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The Embedding Theorems for Lorentz–Morrey Spaces of Many Groups of Variables

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Abstract

In present paper, I intend to introduce our study of new normed function space type of Lorentz-Morrey ℒ<l> (s,G) associated parameters of many groups of variables started in works by A.Dj. Djabrailov. I must note that, this space belongs to spaces type of Lebesgue-Morrey type. As an application, we give some properties for these spaces again. In addition, I have given two needing lemmas and they have been proved. In view of the embedding theorems we study some properties of the functions, which are belonging to these spaces. Although I have dealt with a lot of measurable cases, differentiable function spaces are very difficult in general. The most important cases are Lebesgue-Morrey type spaces with many groups of variables. I begin with the general theory of Mathematical Analysis, I have constructed new normed spaces type of Lorentz- Morrey, gave and proved some characterization of these type of spaces. In addition, specific techniques for introducing some embedding theorems will be given late.

Keywords: Lorentz-Morrey Spaces, Many Groups of Variables, Integral Representation, Embedding Theorems.

Introduction

Let $1 \leq s \leq n;\ s, n -$ are positive integers and $n_1 + \cdots + n_s = n$. Assume that

$$x = (x_1, \dots, x_s) \in R^n\ x_k = \left(x_{k.1}; \dots; x_{k,n_k}\right) \in R^{n_k}\ (k \in e_s = \{1,2,\dots,s\})$$

and we are given a Lebesgue measurable functions $f(x)$. More precisely, $R^n = R^{n_1} \times R^{n_2} \times \cdots \times R^{n_s}$. Thus we consider the fixed, non-negative and integer vectors $l = (l_1, \dots, l_s)$, such that, $l_k = \left(l_{k.1}; \dots; l_{k,n_k}\right)$, $(k \in e_s)$. That is, $l_{k,j} > 0$, $(j = 1, \dots, n_k)$, for all $k \in e_s$. Here we denote by Q the set of the vectors $i = (i_1, \dots, i_s)$, where $i_k = 1,2,\dots, n_k$ and $k \in e_s$. The number of elements of the set Q is equal to $|Q| = \prod_{k=1}^{s}(1 + n_k)$. Therefore, to each vector $i = (i_1, \dots, i_s) \in Q$ we correspond the vector $l^i = \left(l_1^{i_1}; \dots; l_s^{i_s}\right)$ associated with the fixed "positive vector" $l = (l_1, \dots, l_s)$ by the following way $l^0 = (0,0, \dots ,0), l_k^1 = \left(l_{k,1}, 0, \dots ,0\right), \dots , l_k^{i_k} = \left(0,0, \dots, l_{k,n_k}\right)$, for all $k \in e_s$. Then to the vectors e^i, we correspond the vectors $\bar{l}^i = (\bar{l}_1^{i_1}, \bar{l}_1^{i_2}, \dots, \bar{l}_1^{i_s})$, where $\bar{l}_k^{i_k} = \left(\bar{l}_{k,1}^{i_1}, \bar{l}_{k,2}^{i_2}, \dots, \bar{l}_{k,n_k}^{i_k}\right)$ $(k \in e_s)$. Here for every $k \in e_s$, $\bar{l}_{k,j}^{i_k}$ is the greatest integer less than $l_{k,j}^{i_k}$ if $l_{k,j}^{i_k} > 0$, and $\bar{l}_{k,j}^{i_k} = 0$, if $l_{k,j}^{i_k} = 0$. [13, 14, 15, 17, 19]

Materials and Methods

In this paper we introduce and study the new function space

$$\mathcal{L}^{<l>}_{p,\theta,a,\varkappa,\tau}(s,G) \qquad (1)$$

of several groups of variables of Lorentz-Morrey type, where the analysis is based on a setting space, related methods of the integral representation and differential properties of some classes of such function.

Definition 1. We denote by $\mathcal{L}^{<l>}_{p,\theta,a,\varkappa,\tau}(s,G)$ Lorentz-Morrey space type of locally summable function f on $G \subset R^n$, with finite norm ($1 \le p < \infty, 1 \le \theta \le \infty$)

$$\|f\|_{p,a,\varkappa,\tau:G} = \|f\|_{\mathcal{L}_{p,a,\varkappa,\tau}(G)} =$$

$$\begin{cases} \left\{\int_0^\infty \left[\prod_{k\in e_s}[t_k]_1^{-\frac{|\varkappa_k|a}{p}} \times \|f^*\|_{p,G_{t^\varkappa}(x)}\right]^\tau \prod_{k\in e_s}\frac{dt_k}{t_k}\right\}^{1/\tau} \\ sup_{0<t<\infty}\left(\prod_{k\in e_s}[t_k]_1^{-\frac{|\varkappa_k|a}{p}} \times \|f^*\|_{p,G_{t^\varkappa}(x)}\right) \end{cases}, \qquad (2)$$

$D^{\bar{l}^i}f = D_1^{l_1^{i_1}} \cdots D_s^{l_s^{i_s}}f$, $\quad D_k^{l_k^{i_k}}f = D_{k,1}^{l_k^{i_k}} \cdots D_{k,n_k}^{l_k^{i_k}}f$; $G_{t^\varkappa} = G \cap I_t(x)$; $\quad I_{t^\varkappa}(x) = I_{t_1^{\varkappa_1}} \times I_{t_2^{\varkappa_2}} \times \cdots \times I_{t_s^{\varkappa_s}}$; $I_{t_k^{\varkappa_k}}(x_k)=\left\{y_k : |y_k - x_k| < \frac{1}{2}t_k^{|\varkappa_k|},\ k \in e_s\right\}$, where $|\beta_k| = \sum_{j=1}^{n_k}\beta_{k,j}^{i_k}$, $\frac{dt_k}{t_k} = \prod_{j\in e_k^i}\frac{dt_{k,j}}{t_{k,j}}$ and $0<\beta_{k,j}^{i_k} = l_{k,j}^{i_k} - \bar{l}_{k,j}^{i_k} \le 1$ for $l_k^{i_k} > 0$, and $\beta_{k,j}^{i_k} = 0$, if $l_{k,j}^{i_k} = 0$, $t = (t_1, \dots, t_s)$, $t_k = (t_{k.1}; \dots; t_{k,n_k})$, $\omega = (\omega_1, \dots, \omega_s)$, $\omega_k = (\omega_{k.1}; \dots; \omega_{k,n_k})$. When $\omega_{k.j} = 1$ for $k \in e^i$, then $\omega_{k.j}=0$, $k\in e_s / e^i$; $e^i = supp\bar{l}^i = supp\omega$. Hence, let $t_0 = (t_{0,1}, \dots, t_{0,s})$, $t_{0,k} = (t_{0,k,1}, \dots, t_{0,k,n_k})$ be a fixed positive vector, and $\varkappa\in (0,\infty)^n$, $a\in [0,1]$, $\tau \in [1,\infty]$, $[t_k]_1 = \min\{1, t_k\}$, $k \in e_s$.

Let us give some characterization of $\mathcal{L}_{p,a,\varkappa,\tau}(G)$:

1) $\|\cdot\|_{p,a,\varkappa,\tau:G}$ is a qiasi-norm.

2) We must note that, for every $\tau > 0$

$$\mathcal{L}_{p,a,\varkappa,\tau}(G) = \mathcal{L}_{p,a,\varkappa}(G)$$

3) The space $\mathcal{L}_{p,a,\varkappa,\tau}(G)$ is complete.

4) For c>0 we have

$$\|f\|_{p,a,c\varkappa,\tau:G} = \frac{1}{c^{\frac{s}{\tau}}}\|f\|_{p,a,\varkappa,\tau:G}.$$

5) For any $\varkappa=(\varkappa_1, \dots, \varkappa_n) > 0$ we get:

a) $\|f\|_{p,0,\varkappa,\infty:G} = \|f\|_{p,G}$;

b) $\|f\|_{p,1,\varkappa,\tau:G} \ge \|f\|_{\infty,G}$.

6) If $p \le q, \frac{1-b}{q} \le \frac{1-a}{p}$, $1 \le \tau_1 \le \tau_2 \le \infty$ then

$$\mathcal{L}_{q,b,\varkappa,\tau_1}(G) \subset_> \mathcal{L}_{p,a,\varkappa,\tau_2}(G)$$

and

$$\|f\|_{p,a,\varkappa,\tau_2:G} \le \|f\|_{q,b,\varkappa,\tau_1:G}. \qquad (3)$$

Results and Discussion

Theorem 1: Let f and g be two functions in $\mathcal{L}_{p,a,\varkappa,\tau}(G)$. Then for all $\lambda_1, \lambda_2, \lambda_3 \geq 0$, we have:

a) D_f is decreasing and continuous from the right;

b) $|g| \leq |f|$ then $D_g(\lambda) \leq D_f(\lambda)$;

c) $D_{cf}(\lambda_1) = D_f\left(\frac{\lambda}{|c|}\right)$ for all $c \in C\backslash\{0\}$;

d) $D_{f+g}(\lambda_1 + \lambda_2) \leq D_f(\lambda_1) + D_g(\lambda_2)$;

e) $D_{fg}(\lambda_1\lambda_2) \leq D_f(\lambda_1) \times D_g(\lambda_2)$;

f) If $|f| \leq \lim inf|f_n|$, implies that $D_{f_n}(\lambda)$ for any $\lambda \geq 0$;

g) If $|f_n| \uparrow |f|$, then $\lim\limits_{n\to\infty} D_{f_n}(\lambda) = D_f(\lambda)$.

Proof:

a) Let $0 \leq \lambda_1 \leq \lambda_2$. Then

$$\left\{t, |f(t)| > \prod_{k\in e_s}[\lambda_{2,k}]_1\right\} \subseteq \left\{t, |f(t)| > \prod_{k\in e_s}[\lambda_{1,k}]_1\right\}.$$

Then by the monotonicity of the measure we get

$$D_f(\lambda_1) \geq D_f(\lambda_2).$$

It means that, the function D_f is decreasing. Let us proof continuous from the right. Let be $\lambda_0 > 0$ and we have to choose $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ such that $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4 \geq \cdots$ and let us define $E_f(\lambda)$ for each

$$E_f(\lambda) = \left\{t, \qquad |f(t)| > \prod_{k\in e_s}[\lambda_k]_1\right\}.$$

Hence, $E_f(\lambda_1) \subseteq E_f(\lambda_2) \subseteq E_f(\lambda_3) \subseteq \cdots$, and by the monotone convergence theorem we get

$$\lim_{n\to\infty} D_f\left(\lambda_0 + \frac{1}{n}\right) = \lim_{n\to\infty} \mu\left(E_f\left(\lambda_0 + \frac{1}{n}\right)\right) =$$

$$\mu\left(\cup_{n=1}^{\infty} E_f\left(\lambda_0 + \tfrac{1}{n}\right)\right) = \mu\left(E_f(\lambda_0)\right) = D_f(\lambda_0).$$

Since $E_f(\lambda_1) \subseteq E_f(\lambda_2) \subseteq E_f(\lambda_3) \subseteq \cdots$, and $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4 \geq \cdots$. This establishes the right continuity.

b) Suppose that, f and g are two functions in $\mathcal{L}_{p,a,\varkappa,\tau}(G)$ and $|g| \leq |f|$. Following

$$\left\{t, |g(t)| > \prod_{k\in e_s}[\lambda_k]_1\right\} \subseteq \left\{t, |f(t)| > \prod_{k\in e_s}[\lambda_k]_1\right\}.$$

According to the monotonicity of a measure we hold following

$$\mu\left\{t, |g(t)| > \prod_{k\in e_s}[\lambda_k]_1\right\} \leq \mu\left\{t, |f(t)| > \prod_{k\in e_s}[\lambda_k]_1\right\}.$$

It means that, $D_g(\lambda) \leq D_f(\lambda)$.

c) Let $f \in \mathcal{L}_{p,a,\varkappa,\tau}(G)$ and $c \in C\backslash\{0\}$. Following

$$\left\{t, |f(ct)| > \prod_{k\in e_s}[\lambda_k]_1\right\} =$$

$$\left\{t, |c||f(t)| > \prod_{k\in e_s}[\lambda_k]_1\right\} =$$

$$\left\{t, |f(t)| > \frac{1}{|c|}\prod_{k\in e_s}[\lambda_k]_1\right\}.$$

It implies, that

$$\mu\left\{t, |cf(t)| > \prod_{k\in e_s}[\lambda_k]_1\right\} = \mu\left\{t, |f(t)| > \prod_{k\in e_s}\frac{1}{|c|}[\lambda_k]_1\right\}$$

Moreover, we get $D_{cf}(\lambda_1) = D_f\left(\frac{\lambda}{|c|}\right)$.

d) Let $f, g \in \mathcal{L}_{p,a,\varkappa,\tau}(G)$ and $\lambda_1, \lambda_2 \geq 0$. Then we have

$$\left\{t, |f(t) + g(t)| > \prod_{k\in e_s}[\lambda_{1,k}]_1 + \prod\nolimits_{k\in e_s}[\lambda_{2,k}]_1\right\} \subseteq$$

$$\left\{t, |f(t)| + |g(t)| > \prod_{k\in e_s}[\lambda_{1,k}]_1 + \prod\nolimits_{k\in e_s}[\lambda_{2,k}]_1\right\} \subseteq$$

$$\left\{t, |f(t) + g(t)| > \prod_{k\in e_s}[\lambda_{1,k}]_1 + \prod\nolimits_{k\in e_s}[\lambda_{2,k}]_1\right\} \subseteq$$

$$\left\{t, |f(t)| > \prod_{k\in e_s}[\lambda_{1,k}]_1\right\} \cup \left\{t, |g(t)| > \prod\nolimits_{k\in e_s}[\lambda_{2,k}]_1\right\}.$$

That is,

$$\mu\left\{t, |f(t) + g(t)| > \prod_{k\in e_s}[\lambda_{1,k}]_1 + \prod\nolimits_{k\in e_s}[\lambda_{2,k}]_1\right\} \leq$$

$$\mu\left\{t, |f(t)| > \prod_{k\in e_s}[\lambda_{1,k}]_1\right\} + \mu\left\{t, |g(t)| > \prod\nolimits_{k\in e_s}[\lambda_{2,k}]_1\right\}.$$

Thus, $D_{f+g}(\lambda_1 + \lambda_2) \leq D_f(\lambda_1) + D_g(\lambda_2)$.

e) Let $f, g \in \mathcal{L}_{p,a,\varkappa,\tau}(G)$ and $\lambda_1, \lambda_2 \geq 0$. Then we have

$$\left\{t, |f(t)g(t)| > \prod_{k\in e_s}[\lambda_{1,k}]_1 \cdot \prod_{k\in e_s}[\lambda_{2,k}]_1\right\} =$$

$$\left\{t, |f(t)||g(t)| > \prod_{k\in e_s}[\lambda_{1,k}]_1 \cdot \prod_{k\in e_s}[\lambda_{2,k}]_1\right\} \subseteq$$

$$\left\{t, |f(t)| > \prod_{k\in e_s}[\lambda_{1,k}]_1\right\} \cup \left\{t, |g(t)| > \prod_{k\in e_s}[\lambda_{2,k}]_1\right\}.$$

It means that, $D_{f+g}(\lambda_1\lambda_2) \leq D_f(\lambda_1)+D_g(\lambda_2)$.

According to [4] one can easy proof f and g. [1, 2, 4, 7, 18]

Remark: (Completeness). The normed space $\mathcal{L}_{p,a,\varkappa,\tau}(G)$ is a complete.

Proof. Let $\|f_n\|_{m,n\in N}$ be an arbitrary Cauchy sequence in $\mathcal{L}_{p,a,\varkappa,\tau}(G)$. Then we hold

$$\|f_m - f_n\|_{p,a,\varkappa,\tau:\, G} \to 0, \text{ as } m,n \to \infty$$

and according to corollary 2.16 [4] we get

$$\|f_m - f_n\|_{(p,\infty)} \leq \left(\prod_{k\in e_s}[t_k]_1^{-\frac{|\varkappa_k|a}{p}} \times \|f_m - f_n\|\right) \to \infty, \text{as } m,n \to \infty.$$

Thus

$$sup_{0<\lambda<\infty}\left(\prod_{k\in e_s}[\lambda_k]_1^{-\frac{|\varkappa_k|a}{p}} \times D_{f_m - f_n}(\lambda)\right)^{1/\tau} =$$

$$sup_{0<t<\infty}\left(\prod_{k\in e_s}[t_k]_1^{-\frac{|\varkappa_k|a}{p}} \times (f_m - f_n)^*(t)\right)^{1/\tau} \to 0, \text{ as } m,n \to \infty.$$

It follows now by theorem 1.6(g) in [4] that

$$(f - f_0)^*(t) \leq \lim_{k\to\infty} inf\left(f_{n_k} - f_{n_0}\right)^*(t), \text{ for all } t>0.$$

Due to Fatou Lemma, we have

$$(f - f_0)^{**}(t) \leq \lim_{k\to\infty} inf\left(f_{n_k} - f_{n_0}\right)^{**}(t), \text{ for all } t>0.$$

Once again by Fatou's Lemma we hold

$$\|f - f_{n_0}\|_{p,a,\varkappa,\tau} =$$

$$\left\{\int_0^\infty \left[\prod_{k\in e_s}[t_k]_1^{-\frac{|\varkappa_k|a}{p}} \times \left(f - f_{n_0}\right)^{**}(t)\right]^\tau \prod_{k\in e_s}\frac{dt_k}{t_k}\right\}^{1/\tau} \leq$$

$$\left\{\int_0^\infty \left[\prod_{k\in e_s}[t_k]_1^{-\frac{|\varkappa_k|a}{p}} \times \lim_{k\to\infty} inf\left(f_{n_k} - f_{n_0}\right)^{**}(t)\right]^\tau \prod_{k\in e_s}\frac{dt_k}{t_k}\right\}^{1/\tau} \leq$$

$$\lim_{k\to\infty} inf \left\{ \int_0^\infty \left[\prod_{k\in e_s} [t_k]_1^{-\frac{|\varkappa_k|a}{p}} \times \left(f_{n_k} - f_{n_0}\right)^{**}(t) \right]^\tau \prod_{k\in e_s} \frac{dt_k}{t_k} \right\}^{1/\tau} \le$$

$$\lim_{k\to\infty} inf \left\|f_{n_k} - f_{n_0}\right\|_{p,a,\varkappa,\tau} \le \varepsilon, \text{for } n_k > n_0.$$

Since $f = \left(f - f_{n_0}\right) + f_{n_0} \in \mathcal{L}_{p,a,\varkappa,\tau}(G)$. This implies that $\mathcal{L}_{p,a,\varkappa,\tau}(G)$ is complete.

$$sup_{0<t<\infty}\left(\prod_{k\in e_s}[t_k]_1^{-\frac{|\varkappa_k|a}{p}} \times \{t: |f_m(t) - f_n(t)| > \prod_{k\in e_s}[t_k]_1\}\right)^{1/\tau} =$$

$$sup_{0<t<\infty}\left(\prod_{k\in e_s}[t_k]_1^{-\frac{|\varkappa_k|a}{p}} \times D_{f_m(t)-f_n(t)}\right)^{1/\tau} \to 0,\ \ m,n \to \infty.$$

It proves that $\{t: |f_m(t) - f_n(t)| > \prod_{k\in e_s}[t_k]_1\} \to 0,\ m,n \to \infty$, for any t>0. We proved that given $\|f_n\|_{n,n\in N}$ sequence is a Cauchy sequence.

Let be $\varepsilon > 0$ arbitrary, since $\|f_n\|_{n,n\in N}$ is a Cauchy sequence, then there exits $n_0 \in N$ such that

$$\left\|f_n - f_{n_0}\right\|_{p,a,\varkappa,\tau} < \varepsilon, (n > n_0)$$

and $(f_{n_k} - f_{n_0})$ convergence to $(f - f_0)$.

$$min_G\left[(f_j - f_k)_n^*\right]^{-\frac{|\varkappa_k|a}{p}} \le \left[(f_j - f_k)_n^*(y)\right]^{-\frac{|\varkappa_k|a}{p}}, y \in G \Rightarrow$$

$$\left[(f_j - f_k)_n^*\right]^{-\frac{|\varkappa_k|a}{p}} \le \left[(f_j - f_k)_n^*(y)\right]^{-\frac{|\varkappa_k|a}{p}}, y \in G \Rightarrow$$

$$\left[(f_j - f_k)_n^*(t)\right]^{-\frac{|\varkappa_k|a}{p}} \gamma(t) \le \left[(f_j - f_k)_n^*(y)\right]^{-\frac{|\varkappa_k|a}{p}} \gamma(y) \Rightarrow$$

$$\int_0^\infty \left\{\left[(f_j - f_k)^{**}(t)\right]^\tau \gamma(y)\right\} \prod_{k\in e_s} \frac{dt_k}{t_k} \le$$
$$\int_0^\infty \left\{\left[(f_j - f_k)^{**}(y)\right]^\tau \gamma(y)\right\} \prod_{k\in e_s} \frac{dt_k}{t_k} \Rightarrow$$

$$\left[(f_j - f_k)^{**}(t)\right]^\tau \int_0^\infty \{\gamma(y)\} \prod_{k\in e_s} \frac{dt_k}{t_k} \le$$

$$\int_{R^n} \left\{\left[(f_j - f_k)_n^{**}(y)\right]^\tau \gamma(y)\right\} \prod_{k\in e_s} \frac{dt_k}{t_k}.$$

This way we hold

$$\left[(f_j - f_k)^{**}(t)\right]^\tau \int_G \gamma(y) \prod_{k\in e_s} \frac{dt_k}{t_k} \le$$

$$\left\|f_j - f_k\right\|_{\mathcal{L}_{p,a,\varkappa,\tau}(G)}^\tau.$$

Hence

$$\left(f_j - f_k\right)_n^* \to 0 \Rightarrow D_{\left(f_j - f_k\right)_n^*} \to 0 \Rightarrow D_{\left(f_j - f_k\right)} \to 0.$$

That means $(f_k)_k$ is Cauchy in measure. Then there exists a subsequence $\left(f_{k_j}\right)$ that converges pointwise to a measurable function f. According to Property 3.15(e) in [4] and Fatou's lemma we are able to finish that $f \in \mathcal{L}_{p,a,\varkappa,\tau}(G)$. Moreover

$$f = \lim_{j\to\infty} f_{k_j} \Rightarrow f_n^* = \lim_{j\to\infty} \left(f_{k_j}\right)_n^* \Rightarrow (\text{ Property } 3.15(\text{e}))$$

$$\int_{R^n} \{[f_n^*(t)]^\tau \gamma(t)\} \prod_{k\in e_s} \frac{dt_k}{t_k} \leq$$

$$\lim_{j\to\infty} inf \int_{R^n} \{[f_n^*(t)]^\tau \gamma(t)\} \prod_{k\in e_s} \frac{dt_k}{t_k}$$

$$(\text{Fatou's lemma}) \leq c < \infty \Rightarrow f \in \mathcal{L}_{p,a,\varkappa,\tau}(G).$$

Besides

$$\lim_{j\to\infty} \left|f_{k_j}(t) - f_j(t)\right| = \left|f(t) - f_j(t)\right|, t \in R^n.$$

Taking Fatou's lemma and the fact that $(f_k)_k$ is a Cauchy sequence, we obtain following

$$\|f - f_i\|_{\mathcal{L}_{p,a,\varkappa,\tau}(G)} = \left\|f - f_{k_j} + f_{k_j} - f_i\right\|_{\mathcal{L}_{p,a,\varkappa,\tau}(G)} \leq$$

$$c\left(\left\|f - f_{k_j}\right\|_{\mathcal{L}_{p,a,\varkappa,\tau}(G)} + \left\|f_{k_j} - f_i\right\|_{\mathcal{L}_{p,a,\varkappa,\tau}(G)}\right) =$$

$$c\left(\left\|f - f_{k_j}\right\|_{\mathcal{L}_{p,a,\varkappa,\tau}(G)} + \left\|f_i - f_{k_j}\right\|_{\mathcal{L}_{p,a,\varkappa,\tau}(G)}\right) \to 0.$$

Conclusions and Recommendations

In present paper, I intent to introduce my study of new normed function space type of Lorentz-Morrey, associated parameters of many groups of variables started in works by Allahveran Djabrailov. As an application, I give some properties for these spaces again. In addition, I have given two needing lemmas and they have been proved. In view of the embedding theorems, I study some properties of the functions, which are belonging to these spaces. Although I have dealt with a lot of measurable cases, differentiable function spaces are very difficult in general. The most important cases are Lebesgue-Morrey type spaces with many groups of variables. I begin with the general theory of Mathematical Analysis, I have constructed new normed spaces type of Lorentz-Morrey, gave and proved some characterization of these type of spaces. In addition, specific techniques for introducing some embedding theorems will be given late.

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