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Modeling the Elasticity of Ulaanbaatar Development Index in Connection with its Sub-Indices

Ganlkhagva Gantigmaa[1]., Badrakh Otgonsuvd[1]., Dulamragchaa Uuganbaatar[1]., Rentsen Enkhbat[1*]., Natsagdorj Tungalag[2]., Altangerel Munkh-Erdene[1]., Munkhnast Munkh-Erdene[1]

[1]*Institute of Mathematics and Digital Technology, Mongolian Academy of Sciences, Ulaanbaatar 13330, Mongolia*
[2]*National University of Mongolia, Ulaanbaatar 210646, Mongolia*

[*]**Corresponding author:** Rentsen Enkhbat, 1Institute of Mathematics and Digital Technology, Mongolian Academy of Sciences, Ulaanbaatar 13330, Mongolia.

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Abstract
This research presents the results of four simulations using the Cobb-Douglas function to model the impact of 86 sub-indices on the Ulaanbaatar Development Index. The 86 sub-indices and the general index for 2023, as calculated by scientific organizations, serve as the primary data. To estimate the elasticities of the city indices, the Cobb-Douglas function was reformulated and analyzed using the least squares method. The numerical computations were performed in Python with the CVXPY package.

Keywords: City Indicators, Urban Livability

Introduction

Many indices are used globally to assess different aspects of city development and urban livability, such as the City Prosperity Index, Global Power City Index, and Mercer Quality of Living Index. These indices are built from sub-indices focusing on areas like infrastructure, quality of life, environmental sustainability, economic performance, accessibility, and healthcare.

The municipality of Ulaanbaatar calculated the City Development Index for the first time in 2023. It comprises 86 sub-indices organized into four broad groups and six distinct categories. The government's aim in developing this index is to promote a city that is competitive, sustainable, accessible, and offers a high quality of life. Meanwhile, citizens are encouraged to fulfill their responsibilities while benefiting from these improvements [1].

Understanding the relationship between sub-indices and the overall City Development Index is essential for identifying the key drivers of urban development goals. This study applies the Cobb-Douglas function to specifically quantify how changes in individual sub-indices influence the overall CDI(general index). By doing so, it enables policymakers and stakeholders to identify and prioritize targeted initiatives that maximize the effectiveness of urban development strategies, ensuring sustainable and measurable improvements in the city's growth and quality of life.

Currently, there doesn't appear to be a published research paper that explicitly uses the Cobb-Douglas function to calculate the elasticity of a city development index (or similar indices) in direct correlation with its sub-indices. However, the following methodologies have been studied previously to demonstrate the relationship between development indicators and their influential factors.

An improved simulated annealing algorithm is employed for parameter estimation to refine the traditional Cobb-Douglas production function. This approach incorporates policy factors that influence growth at different stages, thereby improving the accuracy of estimating the contribution rates of various factors on economic growth [2].

The methodology of using probability-weighted averages of population sizes within local labor market areas has shown stronger correlations with location quotients for knowledge-intensive business services—industries that are highly dependent on urbanization economies [3].

The use of multilevel logistic regression revealed significant correlations between higher Urban Liveability Index scores and an increased likelihood of using active and public transport, along with a decreased reliance on private vehicles [4]. The Analytic Hierarchy Process is also employed to determine the relative weights of various indicators in the development of the Liveable.City.Index [5].

Research Methodology

The model of the general city index, taking into account the elasticities in the indices, is formulated as following:

$$f = k_1^{\alpha_1} k_2^{\alpha_2} \dots k_n^{\alpha_n} \quad (1)$$

were, α_i – elasticity coefficients of k_i-th index. In other words, a 1% change in the k_i-th index results in a change of α_i percent in the general city index f_ .

From the given 86 sub-indices, the general city index is found by the following formula.

$$f=0.35 \sum_1^{43} k_i + 0.15 \sum_{44}^{86} k_i \quad (2)$$

where, f – general city index, k_i –sub-indices, i=1, 2…, 86.

The elasticities are solved as follows by taking the logarithms from both sides of (1) and using the least squares method.

$$\ln f = \ln k_1^{\alpha_1} k_2^{\alpha_2} \dots k_n^{\alpha_m} \Rightarrow \ \ln f = \alpha_1 \ln k_1 + \alpha_2 \ln k_2 \dots \alpha_m \ln k_n$$

For $k_1^j, k_2^j, \dots, k_n^j$, we compose

$$\ln f_j = \sum_{i=1}^n \alpha_i \ln k_i^j \,, j = 1,2,..,m.$$

Finding α_i, i=1,2,..,n reduces to following optimization problem,

$$F = \sum_{j=1}^m \left(\sum_{i=1}^n \alpha_i \ln k_i^j - \ln f_j\right)^2 \to min \quad (3)$$

In order to find α_p, we take derivatives with respect to α_p and equalize to zero.

$$\frac{dF}{d\alpha_p} = 2 \sum_{j=1}^m \left(\sum_{i=1}^n \alpha_i \ln k_i^j - \ln f_j\right) \ln k_p^j = 0, \text{ p=1, 2…, n}$$

We simplify the above equations in the following:

$$\sum_{i=1}^n \alpha_i \left(\sum_{j=1}^m \ln k_i^j \ln k_p^j\right) = \sum_{j=1}^m \ln f_j \ln k_p^j, \text{ p=1, 2…, n.} \quad (4)$$

Let us assume the change in the i-th index is $\Delta\tilde{k}_i$ and calculate it as follows:

$$\tilde{k}_i = k_i + \Delta\tilde{k}_i, \ i=1,2\dots,n$$
$$\Delta\tilde{k}_i = 0.01i, \ i=1,2\dots,86$$
$$\Delta\tilde{k}_i = 0.02i, \ i=1,2\dots,86$$
……………………………….
……………………………….
$$\Delta\tilde{k}_i = 0.20i, \ i=1,2\dots,86$$

The perturbed equations of (1) and (2) are calculated as shown below:

$$ln\tilde{f} = \alpha_1 \ln \tilde{k}_1 + \alpha_2 \ln \tilde{k}_2 + \dots + \alpha_n \ln \tilde{k}_n$$

$$\tilde{f}=0.35 \sum_1^{43} \tilde{k}_i + 0.15 \sum_{44}^{86} \tilde{k}_i$$

The elasticities are calculated through the following four simulations.

Case1.

$$F(\alpha_1, \alpha_2 \dots, \alpha_n) = \sum_{i=1}^n (\alpha_i \ln k_i - \ln f_i)^2 \to min$$

Case2.

$F(\alpha_1, \alpha_2 \dots, \alpha_n) = \sum_{i=1}^n \left(\alpha_i \ln k_i^j - \ln f_i\right)^2 + \sum_{j=1}^m \sum_{i=1}^n \left(\alpha_i \ln \tilde{k}_i^j - \ln \tilde{f}_j\right)^2 \to min$, where $\Delta\tilde{k}_i=0.01i$, i=1,2…,86.

Case3.

$F(\alpha_1, \alpha_2 \dots, \alpha_n) = \sum_{i=1}^n (\alpha_i \ln k_i - \ln f_i)^2 + \sum_{j=1}^m \sum_{i=1}^n \left(\alpha_i \ln \tilde{k}_i^j - \ln \tilde{f}_j\right)^2 \to min$, where $\Delta\tilde{k}_i=0.20i$, i=1,2…,86.

Case4.

$F(\alpha_1, \alpha_2 \dots, \alpha_n) = \sum_{i=1}^{86} (\alpha_i \ln k_i - \ln f_i)^2 + \sum_{j=1}^{20} (\sum_i^{86} \alpha_i \ln(\tilde{k}_j + \Delta\tilde{k}_j) - \ln f_j) \to min$, where $\Delta\tilde{k}_i$=0.01i, 0.02i,...0.20i. i=1,2…,86.

Since some indices may affect positively and negatively to the general index, all optimization problems for all cases (1-4) are unconstrained convex optimization problems.

Results

Calculations are performed using the CVXPY(Convex Programming in Python) library, which contains specially designed methods for finding solutions to convex and static optimization problems. OSPQ (Operator Splitting Quadratic Programming) and ADMM (Alternating Direction Method of Multipliers) methods were applied, and a total of 50 iterations were completed.

Annex 1 lists the 86 indices along with their corresponding names and positive/negative elasticities.

Case	Case 1	Case 2	Case 3	Case 4	Case	Case 1	Case 2	Case 3	Case 4
Min.value	**1.55E-15**	**1.72E-15**	**1.11E-13**	**2.41E-09**	**Min.value**	**1.55E-15**	**1.72E-15**	**1.11E-13**	**2.41E-09**
α1	0.007	0.0105	0.0356	0.0056	α44	-0.0275	-0.0273	-0.0371	-0.0466
α2	0.011	0.0161	0.0338	0.0148	α45	0.0216	0.0213	0.0217	0.0353
α3	0.0198	0.0289	0.0277	0.0223	α46	0.0138	0.0119	0.0242	0.0336

α4	-0.0312	-0.0337	-0.0287	-0.0207	α47	0.0174	0.016	0.0231	0.0347
α5	0.0231	0.0302	0.0347	0.0303	α48	0.0227	0.0222	0.0209	0.0297
α6	0.0126	0.0176	0.0314	0.0241	α49	0.0243	0.0239	0.0199	0.0225
α7	0.0288	0.0392	0.0135	0.0511	α50	0.0073	0.004	0.0257	0.0263
α8	-0.0226	-0.0242	-0.0465	-0.0482	α51	0.0068	0.0041	0.0258	0.0256
α9	-0.0128	-0.0135	-0.0564	-0.0486	α52	0.0047	0.0042	0.0263	0.0229
α10	-0.0381	-0.0404	-0.0153	-0.0354	α53	0.01	0.0064	0.0248	0.0284
α11	-0.0113	-0.0118	-0.0566	-0.0555	α54	0.0124	0.0092	0.0241	0.0295
α12	0.0235	0.0311	0.023	0.0413	α55	0.014	0.011	0.0236	0.0295
α13	-0.0277	-0.0292	-0.0401	-0.065	α56	0.0206	0.0192	0.0084	0.0037
α14	0.0212	0.0275	0.0248	0.0358	α57	0.0211	0.0187	0.0215	0.0247
α15	0.006	0.0066	0.0309	0.0272	α58	0.0288	0.0274	0.016	0.0093
α16	0.0127	0.0156	0.0281	0.0182	α59	0.0088	0.0048	0.0248	0.026
α17	0.0132	0.0161	0.0276	0.0172	α60	0.005	0.0049	0.0257	0.0229

Conclusions

After 4 different simulations, the Case1 has the minimum and the best result. This leads to the conclusion that the perturbations should be put in different forms.

If we summarize the results of the first simulation:

- 17 sub-indices, such as Commute time and Job security, have elasticities that are negatively correlated with the general index (Annex1).

The sub-indices with the greatest positive impact appear to be Agriculture, Accessibility of Green Spaces and Public Amenities, Investments in The Health Sector, Investments in Low Greenhouse Gas Emission Energy Production, Economic Returns from Land Use and Integrated System for Reducing Potential Risks and Threats. A 1% sub-increase in these indices would lead to approximately a 0.03% increase in the general index [6].

On the other hand, the sub-indices with the strongest negative influence are the Consumer Price and Investments in Tourism and Cultural Events. A 1% increase in these sub-indices is estimated to reduce the general index by approximately 0.04%.
The least sensitive sub-indices are Accessibility of Food Supply and Accessibility of Education, with elasticities of approximately 0.005% [7].

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Annex 1

	General Index	0.433			
		The year 2023			The year 2023
	Stability condition	0.412		Accessibility	0.488
1	Average Life Expectancy	0.710	44	General Health Indicators	-0.358
2	Fertility Rate	0.587	45	Accessibility of Private Health Care Services	0.331
3	Migration	0.381	46	Accessibility of Public Health Care Services	0.511
4	Commute Time and Traffic Jams	-0.268	47	Accessibility of Public Transportation Services	0.429
5	Share of Renewable Energy in Total Consumption	0.031	48	Availability of Pedestrian and Bicycle Paths	0.295
6	Employment Rate	0.542	49	Accessibility of The Road Network	0.246
7	Agriculture	0.104	50	Accessibility of Local Road Connections	0.700
8	Job Security (as measured by the unemployment rate)	-0.435	51	Accessibility of Engineering Infrastructure	0.717
9	Poverty Level	-0.628	52	Accessibility of Food Supply	0.797
10	Consumer Price Index	-0.140	53	Accessibility of Commercial and Public Facilities	0.615
11	Income Disparity	-0.660	54	Accessibility of Consumer Goods and Services	0.547
12	GDP per Capita (at 2015 par prices)	0.270	55	Accessibility of Housing	0.505
13	Tax Burden	-0.355	56	Cultural Accessibility	0.020

14	Personal Physical Security Rating	0.344
15	IntegrityAssessment	0.745
16	Responsibility and Capacity of Civil Servants of The City (ethics and responsibility)	0.539
17	Education Level	0.526
18	Digital Transformation of The City	0.398
19	Legal Framework for The Policy Development	0.491
20	Capital Policy Development, Planning and Implementation (optimal, effective, impactful, transparent)	0.226
21	Risk Management Plan	0.280
	Quality of Environment	**0.490**
22	Quality of Public Health Care	0.498
23	Quality of Personal Health Care	0.602
24	Particulate Pollution/particles (PM2.5/ PM10)	-0.321
25	Quality of Water Supply	0.378
26	Air Quality Index	0.319
27	Quality of Public Transport Services	0.396
28	Quality of The Road Network	0.394
29	Power Supply Quality	0.787
30	Quality of Food Supply	0.733
31	Tax Rates	-0.330
32	Fulfillment of Basic Human Needs	0.680
33	Life Satisfaction	0.606
34	Quality of Green Space per Capita	0.207
35	Urban Cultural and Tourism Level /recreation/	0.600
36	Stress Level	-0.471
37	Sanitation Facilities and Their Quality	0.620
38	Level of Waste Management	0.483
39	Quality of Private Education	0.720
40	Quality of General Education	0.545
41	Equality (Gender Inequality)	0.237
42	Quality of Public Services	0.506
43	Disaster Risk	-0.348

57	Accessibility of Sports	0.350
58	Accessibility of Green Spaces and Public Amenities	0.125
59	Accessibility of Neighborhood Association	0.651
60	Accessibility of Education	0.786
61	Accessibility of General Education	0.628
62	Accessibility of Private Education	0.511
63	Accessibility of Public Services	0.492
64	Social Welfare	0.663
65	Location and Accessibility of Risk Reduction Facilities	0.453
	Competitiveness	**0.293**
66	Investments in The Health Sector	0.115
67	Average Salary by Sector	0.566
68	Investments in Low Greenhouse Gas Emission Energy Production	0.111
69	Investments in Infrastructure Sectors	-0.468
70	Utilization of Labor Resources	0.343
71	Trade Logistics	0.210
72	Economic Capability	0.339
73	Economic Returns from Land Use	0.130
74	Types of Investments	-0.280
75	Investments in Tourism and Cultural Events	-0.142
76	Production Share in Total Income	0.250
77	Investments in The Agricultural Sector	-0.300
78	Productivity and Efficiency	0.462
79	Living Conditions	0.244
80	Investments in The Education Sector	-0.194
81	Legal Environment in Finance	0.603
82	Planning Capability	0.275
83	Financial Capability	0.250
84	Investment Freedom	0.450
85	Financial Balance of Budgetary Investment	-0.297
86	Integrated System for Reducing Potential Risks and Threats	0.139