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Extending the Cobb–Douglas Production Function with Okun's Law: Empirical Evidence from Mongolia

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Abstract

This paper develops an integrated framework that extends the classical Cobb–Douglas production function by embedding Okun's Law to examine the role of unemployment in output determination. While the standard Cobb–Douglas model explains output through capital, labor, and technology, it abstracts from cyclical labor-market slack, which may be particularly relevant in small, export- dependent economies. We propose an augmented production specification, $Y = AK\alpha L\beta U-\gamma$, and estimate it using annual macroeconomic data for Mongolia over the period 2010–2024, drawn from the World Bank and national statistical sources. The empirical results indicate that labor input is the dominant determinant of output in long- run production relationships, while capital formation exhibits a positive but statistically insignificant effect. Incorporating unemployment into the level-based production function does not yield a sta- tistically significant impact on output once labor input is controlled for. Estimation of Okun's Law in first differences reveals a negative but small and statistically insignificant relationship between output growth and changes in unemployment, suggesting weak short-run labor-market transmission. Overall, the findings imply that Okun-type dynamics in Mongolia are substantially weaker than those observed in advanced economies, reflecting the dominant influence of external commodity cycles and gradual labor-market adjustment.

Keywords: Cobb–Douglas Production Function, Okun's Law, Unemployment, Economic Growth.

Introduction

The macroeconomic performance of small, resource-dependent economies is strongly shaped by their inte- gration into global commodity markets. Mongolia represents a salient example of such an economy, with growth dynamics heavily influenced by mineral exports particularly copper, coal, and gold and by fluc- tuations in external demand and international commodity prices [1,2]. While export revenues have supported long-term economic expansion, they have also amplified macroeconomic volatility by transmitting external shocks directly into domestic output and labor-market conditions.

Periods of favorable global commodity demand are typically associated with rapid output growth and rising employment, whereas downturns in international markets often lead to sharp output contractions and increases in unemployment. This cyclical pattern underscores the importance of understanding the interaction between production structures and labor-market dynamics in export-dependent economies. In Mongolia, labor-market adjustment tends to be gradual, reflecting public-sector employment buffers, the prevalence of informal labor arrangements, and a high degree of sectoral concentration in mining and related activities.

Traditional growth analysis is commonly conducted within the framework of the Cobb–Douglas pro- duction function, originally formalized by Cobb & Douglas (1928). By linking aggregate output to capital accumulation and labor input, the Cobb–Doug-

las model underpins much of modern growth theory and empirical macroeconomic analysis [3, 4]. Despite its analytical appeal, the standard Cobb–Douglas specification abstracts from cyclical labor- market slack, implicitly treating labor as a fully utilized input and thereby overlooking the potential role of unemployment in shaping observed output dynamics.

In contrast, Okun's Law, first documented by Okun (1962), establishes a systematic empirical rela- tionship between output fluctuations and changes in unemployment. A substantial body of empirical evidence confirms a negative association between output growth and unemployment increases, although the magnitude of the Okun coefficient varies considerably across countries and institutional settings [5-7]. These cross-country differences are particularly pronounced in economies char- acterized by export dependence, sectoral concentration, and limited labor-market flexibility—features that closely describe Mongolia's economic structure.

This paper seeks to bridge the gap between long-run production theory and short-run labor-market dynamics by embedding Okun's Law directly into the Cobb–Douglas production framework. Specifically, we propose an augmented production function of the form $Y = AK^{\alpha}L^{\beta}U^{-\gamma}$,

where Y denotes aggregate output, K capital input, L labor input, U the unemployment rate, and γ captures the elasticity of output with respect to unemployment. This formulation allows unemployment to affect output directly, providing a unified framework that incorporates both structural production relationships and cyclical labor-market conditions [8, 9].

The empirical contribution of this study is to estimate and evaluate this integrated framework using annual data for Mongolia over the period 2010–2024, drawn from the World Bank and national statistical sources. By jointly estimating the output elasticities of capital and labor alongside the unemployment effect, the analysis assesses whether Okun-type dynamics are present in a resource-dependent developing economy and whether their magnitude differs from those observed in advanced economies.

The remainder of the paper is organized as follows. Section 2 presents the theoretical background, re- viewing the Cobb–Douglas production function, Okun's Law, and the integrated production–unemployment framework developed in this study. Section 3 describes the data and empirical methodology and reports the regression results for the baseline production function, the unemployment-augmented specification, and Okun's Law in first differences. Section 4 concludes by summarizing the main findings and dis- cussing their implications for macroeconomic stabilization and labor-market policy in resource-dependent economies [10].

Theoretical Background

Cobb–Douglas Production Function

The Cobb–Douglas production function, originally formalized by Cobb & Douglas (1928), provides a foundational framework for analyzing the relationship between aggregate output and factor inputs. It is commonly specified as

$$Y = AK^{\alpha}L^{\beta}, \tag{2.1}$$

where Y denotes real aggregate output, K represents capital input, L denotes labor input, and A captures total factor productivity (TFP). The parameters α and β measure the output elasticities of capital and labor, respectively.

The Cobb–Douglas specification assumes a constant elasticity of substitution equal to unity, dimin- ishing marginal productivity of each input, and returns to scale determined by the sum α + β. When α + β = 1, the production function exhibits constant returns to scale; values greater (less) than unity imply increasing (decreasing) returns to scale. Owing to its analytical tractability and transparent eco- nomic interpretation, the Cobb–Douglas framework has been extensively employed in growth theory and empirical macroeconomic analysis [11].

Despite its widespread use, the standard Cobb–Douglas model abstracts from cyclical labor-market conditions. Labor input is typically proxied by employment or labor force measures, implicitly assuming full utilization of labor. Consequently, the framework does not explicitly account for unemployment or labor-market slack, which may play a significant role in shaping observed output dynamics—particularly in economies exposed to external shocks and commodity-driven volatility.

Okun's Law

Okun's Law, first documented by Okun (1962), describes a systematic empirical relationship between fluctuations in economic activity and changes in unemployment. In its growth-rate formulation, Okun's Law is commonly expressed as

$$\frac{\Delta Y}{Y} \approx -c\,\Delta u, \tag{2.2}$$

where u denotes the unemployment rate and c is the Okun coefficient measuring the responsiveness of output growth to changes in unemployment.

Early empirical evidence for the United States suggested that a one–percentage-point increase in the unemployment rate is associated with approximately a two–percentage-point decline in real output growth, a regularity often referred to as the "2% rule" [8]. Subsequent studies for advanced economies have broadly confirmed the existence of a strong negative relationship, with estimated Okun coefficients typically ranging between 1.5 and 3, depending on the sample period, econometric specifica- tion, and business-cycle conditions.

An alternative representation is the gap formulation,

$$Y - Y^{*} = -k(u - u^{*}), \tag{2.3}$$

where Y^{*} denotes potential output, u^{*} the natural rate of unemployment, and k the Okun coefficient in gap terms. In this specification, deviations of unemployment from its natural rate are associated with proportional output gaps. Empirical estimates of k are likewise found to be economically meaningful and statistically significant in many advanced economies.

Importantly, Okun's Law is generally interpreted as a short-run, cyclical relationship rather than a structural long-run equilibrium condition. Its empirical strength depends on labor-market flexibility, sectoral composition, and the nature of economic shocks. Economies with flexible labor markets and diversified production structures tend to exhibit stronger Okun relationships, whereas small, open, and resource-dependent economies

often display weaker or more asymmetric unemployment–output linkages. In such economies, output fluctuations are frequently driven by external demand and price movements, while labor-market adjustment occurs gradually and imperfectly.

An Integrated Production–Unemployment Framework

To integrate long-run production structures with short-run labor-market dynamics, this study extends the standard Cobb–Douglas production function by explicitly incorporating unemployment:

$$Y = AK^{\alpha}L^{\beta}U^{-\gamma}, \quad (2.4)$$

where Y denotes aggregate output, K capital input, L labor input, U the unemployment rate, and γ measures the elasticity of output with respect to unemployment. This formulation allows unemployment to affect output directly, capturing labor-market slack that is not fully reflected in aggregate labor input alone.

Taking natural logarithms yields the estimable log-linear specification:

$$\ln Y = \ln A + \alpha \ln K + \beta \ln L - \gamma \ln U + \varepsilon, \quad (2.5)$$

where ε is an error term. Within this framework, the parameter γ admits a clear structural interpre- tation: it measures the percentage reduction in output associated with a one-percent increase in the unemployment rate, thereby embedding Okun's Law within a production-function setting.

Theorem 2.1: Let the augmented production function be given by

$$Y = AK^{\alpha}L^{\beta}U^{-\gamma},$$

and suppose that it exhibits constant returns to scale with respect to capital and labor. Then the elasticity of output with respect to unemployment is equal to $-\gamma$.

Proof. Taking natural logarithms of the production function yields

$$\ln Y = \ln A + \alpha \ln K + \beta \ln L - \gamma \ln U.$$

Differentiating with respect to ln U gives

$$\frac{\partial \ln Y}{\partial \ln U} = -\gamma.$$

Thus, a one-percent increase in the unemployment rate is associated with an approximate γ percent reduction in aggregate output [12].

This result provides a structural interpretation of Okun's Law within a production-function frame- work. While unemployment may exert a limited influence on output in long-run levels once factor inputs are controlled for, its role becomes more pronounced in short-run cyclical adjustments. This distinction motivates the complementary empirical strategy adopted in this study, which combines level-based pro- duction regressions with first-difference specifications to capture both structural and cyclical dynamics.

Empirical Analysis

This section presents the empirical analysis for Mongolia using annual data spanning the period 2010– 2024. To examine the relationship between output, factor inputs, and unemployment, three comple- mentary econometric specifications are estimated. Model 1 evaluates a baseline Cobb–Douglas pro- duction function. Model 2 augments the production framework by explicitly incorporating unemploy- ment. Model 3 estimates Okun's Law in first differences in order to capture short-run cyclical dynamics.

Together, these models allow for a coherent assessment of both long-run structural relationships and short-run labor-market adjustments.

Data and Descriptive Statistics

The empirical analysis employs annual macroeconomic data obtained from the World Bank and the National Statistical Office of Mongolia. Real gross domestic product (GDP) and gross capital formation are measured in constant 2015 U.S. dollars. Labor input is proxied by the total labor force, while unemployment is measured as the official unemployment rate. All variables used in the production- function estimations are expressed in natural logarithms.

Table 1 reports descriptive statistics for the key variables. Over the sample period, real output exhibits moderate variability, reflecting Mongolia's exposure to external commodity cycles. Capital formation displays higher dispersion, consistent with episodic investment patterns associated with large- scale mining projects. Labor input evolves smoothly over time, while the unemployment rate shows notable fluctuations, particularly during periods of external economic stress.

Table 1: Descriptive Statistics

Variable	Mean	Std Dev	Min	Max
ln Y (GDP)	23.17	0.25	22.69	23.51
ln K (Capital)	22.29	0.36	21.75	22.79
ln L (Labor)	14.05	0.07	13.93	14.18
Unemployment rate (%)	5.91	1.49	3.91	9.02

Model 1: Baseline Cobb–Douglas Production Function

Model 1 estimates the standard Cobb–Douglas production function:

$$\ln Y_t = \ln A + \alpha \ln K_t + \beta \ln L_t + \varepsilon_t, \quad (3.1)$$

where Yt denotes real output, Kt capital input, and Lt labor input. The estimation results are reported in Table 2. The model exhibits a strong overall fit, with an R2 of 0.89 and a highly significant F-statistic, rejecting the null hypothesis of joint insignificance. Labor input displays a large and statistically significant elasticity, while the coefficient on capital is positive but not statistically different from zero at conventional significance levels.

Table 2: Cobb–Douglas Production Function

Variable	Coefficient	Std Error	t-statistic	p-value
Intercept	-15.161	3.941	-3.85	0.002
ln K	0.097	0.071	1.37	0.196
ln L	2.575	0.310	8.32	< 0.001
R2				0.892
Adjusted R2				0.874
Observations				15

The estimated output elasticity of labor substantially exceeds unity, indicating that variations in labor input account for a large share of observed output fluctuations over the sample period. This finding is consistent with the structural characteristics of the Mongolian economy, where production—particularly in mining and related service activities—is highly labor intensive and closely linked to employment dynamics.

By contrast, the weak statistical significance of capital formation likely reflects measurement lim- itations and investment dynamics rather than the absence of capital deepening effects. Gross capital formation is an imperfect proxy for the productive capital stock, and its impact on output may be ob- scured by investment volatility, gestation lags, and the episodic nature of large-scale mining projects. These features complicate the identification of short-run capital–output relationships in small, resource- dependent economies.

Model 2: Cobb–Douglas Production Function with Unemployment

Model 2 extends the baseline Cobb–Douglas specification by explicitly incorporating unemployment:

$$\ln Y_t = \ln A + \alpha \ln K_t + \beta \ln L_t - \gamma \ln U_t + \varepsilon_t, \quad (3.2)$$

where Ut denotes the unemployment rate.

The estimation results are reported in Table 3. Relative to Model 1, the inclusion of unemployment leads to a marginal increase in explanatory power, with the R2 rising to 0.90, although the adjusted R2 remains unchanged. Labor input continues to exhibit a large and statistically significant elasticity, while capital formation remains positive but statistically insignificant.

Table 3: Cobb–Douglas Production Function with Unemployment

Variable	Coefficient	Std Error	t-statistic	p-value
Intercept	-17.821	4.770	-3.74	0.003
ln K	0.065	0.078	0.83	0.423
ln L	2.792	0.379	7.37	< 0.001
ln U	-0.115	0.116	-0.99	0.343
R2				0.901
Adjusted R2				0.874
Observations				15

The estimated unemployment coefficient is negative, consistent with theoretical predictions, but sta- tistically insignificant at conventional levels. This finding indicates that unemployment does not exert a strong independent influence on long-run output once labor input is explicitly controlled for. The result reflects both the long-run nature of the production-function framework and the close mechanical relationship between labor force measures and unemployment.

In the Mongolian context, labor-market adjustment is characterized by gradual employment re- sponses, public-sector employment buffers, and widespread informal and underemployed labor. As a result, variations in measured unemployment capture only a limited share of effective labor-market slack, reducing its explanatory power in level-based production regressions. These considerations motivate the complementary analysis of Okun's Law in first differences, which is better suited to capturing short-run cyclical labor-market dynamics [13].

Model 3: Okun's Law in First Differences

To examine short-run cyclical labor-market dynamics, Model 3 estimates Okun's Law in first differences:

$$\Delta \ln Y_t = a - k \Delta U_t + \varepsilon_t, \quad (3.3)$$

where Δ ln Yt denotes real output growth and ΔUt the change in the unemployment rate. The estimation results are reported in Table 4. The estimated Okun coefficient is negative, as predicted by theory, but small in magnitude and statistically insignificant. A one–percentage-point increase in the unemployment rate is associated with an estimated reduction in output growth of approximately 0.3 percentage points.

Table 4: Okun's Law (First-Difference Specification)

Variable	Coefficient	Std Error	t-statistic	p-value
Intercept	5.843	1.399	4.18	0.001
ΔU	-0.305	0.846	-0.36	0.725

R2				0.011
Adjusted R2				-0.072
Observations				14

The very low explanatory power of the regression indicates that short-run output fluctuations in Mongolia are only weakly transmitted through the unemployment channel. This contrasts sharply with evidence from advanced economies, where Okun's Law typically exhibits both economic and statistical significance.

The result reflects structural characteristics of the Mongolian economy. Output volatility is driven primarily by external commodity price movements and export demand, while labor-market adjustment occurs gradually. In addition, unemployment rates do not fully capture cyclical labor-market slack in the presence of public employment buffers, informal employment, and underemployment. Consequently, measured unemployment responds only weakly to short-run output fluctuations, limiting the empirical relevance of Okun's Law in difference form.

Summary of Empirical Findings

The empirical results consistently indicate that labor input is the primary determinant of output in Mongolia's long-run production relationship. Capital formation exhibits a positive but statistically in- significant effect, while unemployment does not exert an independent influence on output levels once labor input is explicitly controlled for. This suggests that unemployment plays a limited role in shaping long-run production outcomes in a resource-dependent economy.

In contrast, the first-difference specification provides evidence of a negative association between out- put growth and changes in unemployment, consistent with Okun's Law. However, the estimated effect is economically small and statistically insignificant, indicating that short-run labor-market adjustments transmit only weakly into output fluctuations. Taken together, these findings highlight the importance of distinguishing between long-run structural production relationships and short-run cyclical dynamics when assessing labor-market effects on output in developing, export-oriented economies.

Conclusion

This paper examined the relationship between output, factor inputs, and unemployment in Mongolia by embedding Okun's Law within the Cobb–Douglas production framework. By integrating long-run production theory with short-run labor-market dynamics, the study provides a unified perspective on output determination in a small, resource-dependent economy.

The empirical results yield three main conclusions. First, labor input is the dominant determinant of long-run output, while capital formation exhibits a positive but statistically insignificant effect, reflecting Mongolia's reliance on labor-intensive mining and service activities and volatile investment cycles. Sec- ond, once labor input is controlled for, unemployment does not exert a statistically significant influence on output in levels, indicating a limited independent role in long-run production.

Third, estimates of Okun's Law in first differences reveal a negative but small and statistically insignificant relationship between output growth and changes in unemployment, suggesting weak short-run labor-market transmission.

Overall, the findings indicate that Okun-type dynamics in Mongolia are substantially weaker than those typically observed in advanced economies. Output fluctuations are driven primarily by external commodity price movements and export demand, while labor-market adjustment is gradual and incom- plete, implying that measured unemployment captures only a limited share of cyclical labor-market slack.

The contribution of this paper lies in providing a structural interpretation of the unemployment–output relationship within a production-function framework and offering new empirical evidence from a resource- dependent developing economy. From a policy perspective, the results suggest that employment stabilization alone is insufficient to smooth output volatility; greater emphasis should be placed on managing commodity cycles, promoting economic diversification, and strengthening productivity-enhancing invest- ment to support sustainable long-run growth.

Data Availability

The data supporting the findings of this study are publicly available from the World Bank and the National Statistical Office of Mongolia (1212.mn).

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