s\neq 1 \text{ and } s \neq -2n, n \in Z^+)$ and $\xi(t)=0(t \in C \text{ and } t \neq 0)$, the number of real roots of the real part of the equation ξ(t)=0(t ∈

C and t ≠ 0) between 0 and T must be approximately equal to $\frac{T}{2\pi}\ln\frac{T}{2\pi}-\frac{T}{2\pi}$,so when the Riemann $\zeta(s)$(s∈C and s≠ 1 and $s \neq -2n, n \in Z^{+}$) function has nontrivial zeroes, then and the Riemann conjecture are perfectly valid. N=$\lim_{T\to+\infty}(\frac{T}{2\pi}\ln\frac{T}{2\pi}-\frac{T}{2\pi})\to\infty$, so the Riemann ζ(s)(s∈C and s≠ 1 and $s \neq -2n, n \in Z^{+}$) function in Re(s)=$\frac{1}{2}$ nontrivial critical line zero have an infinite number, 1921, The British mathematician Hardy has proved that the Riemann ζ(s)(s∈C and s≠ 1 and $s \neq -2n, n \in Z^{+}$) function has an infinite number of non-trivial zeros on the critical line Re(s)=$\frac{1}{2}$, but he did not prove that the non-trivial zeros of the Riemann ζ(s)(s∈C and s≠ 1 and $s \neq -2n, n \in Z^{+}$) function are all on the critical line Re(s)=$\frac{1}{2}$.

Definition: Assuming that a(n) is a uniproduct function, then the Dirichlet series $\sum_{n=1}^{\infty} a(n)n^{-s}$ (s ∈ C and s ≠ 1, n ∈ Z^{+} and n goes through all the positive numbers) is equal to the Euler product

$\prod_p P(p, s)$ (s ∈ C and s ≠ 1, p ∈ Z^{+} and p goes through all the prime numbers) .Where the product is applied to all prime numbers p, it can be expressed as: $1+a(p)p^{-s}+a(p^2)p^{-2s}+\ldots$, this can be seen as a formal generating function, where the existence of a formal Euler product expansion and a(n) being a product function are mutually sufficient and necessary conditions. When a(n) is a completely integrative function, an important special case is obtained,where P(p , s)(s ∈ C and s ≠ 1, p ∈ Z^{+} and p goes through all the prime numbers) is a geometric series, and

$P(p,s) = \frac{1}{1-a(p)p^{-s}}$ (s ∈ C and s ≠ 1, p ∈ Z^{+} and p goes through all the prime numbers) .When a(n)=1,it is the Riemann zeta function, and more generally the Dirichlet feature.

Euler's product formula: for any complex number s, $Rs(s) > 1$ and $s \neq 1$, then $\sum_{n=1}^{\infty} n^{-s} = \prod_p (1-p^{-s})^{-1}$(s ∈ C and s ≠ 1, p ∈ Z^{+} and p goes through all the prime numbers, n ∈ Z^{+} and n goes through all positive numbers) , and when $Rs(s) >$

1 Riemann Zeta function $\zeta(s) = \sum_{n=1}^{\infty} n^{-s}$=$\prod_p (1-p^{-s})^{-1}$(s ∈ C and $Rs(s) > 0$ and s ≠ 1,

n∈ Z^{+}, p ∈ Z^{+}, s ∈C , n goes through all the positive numbers, p goes through all the prime numbers).

Riemann ζfunction expression:

$\zeta(s)=1/1^s+1/2^s+1/3^s+\ldots+1/m^s$ (m tends to infinity, and m is always even).

(1)Multiply both sides of the expression by $(1/2^s)$,

$(1/2^s)\zeta(s)=1/1^s(1/2^s)+1/2^s(1/2^s)+1/3^s(1/2^s)+\ldots+1/m^s(1/2^s)=1/2^s+1/4^s+1/6^s+\ldots+1/(2m)^s$

This is given by (1) - (2)

$\zeta(s)-(1/2^s)\zeta(s)=1/1^s+1/2^s+1/3^s+\ldots+1/m^s-[1/2^s+1/4^s+1/6^s+\ldots+1/(2m)^s]$

The derivation of Euler product formula is as follows:

$\zeta(s)-(1/2^s)\zeta(s)=1/1^s+1/3^s+1/5^s+\ldots+1/(m-1)^s$.

Generalized Euler product formula:

Suppose f(n) is a functionthat satisfies $f(n_1)f(n_2)=f(n_1n_2)$ and $\sum_n |f(n)| < +\infty$ (n_1 and n_2 are both natural numbers), then$\sum_n f(n)=\prod_p[1+f(p)+f(p^2)+f(p^3)+\ldots]$.

Proof:

The proof of Euler product formula is very simple, the only caution is to deal with infinite series and infinite products, can not arbitrarily use the properties of finite series and finite products. What I prove below is a more general result, and the Euler product formula will appear as a special case of this result.

Due to $\sum_{n=1}^{\infty}|f(n)|<+\infty$, so $1+f(p)+f(p^2)+f(p^3)+\ldots$ absolute convergence.Consider the

part of p<N in the continued product (finite product),Since the series is absolutely convergent and the product has only finite terms, the same associative and distributive laws can be used as ordinary finite summations and products.

Using the product property of f(n), we can obtain:

$\prod_{p<N}[1+f(p)+f(p^2)+f(p^3)+\ldots]=\sum f(n)$.The right end of the summation is performed on all natural numbers with only prime factors below N (each such natural number occurs only once in the summation, because the prime factorization of the natural numbers is unique).Since all natural numbers that are themselves below N obviously contain only prime factors below N, So

$\Sigma' f(n) = \sum_{n<N} f(n)+$ R(N),Where R(N) is the result of summing all natural numbers that are

greater than or equal to N but contain only prime factors below N.From this we get: $\prod_{p<N}[1+$ f(p)+f(p2)+f(p3)+...]=n$<N$ f(n) + R(N).For the generalized Euler product formula to hold, it is only necessary to prove $\lim_{n\to\infty}R(N)$ =0,and this is obvious,because |R(N)| $\le\sum_{n\ge N}|f(n)|$,and $\sum_n|f(n)|<+\infty$ sign of

$\lim_{n\to\infty}\sum_{n\ge N}|F(n)|$ =0,thus $\lim_{n\to\infty}R(N)$ =0.Beacuse $1+f(p)+f(p^2)+f(p^3)+\ldots$.= $1+$ $f(p)+f(p)^2+f(p)^3+\ldots=[1-f(p)]^{-1}$, so the generalized Euler product formula can also be written as:

$\sum_n f(n)=\prod_p[1-f(p)]^{-1}$.In the generalized Euler product formula, take $f(n)=n^{-s}$,Then

obviously $\sum_n|f(n)|<+\infty$ corresponds to the condition Rs(s)>1 in the Euler product formula, and the generalized Euler product formula is reduced to the Euler product formula.

From the above proof, we can see that the key to the Euler product formula is the basic property that every natural number has a unique prime factorization, that is, the so-called fundamental theorem of arithmetic.

For any complex number s, χ(n) is the Dirichlet characteristic and satisfies the following properties:

1: There exists a positive integer q such that $\chi(n+q)=\chi(n)$;

2: when n and q are not mutual prime, $\chi(n)=0$;

3: $\chi(a).\chi(b)=\chi(ab)$ for any integer a and b;

Reasoning 3:

If $Re(s)>1$ and $s\neq 1$,then

L(s, χ(n))=$\sum_{n=1}^{\infty}\frac{\chi(n)}{n^s}$ (n$\in Z_+, p\in Z_+, s\in$C and s$\neq$ 1 , n goes through all the positive numbers,

p goes through all the prime numbers, $\chi(n)\in$R

and ($\chi(n)\neq 0$),$a(n)=a(p)=\chi(n)$),P(p , s)=$\frac{1}{1-a(p)p^{-s}}$).

Next we prove the generalized Riemann conjecture when the Dirichlet eigen function χ(n) is any

real number that is not equal to zero, and $\eta(s)=\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^s}$ ($s\in$ C and $Rs(s)>0$ and $s\neq$

1), $\zeta(s)$ is the Riemann $\zeta(s)=\frac{\eta(s)}{(1-2^{1-s})}=\frac{1}{(1-2^{1-s})}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^s}=\frac{(-1)^{n-1}}{(1-2^{1-s})}\prod_p(1-p^{-s})^{-1}$ ($s\in$

C and $Rs(s)>0$ and $s\neq 1$, n$\in Z^+$ and n goes through all the positive integers, $p\in Z^+$and p

goes through all the prime numbers), so

GRH(s, χ(n))=L(s, χ(n))= $\sum_{n=1}^{\infty}\frac{\chi(n)}{n^s}=\sum_{n=1}^{\infty}a(n)n^{-s}=\prod_p P(p,s)=\prod_p(\frac{1}{1-a(p)p^{-s}})$ ($n \in Z^+, p \in Z^+$,s∈C and s≠ 1 , n goes through all the positive integers, p goes through all the prime numbers, χ(n)∈R and ($\chi(n) \neq 0$),a(n) = a(p)= χ(n)),P(p , s)=$\frac{1}{1-a(p)p^{-s}}$).

$a(p)p^{-s}=a(p)p^{-\sigma}\frac{1}{(\cos(t\ln p)+i\sin(t\ln p))}=a(p)(p^{-\sigma}(\cos(t\ln p)-i\sin(t\ln p))$(s ∈ C and s ≠ 1, t ∈ C and t≠0) ,

$(1-a(p)p^{-s})$ =1- $a(p)(p^{-\sigma}(\cos(t\ln p)-i\sin(t\ln p))$ = $1-a(p)p^{-\sigma}\cos(t\ln p)+ia(p)p^{-\sigma}\sin(t\ln p)$(s ∈ C and s ≠ 1, t ∈ C and t ≠ 0) ,

$a(p)p^{-\bar{s}}=a(p)p^{-\sigma}\frac{1}{(\cos(t\ln p)-i\sin(t\ln p))}=a(p)(p^{-\sigma}(\cos(t\ln p)+i\sin(t\ln p))$(s ∈ C and s ≠ 1, t ∈ C and t ≠ 0) ,

$(1-a(p)p^{-\bar{s}})$=1-$a(p)p^{-\sigma}\cos(t\ln p)-ia(p)p^{-\sigma}\sin(t\ln p)$(s ∈ C and s ≠ 1, t ∈ C and t ≠ 0) , because

$a(p)p^{-s})=\overline{1-a(p)p^{-\bar{s}}}$ (s ∈ C and s ≠ 1, p ∈ Z^+ and p is a prime integer),

so

$(1-a(p)p^{-s})^{-1}=\overline{(1-a(p)p^{-\bar{s}})^{-1}}$ (s ∈ C and s ≠ 1, p ∈ Z^+ and p is a prime number),

so

$\prod_p(1-a(p)p^{-s})^{-1}=\overline{\prod_p(1-a(p)p^{-\bar{s}})^{-1}}$

(s ∈ C and s ≠ 1, p ∈ Z^+ and p goes through all the prime numbers)) .

becuse L(s, χ(n))=$\sum_{n=1}^{\infty}a(n)n^{-s}=\prod_p(1-a(p)p^{-s})^{-1}$ (s ∈ C and s ≠ 1)and

L($\bar{s}$, χ(n))=$\sum_{n=1}^{\infty}a(n)n^{-\bar{s}}=\prod_p(1-a(p)p^{-\bar{s}})^{-1}$ (s ∈ C and s ≠ 1),

(s ∈ C and s ≠ 1, n ∈ Z^+ and n goes through all the positive integers, p ∈ Z^+ and p goes through all the prime numbers)).

For the Generalized Riemann function L(s, χ(n))=$\sum_{n=1}^{\infty}\frac{\chi(n)}{n^s}=\sum_{n=1}^{\infty}a(n)n^{-s}=\prod_p\frac{1}{1-a(p)p^{-s}}$

(χ (n) ∈ R and ($\chi(n) \neq 0$, a(n) = a(p) = χ(n)), P(p , s) = $\frac{1}{1-a(p)p^{-s}}$, s ∈ C and s ≠ 1, n ∈ Z^+ and n goes through all the positive integers, p ∈ Z^+ and p goes through all the prime numbers)) .so $L(s,\chi(n))=\overline{L(\bar{s},\chi(n))}$

(s ∈ C and s ≠ 1, n ∈ Z^+ and n goes through all the positive integers).

$a(p)p^{1-s}$ = $a(p)p^{(1-\sigma-ti)}$ = $a(p)p^{1-\sigma}x^{-ti}$ = $a(p)p^{1-\sigma}(\cos(\ln p)+i\sin(\ln p))-t$=a(p)p1−σ(costlnp−isin(tlnp))(s∈C and s≠1,t∈C and t≠0)

(s ∈ C and s ≠ 1, t ∈ C and t ≠ 0,, p ∈ Z^+ and p goes through all the prime numbers),

$a(p)p^{1-\bar{s}}$ = $a(p)p^{(1-\sigma+ti)}$ = $a(p)p^{1-\sigma}p^{ti}$ = $a(p)p^{1-\sigma}(p^{ti})$ = $a(p)p^{1-\sigma}(\cos(\ln p)+i\sin(\ln p))^t$ = $a(p)p^{1-\sigma}(\cos(t\ln p)-i\sin(t\ln p))$(s ∈ C and s ≠ 1, t ∈ C and t ≠ 0, p ∈ Z^+ and p goes through all the prime numbers),

then

$a(p)p^{-(1-s)}=a(p)p^{\sigma-1}\frac{1}{(\cos(t\ln p)-i\sin(t\ln p))}=a(p)(p^{\sigma-1}(\cos(t\ln p)+i\sin(t\ln p))$ (s ∈ C and s ≠ 1, t ∈ C and t ≠ 0, p ∈ Z^+ and p goes through all the prime numbers),

$(1-a(p)p^{-(1-s)})$ =1- $a(p)p^{\sigma-1}(\cos(t\ln p)+i\sin(t\ln p))$ = 1 −

$a(p)p^{\sigma-1}\cos(t\ln p) - a(p)p^{\sigma-1}i\sin(t\ln p)$
$(s \in C$ and $s \neq 1, t \in C$ and $t \neq 0, p \in Z^{+}$ and p goes through all the prime numbers,),
$(1 - a(p)p^{-\bar{s}})$ =1- $a(p)(p^{-\sigma}(\cos(t\ln p) + i\sin(t\ln p))$ = $1 - a(p)p^{-\sigma}\cos(t\ln p) - ia(p)p^{-\sigma}\sin(t\ln p)$ $(s \in C$ and $s \neq 1, t \in C$ and $t \neq 0, p \in Z^{+}$ and p goes through all the prime numbers),

When $\sigma=\frac{1}{2}$, then

$(1 - a(p)p^{-(1-s)}) = (1 - a(p)p^{-\bar{s}})(s \in C$ and $s \neq 1)$,

$(1 - a(p)p^{-(1-s)})^{-1} = (1 - a(p)p^{-\bar{s}})^{-1}(s \in C$ and $s \neq 1)$,

So

$\prod_p(1 - a(p)p^{-(1-s)})^{-1} = \prod_p(1 - a(p)p^{-\bar{s}})^{-1}(s \in C$ and $s \neq 1)$,becuse $L(1 - s, \chi(n)) = \prod_p(1 - a(p)p^{-(1-s)})^{-1}$ and $L(\bar{s}, \chi(n)) = \prod_p(1 - a(p)p^{-\bar{s}})^{-1}$, $n \in Z^{+}, p \in Z^{+}, s \in C$ and $s \neq 1$, n goes through all the positive integers, p goes through all the prime numbers, $\chi(n) \in R$ and $(\chi(n) \neq 0), a(n) = a(p) = \chi(n)$),$P(p, s) = \frac{1}{1-a(p)p^{-s}}$.

So Only $L\left(1 - s, \chi(n)\right) = L\left(\bar{s}, \chi(n)\right)$

$(s \in C$ and $s \neq 1, n \in Z^{+}$ and n goes through all positive integers),
and
Only $L(1 - \bar{s}, \chi(n)) = L(s, \chi(n))(s \in C$ and $s \neq 1)$
$(s \in C$ and $s \neq 1, n \in Z^{+}$ and n goes through all positive integers),

Because $L(s, \chi(n)) = \chi(n)\zeta(s)$ ($s \in C$ and $s \neq 1$, $n \in Z^{+}$ and n goes through all the positive integers), and $L(1 - s, \chi(n)) = \chi(n)\zeta(1-s)$($s \in C$ and $s \neq 1$, $n \in Z^{+}$ and n goes through all the positive integers), so When only $\sigma=\frac{1}{2}$, it must be true that $L(s, \chi(n)) = \overline{L(\bar{s}, \chi(n))}$($s \in C$ and $s \neq 1, n \in Z^{+}$ and n goes through all the positive integers),and it must be true that

$L(1 - s, \chi(n)) = L(\bar{s}, \chi(n))$($s \in C$ and $s \neq 1, n \in Z^{+}$ and n goes through all the positive integers)
Suppose $k \in R$,
$a(p)p^{k-s}$ = $a(p)p^{(k-\sigma-ti)}$ = $a(p)p^{k-\sigma}x^{-ti}$ = $a(p)p^{k-\sigma}(\cos(\ln p) + i\sin(\ln p))-t=a(p)pk-\sigma\cos t\ln p-i\sin t\ln p$ ($s \in C$ and $s \neq 1, t \in C$ and $t \neq 0, k \in R$),
$a(p)p^{k-\bar{s}} = a(p)p^{(k-\sigma+ti)} = a(p)p^{k-\sigma}p^{ti} = a(p)p^{k-\sigma}(p^{ti}) = a(p)p^{k-\sigma}(\cos(\ln p) + i\sin(\ln p))^{t} = a(p)(p^{k-\sigma}(\cos(t\ln p) + i\sin(t\ln p))$ $(s \in C$ and $s \neq 1, t \in C$ and $t \neq 0, k \in R)$,
then

$a(p)p^{-(k-s)} = a(p)p^{\sigma-k}\frac{1}{(\cos(t\ln p)-i\sin(t\ln p))} = a(p)$

$(p^{\sigma-k}(\cos(t\ln p) + i\sin(t\ln p))(s \in C$ and $s \neq 1, t \in C$ and $t \neq 0, k \in R)$,
$(1 - a(p)p^{-(k-s)})$ =1- $(a(p)p^{\sigma-k}(\cos(t\ln p) + i\sin(t\ln p))$ = $1 - a(p)p^{\sigma-k}\cos(t\ln p) - ip^{\sigma-k}\sin(t\ln p)(s \in C$ and $s \neq 1, t \in C$ and $t \neq 0, p \in Z^{+}$ and p is a prime numeber, $k \in R)$,
$(1 - a(p)p^{-\bar{s}})$ =1- $(a(p)p^{-\sigma}(\cos(t\ln p) + i\sin(t\ln p))$ = $1 - a(p)p^{-\sigma}\cos(t\ln p) - ia(p)p^{-\sigma}\sin(t\ln p)(s \in C$ and $s \neq 1, t \in C$ and $t \neq 0$, p is a prime numeber) ,

When $\sigma=\frac{k}{2}(k \in R)$,

then

$(1-a(p)p^{-(k-s)})=(1-a(p)p^{-\overline{s}})(s \in C \text{ and } s \neq 1, p \in Z^{+} \text{ and p is a prime integer}, k \in R)$,

$(1-a(p)p^{-(k-s)})^{-1} = \left(1-a(p)p^{-\overline{s}}\right)^{-1}(s \in C \text{ and } s \neq 1, p \in Z^{+} \text{ and p is a prime integer}, k \in R)$,

so

$\prod_p(1-a(p)p^{-(k-s)})^{-1}=\prod_p(1-a(p)p^{-\overline{s}})^{-1}$ (s ∈C and $\neq 1, k \in R, p \in Z^{+}$ and p goes through all the prime numbers, $k \in R$) ,

becuse $L(k-s, \chi(n))=\prod_p\left(1-a(p)p^{-(k-s)}\right)^{-1})$, and $L(\overline{s}, \chi(n))=0$

($s \in C$ and $s \neq 1$, $t \in C$ and $t \neq 0, k \in R, n \in Z^{+}$ and n goes through all positive integers) , for the generalized Riemann function $L(s, \chi(n))$(s ∈C and $s \neq 1$,n∈ Z^{+} and n goes through all the positive integers, $p \in Z^{+}$ and $\chi(n) \in R$ and $\chi(n) \neq 0$, $a(n) = a(p)= \chi(n)$),$P(p\ ,\ s)=\frac{1}{1-a(p)p^{-s}}$).

So

Only $L\left(k-s, \chi(n)\right)=L\left(\overline{s}, \chi(n)\right)$

($s \in C$ and $s \neq 1$, $t \in C$ and $t \neq 0, k \in R, n \in Z^{+}$ and n goes through all positive integers) ,

and

Only $L\left(k-\overline{s}, \chi(n)\right)=L\left(s, \chi(n)\right)$,

($s \in C$ and $s \neq 1$, $t \in C$ and $t \neq 0, k \in R, n \in Z^{+}$ and n goes through all positive integers) , $k \in R$),

And because Only $L\left(1-s, \chi(n)\right)= L\left(\overline{s}, \chi(n)\right)$

($s \in C$ and $s \neq 1$, $t \in C$ and $t \neq 0, k \in R, n \in Z^{+}$ and n goes through all positive integers) , ,so only k=1 be true.

$$GRH\left(s, \chi(n)\right) = L\left(s, \chi(n)\right) = \sum_{n=1}^{\infty}\frac{\chi(n)}{n^s} = \frac{\chi(n)\eta(s)}{(1-2^{1-s})} = \frac{\chi(n)}{(1-2^{1-s})}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^s}$$

$$= \frac{\chi(n)}{(1-2^{1-s})}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^{\sigma+ti}} = \frac{(-1)^{n-1}}{(1-2^{1-s})}\sum_{n=1}^{\infty}\chi(n)\left(\frac{1}{n^{\sigma}}\frac{1}{n^{ti}}\right) =$$

$$\frac{(-1)^{n-1}}{(1-2^{1-s})}\sum_{n=1}^{\infty}\chi(n)(n^{-\sigma})\frac{1}{(\cos\left(\ln((n)\right)+i\sin(\ln(n)))^{t}}$$

$$= \frac{(-1)^{n-1}}{(1-2^{1-s})}\sum_{n=1}^{\infty}\chi(n)(n^{-\sigma}(\cos(\ln(n)) + \ i\sin(\ln(n)))^{-t})$$

$$= \frac{(-1)^{n-1}}{(1-2^{1-s})}\sum_{n=1}^{\infty}\chi(n)n^{-\sigma}(\cos(t\ln(n)) - i\sin(t\ln(n))\ )$$

($s \in C$ and $s \neq 1$, $t \in C$ and $t \neq 0, k \in R, n \in Z^{+}$ and n goes through all positive integers) ,

$$GRH(\bar{s}, \chi(n)) = L(\bar{s}, \chi(n)) = \sum_{n=1}^{\infty} \frac{\chi(n)}{n^{\bar{s}}} = \frac{\chi(n)\eta(\bar{s})}{(1-2^{1-\bar{s}})} = \frac{\chi(n)}{(1-2^{1-\bar{s}})} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{\bar{s}}}$$

$$= \frac{\chi(n)}{(1-2^{1-\bar{s}})} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{\sigma-ti}} = \frac{(-1)^{n-1}}{(1-2^{1-\bar{s}})} \sum_{n=1}^{\infty} \chi(n)(\frac{1}{n^{\sigma}}\frac{1}{n^{-ti}})$$

$$= \frac{1}{(1-2^{1-\bar{s}})} \sum_{n=1}^{\infty} (\chi(n)\frac{1}{n^{\sigma}}\frac{1}{(\cos(\ln(n)) + i\sin(\ln(n)))^{-t}})$$

$$= \frac{1}{(1-2^{1-\bar{s}})} \sum_{n=1}^{\infty} (\chi(n) n^{-\sigma}(\cos(\ln(n)) + i\sin(\ln(n)))^{t}) =$$

$$\frac{1}{(1-2^{1-\bar{s}})} \sum_{n=1}^{\infty} (\chi(n) n^{-\sigma}(\cos(t\ln(n)) + i\sin(t\ln(n)))$$

($s \in C$ and $s \neq 1$, $t \in C$ and $t \neq 0, k \in R, n \in Z^{+}$ and n goes through all positive integers) ,

$$GRH\left(1-s, \chi(n)\right) = L\left(1-s, \chi(n)\right) = \sum_{n=1}^{\infty} \frac{\chi(n)}{n^{s}} = \frac{\chi(n)\eta(1-s)}{(1-2^{s})} = \frac{\chi(n)}{(1-2^{s})} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{1-\sigma-ti}}$$

$$= \frac{(-1)^{n-1}}{(1-2^{s})} \sum_{n=1}^{\infty} \chi(n)\left(\frac{1}{n^{1-\sigma}}\frac{1}{n^{-ti}}\right)$$

$$= \frac{(-1)^{n-1}}{(1-2^{s})} \sum_{n=1}^{\infty} (\chi(n) n^{\sigma-1}(\cos(t\ln(n)) + i\sin(t\ln(n))) ,$$

($s \in C$ and $s \neq 1$, $t \in C$ and $t \neq 0, k \in R, n \in Z^{+}$ and n goes through all positive integers) ,

Suppose
U=[$\chi(n)1^{-\sigma}\cos(t\ln1) - \chi(n)2^{-\sigma}\cos(t\ln2) + \chi(n)3^{-\sigma}\cos(t\ln3) - \chi(n)4^{-\sigma}\cos(t\ln4) + \ldots$],
V=[$\chi(n)1^{-\sigma}\mathrm{Sin}(t\ln1) - \chi(n)2^{-\sigma}\sin(t\ln2) + \chi(n)3^{-\sigma}\sin(t\ln3) - \chi(n)4^{-\sigma}\sin(t\ln4) + \ldots$] ,
then

$L(s, \chi(n)) = \overline{L(\bar{s}, \chi(n))}$

And n goes through all the positive numbers, so n=1,2,3,... ,let's just plug in, so

$L(s, \chi(n)) = \sum_{n=1}^{\infty} \frac{\chi(n)}{n^{s}}$=[$\chi(n)1^{-\sigma}\cos(t\ln1) - \chi(n)2^{-\sigma}\cos(t\ln2) + \chi(n)3^{-\sigma}\cos(t\ln3) - \chi(n)4^{-\sigma}\cos(t\ln4) + \ldots$]-i[$\chi(n)1^{-\sigma}\sin(t\ln1) - \chi(n)2^{-\sigma}\sin(t\ln2) + \chi(n)3^{-\sigma}\sin(t\ln3) - \chi(n)4^{-\sigma}\sin(t\ln4) + \ldots$]=
U-Vi

($s \in C$ and $s \neq 1$, $t \in C$ and $t \neq 0, k \in R, n \in Z^{+}$ and n goes through all positive integers) ,

U=[$\chi(n)1^{-\sigma}\cos(t\ln1) - \chi(n)2^{-\sigma}\cos(t\ln2) + \chi(n)3^{-\sigma}\cos(t\ln3) - \chi(n)4^{-\sigma}\cos(t\ln4) + \ldots$],

V=[$\chi(n)1^{-\sigma}\sin(t\ln1) - \chi(n)2^{-\sigma}\sin(t\ln2) + \chi(n)3^{-\sigma}\sin(t\ln3) - \chi(n)4^{-\sigma}\sin(t\ln4) + \ldots$] ,
Then

$L(\bar{s}, \chi(n)) = \sum_{n=1}^{\infty} \frac{\chi(n)}{n^{s}}$=[$\chi(n)1^{-\sigma}\cos(t\ln1) - \chi(n)2^{-\sigma}\cos(t\ln2) + \chi(n)3^{-\sigma}\cos(t\ln3) - 4^{-\sigma}\cos(t\ln4) + \ldots$]+i[$\chi(n)1^{-\sigma}\sin(t\ln1) - \chi(n)2^{-\sigma}\sin(t\ln2) + \chi(n)3^{-\sigma}\sin(t\ln3) - \chi(n)4^{-\sigma}\sin(t\ln4) + \ldots$]= U+Vi,

($s \in C$ and $s \neq 1$, $t \in C$ and $t \neq 0$, $k \in R$, $n \in Z^+$ and n goes through all positive integers) ,

U=[$\chi(n)1^{-\sigma}\cos(t\ln 1) - \chi(n)2^{-\sigma}\cos(t\ln 2) + \chi(n)3^{-\sigma}\cos(t\ln 3) - \chi(n)4^{-\sigma}\cos(t\ln 4) + \ldots$],
V=[$\chi(n)1^{-\sigma}\sin(t\ln 1) - \chi(n)2^{-\sigma}\sin(t\ln 2) + \chi(n)3^{-\sigma}\sin(t\ln 3) - \chi(n)4^{-\sigma}\sin(t\ln 4) + \ldots$] ,
$L(s, \chi(n))$ and $L(\overline{s}, \chi(n))$ are complex conjugates of each other,that is

$L(s, \chi(n)) = \overline{L(\overline{s}, \chi(n))}$

($s \in C$ and $s \neq 1$, $t \in C$ and $t \neq 0$, $k \in R$, $n \in Z^+$ and n goes through all positive integers) ,

When $\sigma = \frac{1}{2}$, then$L(s, \chi(n)) = L(1 - s, \chi(n))$= U-Vi

($s \in C$ and $s \neq 1$, $t \in C$ and $t \neq 0$, $k \in R$, $n \in Z^+$ and n goes through all positive integers) ,

U=[$\chi(n)1^{-\sigma}\cos(t\ln 1) - \chi(n)2^{-\sigma}\cos(t\ln 2) + \chi(n)3^{-\sigma}\cos(t\ln 3) - \chi(n)4^{-\sigma}\cos(t\ln 4) + \ldots$],

V=[$\chi(n)1^{-\sigma}\sin(t\ln 1) - \chi(n)2^{-\sigma}\sin(t\ln 2) + \chi(n)3^{-\sigma}\sin(t\ln 3) - \chi(n)4^{-\sigma}\sin(t\ln 4) + \ldots$] .

and When $\sigma = \frac{1}{2}$, then *only* $L\left(1 - s, \chi(n)\right) = L\left(\overline{s}, \chi(n)\right)$is true,

($s \in C$ and $s \neq 1$, $t \in C$ and $t \neq 0$, $k \in R$, $n \in Z^+$ and n goes through all positive integers) ,

$GRH(k - s, \chi(n)) = L(k - s, \chi(n)) = \frac{\chi(n)\eta(k-s)}{(1-2^{1-k+s})} = \frac{\chi(n)}{(1-2^{1-k+s})}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^{k-\rho-ti}} = \frac{(-1)^{n-1}}{(1-2^{1-k+s})}\sum_{n=1}^{\infty}\chi(n)(\frac{1}{n^{k-\sigma}}\frac{1}{n^{-ti}}) = \frac{(-1)^{n-1}}{(1-2^{1-k+s})}\sum_{n=1}^{\infty}(\chi(n)n^{\sigma-k}(\cos(t\ln(n)) + i\sin(t\ln(n))$($s \in C$ and $s \neq 1$, $t \in C$ and $t \neq 0$, $k \in R$, $n \in Z^+$ and n goes through all positive integers) ,

W=[$\chi(n)1^{\sigma-k}\cos(t\ln 1) - \chi(n)2^{\sigma-k}\cos(t\ln 2) + \chi(n)3^{\sigma-k}\cos(t\ln 3) - \chi(n)4^{\sigma-k}\cos(t\ln 4) + \ldots$]
U=[$\chi(n)1^{\sigma-k}\sin(t\ln 1) - \chi(n)2^{\sigma-k}\sin(t\ln 2) + \chi(n)3^{\sigma-k}\sin(t\ln 3) - \chi(n)4^{\sigma-k}\sin(t\ln 4) + \ldots$] .

When $\sigma = \frac{k}{2}$($k \in R$), then

Only $L(k - s, \chi(n)) = L\left(\overline{s}, \chi(n)\right) = W - Ui$.

($s \in C$ and $s \neq 1$, $k \in R$, $n \in Z^+$ and n goes through all positive integers) ,but the Riemann $\zeta(s)$ function only satisfies $\zeta(1-s) = 2^{1-s}\pi^{-s}\cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)$ ($s \in C$ and $s \neq 1$), so when $\zeta(s)=0$($s \in C$ and $s \neq 1$), then only $\zeta(1 - s) = \zeta(s) = 0$($s \in C$ and $s \neq 1$),

and when $\zeta(\overline{s})=0$, then only $\zeta(1 - s) = \zeta(\overline{s}) = 0$($s \in C$ and $s \neq 1$), which is $\zeta(k - s) = \zeta(1 - s) = \zeta(\overline{s})$($s \in C$ and $s \neq 1$),so only k=1 be true.so only $Re(s) = \frac{k}{2} = \frac{1}{2}$ ($k \in R$).So Only $L(1 - s, \chi(n)) = L(\overline{s}, \chi(n))$($s \in C$ and $s \neq 1$, $n \in Z^+$) is true, so only k=1 is true.According the equation $\zeta(1-s) = 2^{1-s}\pi^{-s}\cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)$($s \in C$ and $s \neq 1$)obtained by Riemann,since Riemann has shown that the Riemann $\zeta(s)$($s \in C$ and $s \neq 1$) function has zero, that is, in $\zeta(1-s) = 2^{1-s}\pi^{-s}\cos(\frac{\pi s}{2})\Gamma(s)\zeta(s)$($s \in C$ and $s \neq 1$), $\zeta(s)=0$($s \in C$ and $s \neq 1$) is true. So only when $\sigma = \frac{1}{2}$ and $\zeta(s)=0$($s \in C$ and $s \neq 1$, and $\chi(n) \neq 0$, $n \in Z^+$) , then

$L(s, \chi(n))=\chi(n)\zeta(s)=0$ ($s\in C$ and $s\neq 1, n\in Z^+$ and n traverse all positives integers) is true. Because $L(s, \chi(n))=\chi(n)\zeta(s)$ ($s\in C$ and $s\neq 1, n\in Z^+$ and n traverse all positive integers) and $L(1-s, \chi(n))=\chi(n)\zeta(1-s)$ ($s\in C$ and $s\neq 1, n\in Z^+$ and n traverse all positive integers), so When $\rho=\frac{1}{2}$, it must be true that $L(s, \chi(n))=\overline{L\left(\bar{s}, \chi(n)\right)}$ ($s\in C$ and $s\neq 1, n\in Z^+$ and n traverse all positive integers), and it must be true that $L(1-s, \chi(n))=L(\bar{s}, \chi(n))$ ($s\in C$ and $s\neq 1, n\in Z^+$ and n traverse all positive integers). According $\zeta(1-s)=\zeta(s)=0$ ($s\in C$ and $s\neq 1$) and $\zeta(s)=\zeta(\bar{s})=\zeta(1-\bar{s})=0$ ($s\in C$ and $s\neq 1$), so $L(s, \chi(n))=L(1-s, \chi(n))=0$ ($s\in C$ and $s\neq 1, n\in Z^+$ and n traverse all positive integers) and $L(s, \chi(n))=L(\bar{s}, \chi(n))=L(1-\bar{s}, \chi(n))=0$ ($s\in C$ and $s\neq 1, n\in Z^+$ and n traverse all positive integers),then $s=\bar{s}$ or $s=1-s$ or $\bar{s}=1-s$,so $s\in R$, or $\sigma+ti=1-\sigma-ti$,or $\sigma-ti=1-\sigma-ti$, so $s\in R$, or $\sigma=\frac{1}{2}$ and $t=0$, or $\sigma=\frac{1}{2}$ and $t\in R$ and $t\neq 0$, so $s\in R$, or $s=\frac{1}{2}+0i$,or $s=\frac{1}{2}+ti$($t\in R$ and $t\neq 0$),

because $\zeta\left(\frac{1}{2}\right)\rightarrow+\infty, \zeta(1)\rightarrow+\infty, \zeta(1)$ is divergent, $\zeta\left(\frac{1}{2}\right)$ is more divergent, so drop $s=1$ and $s=0$. Only $s=\frac{1}{2}+ti$ ($t\in R$, and $t\neq 0$) and $s=\frac{1}{2}-ti$ ($t\in R$, and $t\neq 0$) are true, or say $s=\frac{1}{2}+ti$ ($t\in R$ and $t\neq 0$) and $s=\frac{1}{2}-ti$ ($t\in R$ and $t\neq 0$ are true. And beacause only when σ=12 ,the next three equations,

$L\left(\sigma+ti, \chi(n)\right)=0$ ($t\in R$ and $t\neq 0, n\in Z^+$ and n traverse all positive integers), $L\left(1-\sigma-ti, \chi(n)\right)=0$ ($t\in R$ and $t\neq 0, n\in Z^+$ and n traverse all positive integers),and

$L\left(\sigma-ti, \chi(n)\right)=0$ ($t\in R$ and $t\neq 0, n\in Z^+$ and traverse all positive integers) are all true. And because $L(\frac{1}{2}, \chi(n))>0$, so only $s=\frac{1}{2}+ti$($t\in R$ and $t\neq 0$) and $s=\frac{1}{2}-ti$($t\in R$ and $t\neq 0$) are true.The Generalized Riemann conjecture must satisfy the properties of the

$L(s, \chi(n)$ ($s\in C$, and $s\neq 1$ and $s\neq -2n, n\in Z^+$ and n traverse all positive integers)

function, The properties of the $L(s, \chi(n))$ ($s\in C$, and s1 $\neq$ and $s\neq -2n, n\in Z^+$ and n traverse all positive integers)function are fundamental, the Generalized Riemann hypothesis and the Generalized Riemann conjecture must be correct to reflect the properties of the $L(s, \chi(n))$ ($s\in C$, and $s\neq 1$ and $s\neq -2n, n\in Z^+$ and n traverse all positive integers) function , that is, the roots of $L(s, \chi(n))=0$ ($s\in C$, and $s\neq 1$ and $s\neq -2n, n\in Z^+$ and n traverse all positive integers) can only be $s=\frac{1}{2}+ti$($t\in R$ and $t\neq 0$) and $s=\frac{1}{2}-ti$($t\in R$ and $t\neq 0$), that is, Re(s) must only be equal to $\frac{1}{2}$, and Im(s) must be real, and Im(s) is not equal to zero.So the Generalized Riemann hypothesis and the Generalized Riemann conjecture must be correct. According $L(1-s, \chi(n))=L(s, \chi(n))=0$ ($s\in C$, and $s\neq 1$ and $s\neq -2n, n\in Z^+$ and n traverse all positive integers),so the zeros Of $L(s, \chi(n))$ ($s\in C$ and $s\neq 1$ and $s\neq -2n, n\in Z^+$ and n traverse all positive integers) function in the complex plane also correspond to the symmetric distribution of point ($\frac{1}{2}$,0i) on a line perpendicular to the real number line in the complexplane

When $L(1-s, \chi(n)) = L(s, \chi(n)) = 0(s \in C$ and $s \neq 1$ and $s \neq -2n, n \in Z^{+}$ and n traverse all positive numbers), s and 1-s are pair of zeros of the function $L(s, \chi(n))(s \in C$ and $s \neq 1, n \in Z^{+}$ and n traverse all positive numbers) symmetrically distributed in the complex plane with respect to point ($\frac{1}{2}$,0i) on a line perpendicular to the real number line of the complex plane.We got $\overline{L(s, \chi(n))}$ $=L(\overline{s}, \chi(n))$(and $s \neq 1$and n traverse all positive numbers and n traverse all positive integers) before,When t in s=$\frac{1}{2}$+ti(t∈C and $t \neq 0$) defined by Riemann is a complex number, and then s in $\overline{L(s, \chi(n))} = L(\overline{s}, \chi(n))$ ($s \in C$ and $s \neq 1$ and $s \neq -2n, n \in Z^{+}$) is consistent with s in s=$\frac{1}{2}$+ti(t∈C and $t \neq 0$)defined by Riemann, so only $\sigma = \frac{1}{2}$.When $L(s, \chi(n))$ $=L(\overline{s}, \chi(n))= 0$(s∈C and $s \neq 1$, $n \in Z^{+}$),since s and $\overline{s}$ are a pair of conjugate complex numbers, so s and $\overline{s}$ must be a pair of zeros of the Generalized function $L(s, \chi(n))(s \in C$ and $s \neq 1$, and $s \neq -2n, n \in Z^{+}$, and n traverse all positive numbers) in the complex plane with respect to point(ρ,0i) on a line perpendicular to the real number line.s is a symmetric zero of 1-s, and a symmetric zero of $\overline{s}$. By the definition of complex numbers, how can a symmetric zero of the same Generalized Riemann function $L(s, \chi(n))(s \in C$ and $s \neq 1$, and $s \neq -2n, n \in Z^{+}$, and n traverse all positivenumbers) of the same zero independent variable s on a line perpendicular to the real number axis of the complex plane be both a symmetric zero of 1-s on a line perpendicular to the real number axis of the complex plane with respect to point ($\frac{1}{2}$,0i) and a symmetric zero of $\overline{s}$ on a line perpendicular to the real number axis of the complex plane with respect to point (σ, 0i)? Unless σ and $\frac{1}{2}$ are the same value, is also that $\sigma = \frac{1}{2}$, and only 1-s=$\overline{s}$ is true, only s=$\frac{1}{2}$+ti(t∈R and $t \neq 0$) and s=$\frac{1}{2}$-ti(t∈R and $t \neq 0$) are true. Otherwise it's impossible,this is determined by the uniqueness of the zero of Generalized Riemann function $L(s, \chi(n))(s \in C$, and n traverse all positive numbers) on the line passing through that point perpendicular to the real number axis of the complex plane with respect to the vertical foot symmetric distribution of the zero of the line and the real number axis of the complex plane,Only one line can be drawn perpendicular from the zero independent variable s of $L(s, \chi(n))(s \in C$ and $s \neq 1$, and $s \neq -2n, n \in Z^{+}$, and n traverse all positivenumbers)on the real number line of the complex plane, the vertical line has only one point of intersection with the real number axis of the complex plane. In the same complex plane, the same zero point of $L(s, \chi(n))(s \in C$ and $s \neq 1$ and $s \neq -2n, n \in Z^{+}$ and n traverse all positive integers) on the line passing through that point perpendicular to the real number line of the complex plane there will be only one zero point about the vertical foot symmetric distribution of the line and the real number line of the complex plane,so I have proved the generalized Riemann conjecture when the Dirichlet eigen function $\chi(n)$($n \in Z^{+}$ and n traverse all positive numbers) is any real number that is not equal to zero,Since the nontrivial zeros of the Riemannian function $\zeta(s)$(s∈C and $s \neq 1$) and$L(s, \chi(n)(s \in C$ and $s \neq 1$ and $s \neq -2n, n \in Z^{+}$ and n traverse all positive integers) are both on the critical line perpendicular to the real number line of Re(s)=$\frac{1}{2}$ and Im(s)≠ 0, these

nontrivial zeros are general complex numbers of Re(s)=$\frac{1}{2}$ and Im(s)≠ 0,so I have proved the

generalized Riemann conjecture when the Dirichlet eigen function χ(n)($n \in Z^+$ and n traverse all positivel intergers) is any real number that is not equal to zero. The Generalized Riemann conjecture must satisfy the properties of the $L(s, \chi(n))$(s∈C and s≠ $1, n \in Z^+$ and n traverse all positive intergers) function, The properties of the $L(s, \chi(n))$ (s∈C and $s \neq 1$ and $s \neq -2n, n \in Z^+$ and n traverse all positive numbers) function are fundamental, the Generalized Riemann conjecture must be correct to reflect the properties of the

$L\left(s, \chi(n)\right)$ (s ∈

C and $s \neq 1$ and $s \neq -2n, n \in Z^+$ and n traverse all positive intergers) function, that is, the

roots of the

$L(s, \chi(n))=0$(s ∈ C and $s \neq 1$ and $s \neq -2n, n \in Z^+$ and n traverse all positive intergers) can only be s=$\frac{1}{2}$+ti(t∈Rand $t \neq 0$)or s=$\frac{1}{2}$-ti(t∈R and $t \neq 0$), that is, Re(s) can only be equal to $\frac{1}{2}$,

and

Im(s) must be real, and Im(s) is not equal to zero.When $L(s, \chi(n)) = 0$ (s∈C and s≠ 1, and $s \neq -2n, n \in Z^+$ and n traverse all positive numbers n goes through all the positive integers, $\chi(n) \in R$ and $\chi(n) \neq 0$) , $a(n) = a(p) = \chi(n)$, $P(p, s) = \frac{1}{1-a(p)p^{-s}}$) ,then the Generalized Riemann must be correct, and $s = \frac{1}{2} + ti(t \in R$ and $t \neq 0)$ or $s = \frac{1}{2} - ti(t \in R$ and $t \neq 0)$.

For any complex number s,when χ(n) is the Dirichlet characteristic and satisfies the following properties:

1: There exists a positive integer q such that χ(n+q)= $\chi(n)(n \in Z^+)$;

2: when n and q are not mutual prime, χ(n)=0($n \in Z^+$);

3: χ(a) χ(b)= χ(ab) ($a \in Z^+, b \in Z^+$)for any integer a and b; Suppose q=2k($k \in Z^+$),

if n and n+q are all prime number, and if $\chi(Y) = 0$ (Y traverses all positive odd numbers) and $\chi(n+q) = \chi(n) = 0$ (n and n + q traverses all positive odd numbers) ,because n(n traverses all prime numbers) and q=2k($k \in Z^+$) are not mutual prime, then χ(n)=0 ($n \in Z^+$ and n and n + q traverses all prime numbers) andfor any prime number a and b, $\chi(a)\chi(b) = \chi(ab)$ ($a \in Z^+, b \in Z^+$, a traverses all prime numbers and b traverses all prime number, then the three properties described by the Dirichlet

eigenfunction χ(n)($n \in Z^+$ and n traverses all prime numbers). above fit the definition of the Polignac conjecture, the Polignac conjecture states that for all natural numbers k, there are infinitely many pairs of prime numbers (p,p+2k)($k \in Z^+$). In 1849, the French mathematician A. Polignac proposed the conjecture.When k=1, the Polygnac conjecture is equivalent to the twin

prime conjecture.In other words, when $L(s, \chi(n)) = 0$(s ∈C , n∈ Z^+ and n traverses all prime

numbers, χ(n)∈R, $a(n) = a(p) = \chi(n)$),$P(p, s)=\frac{1}{1-a(p)p^{-s}}$), and generalized Riemann

conjecture are true, then the Polygnac conjecture must be completely true, and if the Polignac conjecture must be true, then the twin prime conjecture and Goldbach's conjecture must be true.I proved that the generalized Riemannian conjecture are true, so when L(s, χ (n)) = 0(s ∈C , n∈ Z^+ and n traverses all prime numbers, and $\chi(n) = 0$), P(p , s)=$\frac{1}{1-a(p)p^{-s}}$) and $s = \frac{1}{2} + ti(t \in R \text{ and } t \neq 0)$ or $s = \frac{1}{2} - ti(t \in R \text{ and } t \neq 0)$. I also proved that the Polignac conjecture,twin prime conjecture must be true and Goldbach conjecture are completely true.The Generalized Riemann conjecture are perfectly valid, so the Polygnac conjecture and the twin prime conjecture and Goldbach's conjecture must satisfy the properties of the Generalized Riemann L(s, χ (n)) function and the Riemann ζ(s)(s ∈ C and s ≠ 1) function, so the Polignac conjecture,twin prime conjecture must be true and Goldbach conjecture is completely true,and the Riemann conjecture and the Generalized Riemann conjecture are completely correct.

Reasoning 5:

In order to explain why the zero of the Landau-Siegel function exists under special conditions,we

need to start with the Riemann conjecture. I have solved the Riemann conjecture for the Dirichlet

feature χ (n)≡1(n ∈ Z^+ and n traverses all positive integers) and the generalized Riemann conjecture for the Dirichlet feature χ (n)≠0(n ∈ Z^+ and n traverses all positive integers), I

propose a special form of Dirichlet L(s, χ (p))(s∈C and s ≠ 1, χ (p) ∈R and χ (p)≠0, p ∈ Z^+ and p traverses all odd primes,including 1) function problem. Let me first explain to you what Landau-Siegel zero conjecture is. As you may know, the Landau-Siegel zero point problem, named after Landau and his student Siegel, boils down to solving whether there are abnormal real zeros in the Dirichlet L function. So let's look again at what the Dirichlet L function is. Look at the abve proof process,which is the expression of Dirichlet L(s, χ (n))(s∈C and s≠ 1, n ∈ Z^+ and n traverses all positive integers)

$L(s, \chi(n)) = \sum_{n=1}^{\infty} \frac{\chi(n)}{n^s}$ (s ∈ C and s ≠ 1, n ∈ Z^+ and n goes through all positive integers) .

I shall first introduce the Dirichlet L(s, χ (n))(s∈C and s ≠ 1, n ∈ Z^+ and n traverses all positive integers) function and explain its relation to the Riemannn ζ(s)(s∈C and s ≠ 1) function. χ (n)(n ∈ Z^+ and n traverses all positive integers) is a characteristic value of a Dirichlet function, which is all real numbers, and χ (n)(n ∈ Z^+ and n traverses all positive integers) is a real function. The L(s, χ (n))(s∈C and s ≠ 1, χ (n) ∈R, n∈ Z^+ and n traverse all positivel numbers) function can be analytically extended as a meromorphic function over the entire complex plane.

John Peter Dirichlet proved that L(1, χ (n))≠0(s∈C and s≠ 1, χ (n)∈R and χ (n)≠0, n∈Z^+ and n

traverse all positivel numbers) for all χ (n)(n∈Z^+ and n traverse all positivel numbers), and thus proved Dirichlet's theorem. In number theory, Dirichlet's theorem states that for any positive integers a,d, there are infinitely many forms of prime numbers, such as a+nd, where n is a positive integer, i.e., in the arithmetic sequence a+d,a+2d,a+3d,... There are an infinite number of prime numbers-there are an infinite number of prime modules d as well as a . If χ (n)(n ∈ Z^+ and n traverses all positive integers) is the main feature, then L(s, χ (n))(s∈C and s≠1, χ (n)∈R, n∈$Z^+$and n traverses all positive integers) has a unipolar point at s=1. Dirichlet defined the properties of the characteristic

function χ (n)(n∈$Z^+$and traverses all positive integers) in the Dirichlet function L(s, χ (n))(s∈C and s≠ 1, χ (n) ∈R, n∈$Z^+$and n traverses all positive integers) :

1: There is a positive integer q such that χ (n+q)= χ (n)(n∈$Z^+$and n traverses all positive integers);

2: when n(n∈ Z^+ and n traverses all natural numbers) and q are non-mutual primes, ✗(n)≡0(∈Z^+ and n traverses all positive integers);

3: For any integer a and b, ✗(a).✗(b)=✗(ab)(a is a positive integer, b is a positive integer);

From the expression of the Dirichlet function L(s,✗(n))(s∈C and s≠ 1,X(n)∈R, n∈Z^+ and n takes all positive integers), it is easy to see that when the Dirichlet characteristic real function ✗(n)=1(s∈C and s≠ 1, n∈$Z^+$and n takes all positive integers), Then the Dirichlet L(s,1)(s∈C and s≠ 1, ✗(n)∈R, n∈Z^+ and n traverses all positive integers) becomes the Riemann ζ(s)(s∈C and s≠ 1) function, so the Riemann ζ(s)(s∈C and s≠ 1) function is a special function of functions ✗ (n)≠1(n∈ Z^+ and n traverse all positivel numbers), they are called nontrivial eigenfunctions of the Dirichlet function L(s,✗(n))(s∈C and s≠ 1, ✗(n)∈R , n∈Z^+ and n traverse all positivel integers). When the independent variable s in the expression of the Dirichlet function L(s,✗(n))(s∈C and s≠ 1, ✗(n)∈R, n∈Z^+ and n traverse all positive inteegers) is a real number β, then for all eigenfunction values ✗(n)(n∈Z^+ and n traverses all positive integers), L(β,✗(n))(β ∈R, ✗(n) ∈R, n∈Z^+ and n traverses all positive integers) is called the Landau-Siegel function. Visible landau-siegel function L(β,✗(n))(β∈R and $\beta\neq -2n$, $n\in Z^+$, ✗(n)∈R, and n traverses all positive integers) is dirichlet function L(s,✗(n)) (s∈C and s≠ 1 and $s\neq -2n$,$n\in Z^+$, ✗(n) ∈R and n traverses all positive integers) of a special function, landau-siegel guess is landau and siegel they guess L(β, ✗ (n))(β ∈R and $\beta\neq -2n$, $n\in Z^+$, ✗ (n) ∈ R, ,and n traverses all positive integers) has no zero, So Landau and Siegel's conjecture that L(β, ✗(n))≠0(β∈R and β $\neq -2n$, $n\in Z^+$, ✗(n)∈R, n∈Z^+ and n traverses all positive integers) is easy to understand, right? Well, now that you know what the Landau and Siegel null conjecture is all about, let's continue to see how I'm going to solve the Landau and Siegel null conjecture. Look at the proof above：

$$\mathrm{GRH}\left(s,\text{✗}(n)\right)=L\left(s,\text{✗}(n)\right)=\sum_{n=1}^{\infty}\frac{\text{✗}(n)}{n^{s}}=\frac{\text{✗}(n)\eta(s)}{(1-2^{1-s})}=\frac{\text{✗}(n)}{(1-2^{1-s})}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^{s}}$$

$$=\frac{\text{✗}(n)}{(1-2^{1-s})}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^{\sigma+ti}}=\frac{(-1)^{n-1}}{(1-2^{1-s})}\sum_{n=1}^{\infty}\text{✗}(n)\left(\frac{1}{n^{\sigma}}\frac{1}{n^{ti}}\right)=$$

$$\frac{(-1)^{n-1}}{(1-2^{1-s})}\sum_{n=1}^{\infty}\text{✗}(n)(n^{-\sigma})\frac{1}{(\cos\left(\ln((n)\right)+i\sin(\ln(n)))^{t}}$$

$$=\frac{(-1)^{n-1}}{(1-2^{1-s})}\sum_{n=1}^{\infty}\text{✗}(n)(n^{-\sigma}(\cos(\ln(n))+\ i\sin(\ln(n)))^{-t})$$

$$=\frac{(-1)^{n-1}}{(1-2^{1-s})}\sum_{n=1}^{\infty}\text{✗}(n)n^{-\sigma}(\cos(t\ln(n))-i\sin(t\ln(n))\ \ )$$

($t \in C$ and $t \neq 0, s \in C$ and $s \neq 1, n \in Z^+$ and n goes through all positive integers) ,because $\zeta(s)=2^s\pi^{s-1}\mathrm{Sin}(\frac{\pi s}{2})\Gamma(1-s)\zeta(1-s)$($s\in C$ and $s\neq 1$)(Formula 7) ,so if $\beta\in R$ and $\beta= -2n(n \in Z^+)$,then $\zeta(s)=0$.So $L(\beta, \chi(n))=$

$\frac{(-1)^{n-1}}{(1-2^{1-\beta})}\sum_{n=1}^{\infty} \chi(n)(n^{-\beta}(\cos(0 \times \ln(n)) + i\sin(0 \times \ln(n)))=\frac{(-1)^{n-1}}{(1-2^{1-\beta})}\sum_{n=1}^{\infty}(\chi(n)n^{-\beta}) =$

$\frac{1}{\left(1-2^{1-\beta}\right)}(\chi(1)1^{-\beta} - \chi(2)2^{-\beta} + \chi(3)3^{-\beta} - \chi(4)4^{-\beta} + \cdots)$, " $\times$ " is the symbol for multiplication, because the real exponential function of the real number has a function value greater than zero, because $\chi(n) \in R$ and $\chi(1) = \chi(2) = \chi(3) = \chi(3), \ldots$, so $n^{-\beta} > 0(n \in Z^+$ and n traverses all positive integers) and $1^\beta - 2^\beta < 0, 3^\beta - 4^\beta < 0, 5^\beta - 6^\beta < 0, \ldots, (n-1)^\beta - (n)^\beta < 0, \ldots, or\ 1^\beta - 2^\beta > 0, 3^\beta - 4^\beta > 0, 5^\beta - 6^\beta > 0, \ldots, (n-1)^\beta - (n)^\beta > 0$, and $\frac{1}{\left(1-2^{1-\beta}\right)} \neq 0$, it can be known that if $\chi(n) \neq 0(\chi(n) \in R, n \in Z^+$ and n traverses all positive integers), and $\beta\in R$ and $\beta\neq -2n(n \in Z^+)$, then $L(\beta, \chi(n))\neq 0$($\beta\in R$ and $\beta\neq -2n, n \in Z^+$, $\chi(n) \in R$ and n traverses all positive integers) and $L(\beta, 1)\neq 0$($\beta\in R$ and $\beta \neq -2n, n \in Z^+$, and n traverses all positive integers), so for Riemann $\zeta(s)$($s\in C$ and $s\neq 1$and $s \neq -2n, n \in Z^+$) functions, its corresponding landau-siegel function $L(\beta, \chi(n))$($\beta\in R$ and $\beta\neq -2n, n \in Z^+$, $\chi(n) \in R$ and n traverses all positive integers) of pure real zero does not exist, If $s\neq$-2n($n \in Z^+$), the other Landau-Siegel functions $L(\beta, \chi(n))$($\beta\in R$ and $\beta\neq$-2n, $n \in Z^+$, $\chi(n) \in R$ and n traverses all posite integers) also do not exist pure real zeros, this means that if s $\neq$-2n($n \in Z^+$),then the Riemann $\zeta(s)$($s\in C$ and $s\neq 1$ and $s \neq -2n, n \in Z^+$) function does not have a zero of a pure real variable s, and this means that if $s\neq$-2n($n \in Z^+$),then the generalized Riemannian $L(s, \chi(n))=0$($s\in C$ and $s\neq 1$, and $s \neq -2n, n \in Z^+$, $\chi(n) \in R$ and n traverses all positive integers) function also has no pure real zeros of the variable s,then the generalized Riemann conjecture $L(s, \chi(n))=0$($s\in C$ and $s\neq 1$, and $s \neq -2n, n \in Z^+$, $\chi(n) \in R$ and n traverses all positive integers) satisfies $s=\frac{1}{2}$+ti($t\in R$,$t\neq 0$) and $s=\frac{1}{2}$-ti($t\in R$,$t\neq 0$) is sufficient to prove that the twin primes, Polignac's conjecture and Goldbach's conjecture are almost true. And if $\chi(n) = 0(n \in Z^+$ and n traverses all positive integers) or $\beta \in R$ and $\beta = -2n(n \in Z^+)$, then $L(\beta, \chi(n))=0$($\beta\in R$ and $\beta\neq -2n, n \in Z^+$, $\chi(n) \in R$ and n traverses all positive integers) and $L(\beta, 1)=0$($\beta\in R$ and $\beta\neq -2n, n \in Z^+$, and n traverses all positive integers), so for Riemann $\zeta(s)$($s\in C$ and $s\neq 1$) functions, its corresponding landau-siegel function $L(\beta, \chi(n))$($\beta \in R$, $\chi(n) \in R$, and $s \neq -2n$, $n \in Z^+$,and n traverses all positive integers) of pure real zero exist, this means that the Riemann $\zeta(s)$($s\in C$ and $s\neq 1$) function have a zero of a pure real variable s, and the generalized Riemann conjecture L(s, ✗ (n))=0(s∈C and s≠ 1, ✗ (n) ∈R ,and $s \neq -2n$, $n \in Z^+$ and n traverses all positive integers) is sufficient to prove that the twin primes, Polignac's conjecture and Goldbach's conjecture are completely true.

when ✗ (n)≠1($n \in Z^+$ and n traverses all positive integers) and ✗ (n)≠0($n \in Z^+$ and n traverses all positive integers), because the real exponential function of the real number has a function value greater than zero, so

$n^{-\beta} > 0$($n \in Z^+$ and n traverses all positive integers) and $1^\beta - 2^\beta < 0, 3^\beta - 4^\beta < 0, 5^\beta - 6^\beta < 0, \dots, (n-1)^\beta - (n)^\beta < 0, \dots, or\ 1^\beta - 2^\beta > 0, 3^\beta - 4^\beta > 0, 5^\beta - 6^\beta > 0, \dots, (n-1)^\beta - (n)^\beta > 0$ and $|\frac{1}{(1-2^{1-\beta})}| \neq 0$, it can be known that when ✗ (n)=1(n∈ Z^+ and n traverse all positivel numbers), then L(β,1)≠0(β ∈ R and $\beta \neq -2n$, $n \in Z^+$, ✗ (n) ∈ R and ✗ (n)=1, n traverses all positive integers) so for Riemann ζ(s)(s∈C and s≠ 1) functions, its corresponding landau -siegel function L(β, ✗ (n))(β ∈ R, ✗ (n) ∈ R and ✗ (n) ≠0, $n \in Z^+$ and n traverses all positive integers) of pure real zero does not exist, this means that the generalized Riemann L(β, ✗ (n))(β∈R, ✗ (n)∈R and $n \in Z^+$ and n traverses all positive integers) function does not have a zero of a pure real variable s, and the generalized Riemann conjecture L(s, ✗ (n))=0(s∈C and s≠ 1, ✗ (n)∈R and ✗ (n)≡1 and $n \in Z^+$ and n traverses all positive integers) satisfies $s=\frac{1}{2}+ti$(t ∈ R,t≠0) and $s=\frac{1}{2}-ti$(t ∈ R,t≠0) is sufficient to prove that the twin primes, Polignac's conjecture, Goldbach's conjecture are almost true.

When ✗ (n) ≠1 ($n \in Z^+$ and n traverses all positive integers) and ✗ (n) ≠0 ($n \in Z^+$ and n traverses all positive integers), because the real exponential function of the real number has a function value greater than zero, so

$n^{-\beta} > 0$($n \in Z^+$ and n traverses all positive integers) and $1^\beta - 2^\beta < 0, 3^\beta - 4^\beta < 0, 5^\beta - 6^\beta < 0, \dots, (n-1)^\beta - (n)^\beta < 0, \dots, or\ 1^\beta - 2^\beta > 0, 3^\beta - 4^\beta > 0, 5^\beta - 6^\beta > 0, \dots, (n-1)^\beta - (n)^\beta > 0$ and $|\frac{1}{(1-2^{1-\beta})}| \neq 0$, it can be known that when ✗ (n) ≠1($n \in Z^+$ andn traverses all positive integers) and ✗ (n) ≠ 0($n \in Z^+$ and n traverses all positive integers), then L(β, ✗ (n))≠0(β∈R and $\beta \neq -2n$, $n \in Z^+$, ✗ (n)∈R and ✗ (n) ≠1 and ✗ (n) ≠0 and n traverses all positive integers) ,so for generalized Riemann L(s, ✗ (n)) (s∈C and s≠ 1 and $s \neq -2n$, $n \in Z^+$ and n traverses all positive integers) functions, its corresponding landau-siegel function L(β, ✗ (n))(β∈R and $\beta \neq -2n$, $n \in Z^+$, ✗ (n)∈R and ✗ (n) ≠1 and ✗ (n) ≠0, $n \in Z^+$ and n traverses all positive integers) of pure real zero does not exist, this means that the generalized Riemann L(s, ✗ (n))(s∈C and s≠ 1 and $s \neq -2n$, $n \in Z^+$ and n traverses all positive integers) function does not have a zero of a pure real variable s. and the generalized Riemann conjecture L(s, ✗ (n))=0(s∈C and s≠ 1 and $s \neq -2n$, $n \in Z^+$, ✗ (n)∈R and ✗ (n)≠1 and ✗ (n)≠0 and n traverses all positive integers) satisfies $s=\frac{1}{2}+ti$(t∈R,t≠0) and $s=\frac{1}{2}-ti$(t∈R,t≠0)is sufficient to prove that the twin primes, Polignac's conjecture, Goldbach's conjecture are all almost true.

When ✗ (n) ≡0 ($n \in Z^+$ and n traverses all positive integers), because the real exponential function of the real number has a function value greater than zero, so $n^{-\beta} > 0$($n \in Z^+$ and n traverses all positive integers) and ✗ $(1)1^\beta = 0$, ✗ $(2)2^\beta = 0$, ✗ $(3)3^\beta = 0$, ✗ $(4)4^\beta = 0$, ✗ $(5)5^\beta = 0$, ✗ $(6)6^\beta = 0, \dots$, ✗ $(n-1)(n-1)^\beta = 0$, ✗ $(n)n^\beta = 0, \dots$, and $|\frac{1}{(1-2^{1-\beta})}| \neq 0$, it can be known that when ✗ (n) ≡0 ($n \in Z^+$ and n traverses all positive integers), then L(β, ✗ (n))≠0(β∈R and $\beta \neq -2n$, $n \in Z^+$, ✗ (n)∈R and $\chi(n) \equiv 1$, $n \in Z^+$ and n traverses all positive integers) and L(β, 1)≠0(β∈R and β ≠ $-2n$, $n \in Z^+$, and n traverses all positive integers), so for generalized Riemann L(s, $\chi(n)$)(s∈C and s≠ 1 and s ≠ $-2n, n \in Z^+$ and n traverses all positive integers) functions, its corresponding landau-siegel function L(β,0)(β∈R and β≠ $-2n$, $n \in Z^+$, χ(n)∈R and $\chi(n) \equiv 0$ and n traverses all positive integers) of pure real zero exists, This means that the generalized Riemann L(s, $\chi(n)$)(s∈C and s≠ 1and s ≠ $-2n, n \in Z^+$ and n traverses all positive integers) function has a zero of a pure real variable s, that means the twin prime conjecture, Goldbach's conjecture, Polignac's conjecture are completely true.

When χ(p)≡0(p ∈ Z^+ and p traverses all odd primes, including 1), then L(s, χ(p))=0(s∈C and s≠ 1and s ≠ $-2n, n \in Z^+$, χ(p) ∈R and χ(p)≡0, p ∈ Z^+ and p traverses all odd primes, including 1) was established. At the same time L(s, χ(p))(s∈C and s≠ 1and s ≠ $-2n, n \in Z^+$, χ (p) ∈ R and χ (p) ≡ 0, p ∈ Z^+ and p traverses all odd primes, including 1) the corresponding landau-siegel function

L(β,0)(β∈R and β≠ $-2n$, $n \in Z^+$, χ(p)∈R and χ(p)≡0, p ∈ Z^+ and p traverses all odd primes, including 1) expression as shown as follows: L(β, χ (p))=$\frac{(-1)^{n-1}}{(1-2^{1-\beta})}\sum_{p=1}^{\infty}\chi(p)p^{-\beta}(\cos(0\times$ $\ln p)+i\sin(0\times\ln(p)))=$

$\frac{(-1)^{n-1}}{(1-2^{1-\beta})}\sum_{p=1}^{\infty}(\chi(p)p^{-\beta}) = \frac{(-1)^{n-1}}{(1-2^{1-\beta})}[\chi(1)1^{-\beta} - \chi(2)2^{-\beta} + \chi(3)3^{-\beta} - \chi(5)5^{-\beta} + \chi(7)7^{-\beta} + \ldots$ $-\chi(p)p^{-\beta} + \cdots]$(β ∈R, p ∈ Z^+ and p traverses all primes, including 1), " × " is the symbol for multiplication.

When χ(p)≡0(p ∈ Z^+ and p traverses all odd primes, including 1), then L(s, χ(p))≡0(s∈C and s≠ 1 and s ≠ $-2n, n \in Z^+$, χ(n) ∈R and χ(p)≡0, p trav erses all odd primes, including 1) was established. At the same time L(s, χ (p))(s∈C and s ≠ 1 and s ≠ $-2n, n \in Z^+$, χ (p) ∈ R and χ(p)≡0, p ∈ Z^+ and p traverses all primes, including 1) the corresponding landau-siegel function L(β,0)=0(β∈R and β≠ $-2n$, $n \in Z^+$, χ(p)∈R and χ(p)≡0, p∈ Z^+ and p traverses all primes, including 1), this means that the generalized Riemann L(s, $\chi(n)$)(s∈C and s≠ 1 and s ≠ $-2n, n \in Z^+$ and n traverses all positive integers) function has a zero of a pure real variable s, that means the twin prime conjecture, Goldbach's conjecture, Polignac's conjecture are all completely true.

Now I summarize the Dirichlet function L(s, χ(n))(s∈C and s≠ 1 and s ≠ $-2n, n \in Z^+$, χ(n) ∈R, and n traverses all positive integers) as follows:

1: When χ(n)≡1(s ∈ C and s ≠ 1 and s ≠ $-2n, n \in Z^+$ and n traverses all positive integers), the generalized Riemannian hypothesis and the generalized Riemannian conjecture degenerate to the ordinary Riemannian hypothesis and the ordinary Riemannian conjecture, whose nontrivial zeros s satisfy s=$\frac{1}{2}$+ti(t∈R and t≠0), and ordinary Riemann ζ(s)=L(s, χ(n))(s∈C and s≠ 1 and s ≠ $-2n, n \in Z^+$, χ (n)∈R and χ (n) ≡ 1, $n \in Z^+$ and n traverses all positive integers) the corresponding Landau-siegel function L(β, χ(n))≠0(β∈R,and β≠ $-2n$, $n \in Z^+$, χ(n)∈R and χ(n)≡1 and n traverses all positive integers), ordinary Riemann hypothesis and ordinary Riemann hypothesis all hold, and for Riemann ζ(s)(s∈C and s≠ 1 and s ≠ $-2n, n \in Z^+$) function, its corresponding Landau-Siegel function L(β,1)(β ∈ R and β ≠ $-2n$, $n \in Z^+$, χ (n)∈R and χ(n)≡1, $n \in Z^+$ and n traverses all positive integers) does not exist pure real zero, which also shows that Riemann ζ(s)(s∈C and s≠ 1 and s ≠ $-2n, n \in Z^+$) function does not exist zero when variable s is a pure real zero.

2: When χ (n) ≡ 0(n ∈ Z^+ and n traverses all positive odd numbers,including 1), then χ (p)≡0(p ∈ Z^+ and p traverses all odd primes, including 1), a special Dirichlet function L(s, χ

(p))(s∈C and s$\neq$ 1 and $s \neq -2n, n \in Z^+$, χ(p)∈R and χ(p)≡0, $p \in Z^+$ and p traverses all odd primes, including 1) has zero, and when zero is obtained, the independent variable s is any complex number. This special dirichlet function L(s, χ(p))(s∈C and s$\neq$ 1 and $s \neq -2n, n \in Z^+$, χ(p)∈R and χ(p)≡0, $p \in Z^+$ and p traverses all odd prime, including 1) the corresponding Landau-siegel function L(β,0)=0(β∈R and $\beta \neq -2n$, $n \in Z^+$, χ(p)∈R and χ(p)≡0, $p \in Z^+$ and p traverses all odd prime, including 1) holds, so for this particular Dirichlet function L(s, χ (p))=0(s∈C and $s \neq 1$ and $s \neq -2n, n \in Z^+$, χ (p)∈R and χ (p) ≡ 0, $p \in Z^+$ and p traverses all odd primes, including 1) holds.The existence of a pure real zero of the corresponding Landau-Siegel function L(β,0)(β∈R and $\beta \neq -2n$, $n \in Z^+$, χ(p)∈R and χ(p)≡0, $p \in Z^+$ and p traverses all odd prime numbers, including 1) shows that the twin prime numbers, Polignac conjecture and Goldbach conjecture are all completely true.

3: When χ(n)≠1 and χ(n)≠0(n $\in Z^+$ and n traverses all positive integers), Dirichlet function L(s, χ (n))(s∈C and s$\neq$ 1 and $s \neq -2n, n \in Z^+$, χ(n)∈ and χ(n)≠0 and χ(n)≠1, $n \in Z^+$ and n traverses all positive integers) has zero, it's nontrivial zero meet s=$\frac{1}{2}$+ti(t∈R and t≠0) and s=$\frac{1}{2}$-ti(t∈R and t≠0). For dirichlet function L(s, χ (n))(s∈C and $s \neq 1$and $s \neq -2n, n \in Z^+$, χ(n)∈R and χ(n)≠0, $n \in Z^+$ and n traverses all positive integes), it's corresponding Landau-siegel function L(β, χ(n))(β∈R and $\beta \neq -2n$, $n \in Z^+$, χ(n)∈R and χ(n)≠0 and χ(n)≠1, $n \in Z^+$ and n traverses all positive integers) of pure real zero does not exist, In other words, it shows that the Dirichlet function L(s, χ (n))(s∈C and $s \neq 1$and $s \neq -2n, n \in Z^+$,, χ (n)∈R and χ(n)≠0 and χ(n)≠1, $n \in Z^+$ and n traverses all positive integers) does not exist for the zero of a pure real variable s, so if χ(n)≠0 and χ(n)≠1 ($n \in Z^+$ and n traverses all positive integers), then both the generalized Riemannian hypothesis and the generalized Riemannian conjecture hold and the Generalized Riemann L(s, χ (n))(s∈C and s$\neq$ 1,and $s \neq -2n, n \in Z^+$, χ(n)∈R and χ(n)≠0 and χ(n)≠1, $n \in Z^+$ and n traverses all positive intege) function of nontrivial zero s also .0meet s=$\frac{1}{2}$+ti(t∈R and t≠0) and s=$\frac{1}{2}$-ti(t∈R,t≠0).Now we know that merely proving that the nontrivial zero s of the Riemann conjecture L(s,1)=0(s∈C and s$\neq$ 1and $s \neq -2n, n \in Z^+$, χ (n)∈R and χ (n) ≡ 1, $n \in Z^+$ and n traverses all positive integers) and the generalized Riemann conjecture L(s, χ (n))=0(s∈C and s$\neq$ 1and $s \neq -2n, n \in Z^+$, χ (n)∈R and χ (n)≠1 and χ (n)≠0 and n traverses all positive integers) satisfies s=$\frac{1}{2}$+ti(t∈R,t≠0) and s=$\frac{1}{2}$-ti(t∈R,t≠0) is sufficient to prove that the twin primes, Polignac's conjecture, Goldbach's conjecture are all almost true.

Conclusion

After the Riemann hypothesis and the Riemann conjecture and the Generalized Riemann hypothesis and the Generalized Riemann conjecture are proved to be completely valid, the research on the distribution of prime numbers and other studies related to the Riemann hypothesis and the Riemann conjecture will play a driving role. Readers can do a lot in this respect.

Thanks

Thank you for reading this paper.

Contribution

The sole author, poses the research question, demonstrates and proves the question.

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References

1. Riemann: On the Number of Prime Numbers Less than a Given Value
2. John Derbyshire (America): PRIME OBSESSION P218, berhard riemann and the greatest unsolved problem in mathmatics, Translated by Chen Weifeng, Shanghai Science and Technology Education Press, China,https://www.doc88.com/p-54887013707687.html;
3. Xie Guofang: On the number of prime numbers less than a given value - Notes to Riemann's original paper proposing the Riemann conjecture
4. Lu Changhai: A Ramble on the Riemann Conjecture