

Viscous Dissipation and Thermal Radiation Effects of Boundary Layer Flow of an Electrically Conducting Fluid over a Shrinking Sheet with Suction

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Abstract

A detailed scientific examination is made to reflect the influences of viscous dissipation and thermal radiation on the boundary layer flow of an electrically conducting Newtonian fluid over a linearly shrinking surface with mass suction parameter ($S > 0$). Using similarity transformation, the controlling non-linear partial differential equations are shifted into non-linear ordinary differential equations, and then they have been computed numerically by means of Runge-Kutta scheme along with shooting technique on Python software. The result's accuracy has been tested against newly conferred outcomes, and appreciable accord has been touched on obtainable parameters. The essences of remarkable parameters in velocity and temperature fields have been sketched completely. From this inquisition, it has been realized that the magnitude of velocity decreases with rising mass suction and magnetic interaction and also the temperature upsurges by Eckert number, heat source and thermal radiation parameter. The local skin friction and rate of heat transfer have been derived, computed and presented through tabular form for specified values and plots on graph. The flow of fluid over shrinking surface has a role in industries particularly to pack the product.

Keywords: Thermal Radiation, Mhd, Boundary Layer, Shrinking Sheet, Viscous Dissipation, Mass Suction

Nomenclature

B_0 Constant applied magnetic field
 C Shrinking constant(s^{-1})
 C_f Coefficient of local skin friction
 c_p Specific heat at constant pressure($Jkg^{-1}K^{-1}$)
 Ec Eckert number
 f Dimensionless stream function
 k Thermal conductivity($Wm^{-1}K^{-1}$)
 ke Mean absorption coefficient(m^{-1})
 M Magnetic parameter
 Pr Prandtl number
 Q_0 Volumetric rate of heat generation($JK^{-1}m^{-3}$)
 S Mass Suction parameter
 q_r Radiative heat flux(Wm^{-2})

T Temperature of the fluid(K) T_w Temperature at the wall
 T_∞ Free stream temperature
 u, v Velocity components of the fluid along the x and y directions, respectively.(ms^{-1})
 x, y Cartesian coordinates along the surface and Normal to it, respectively.(m)
 R Thermal Radiation parameter

Greek letters

ρ Density of the fluid(kgm^{-3})
 μ Dynamic Viscosity of the fluid($kgm^{-1}s^{-1}$)
 σ_e Electrical conductivity(Sm^{-1})
 η Dimensionless similarity variable
 ν Kinematic viscosity(m^2s^{-1})
 ψ Stream function

| | |
|------------|---|
| λ | Heat source/sink parameter |
| θ | Dimensionless temperature |
| σ_s | Stefan-Boltzmann constant ($Wm^{-2}K^{-4}$) |

Introduction

A kind of ideal substances in which they cannot sustain any shearing stresses during rigid body motions (including the state of rest) is known as fluid [1]. With the growing of importance in an incompressible fluid flow on stretching/shrinking sheet to many engineering problems, researchers centered more on it. Hence several scholars adhered towards the laminar fluid flows within boundary layer since many industries laying on such kind of fluid. Specifically, glass-fiber production, hot rolling, wire and plastic film drawing, aeronautics, biofluid dynamics and food processing are some of the applications [2].

Owing to a stretching boundary, the viscous flow in a fluid happens in expanding or contracting surfaces such as extrusion of sheet material from a dye and the elongation of pseudopods [3]. Crane started to consider time independent laminar boundary layer movement of a Newtonian fluid over a linearly stretching plane area first and the author obtained a similar solution with the closed analytical form [4]. Gupta and Gupta presented detailed examination of heat and mass relocation for the Newtonian boundary layer flow on a stretching surface with suction or blowing [5]. Wang explored the movement in three dimensional over the stretching flat surface [6]. For the significance impact of magnetic field, recently the study of magnetohydrodynamic (MHD) on heat and mass transfer causes special attention. The MHD effect of an incompressible electrically conducting fluid over a stretching surface first considered by Pavlov in the existence of magnetic field at right angle to the surface [7]. Jaber overlooked the impact of viscous dissipation and joule heating on MHD flow having non constant character of fluid over a stretching erect surface [8]. The interference of MHD in bio-nano-convective slip flow with Stefan blowing effects over a rotating disc analyzed by Fatema [9].

At the present time, the boundary layer flow through shrinking surface has been arouse an essential attention. In many industries, the most fashionable use of shrinking surface is shrink packing of products with different methods of shrinking. When goods of the product enclosed by plastic material, their shape and size checked depending upon the operation of heat, shrink and tight. The key role on achievement of this process is shrinking film which decrease the size of film when heat is applied. Finally, to end up the process, plastic film shrunk to envelop the products. Based on the application, there are several kinds of ploys. Shrink tunnel is one of the best mechanism in which warm air rounded by a hot chamber placed on the top of a conveyor. This conveyors hold the result through chamber. The main desire to get a good shrink tunnel is that fitting constant air temperature and conveyor speed through circulation of air symmetrically. This idea elaborated more by Muhaimin et al. and Vajravelu [10,11].

As Bachok et al. talked over, there are two situations probably happen, concerning the flow near shrinking surface [12]. The one is arresting complete suction on boundary which was examined by Miklavcic and Wang [3]. They verified existence and uniqueness of the problem in shrinking sheet case and studied the impacts on the numerical solution with suction (injec-

tion) rates. For the other condition, Wang had extended it by taking account a stagnation point flow. As a result, the velocity on shrinking surface is restricted on the boundary layer [13]. Thus, Bhattacharyya [14] reported the effect of slip on boundary layer stagnation point and heat transfer towards a shrinking sheet. Under this study, they explained cases where the solution of simplified problem exists, does not exist, unique or dual. With the absence of slip and adding thermal radiation, Bhattacharyya and Layek presented the effects of suction/blowing on steady boundary layer stagnation point flow and heat transfer towards a shrinking sheet [15]. They observed that the existence and uniqueness of boundary layer solution is affected by suction/blowing over permeable surface. By considering non Newtonian power-law fluid, Bhattacharyya examined boundary layer stagnation-point flow and heat transfer over a non-linearly shrinking/stretching sheet with thermal radiation by Lie symmetry analysis [16]. They investigated that for both stretching and shrinking surface, the power-law index diminish the velocity boundary layer thickness. In addition Prandtl number and radiation parameter decrease the fatness of thermal boundary layer. The heat and mass transfer analysis for boundary layer stagnation-point flow towards a heated permeable stretching surface with heat absorption/generation and suction/blowing reflected by Layek [17]. The solution of mass transfer on a shrinking surface has been deliberated by Fang and Fang [18-20]. Fang and Zhang also got a precise analytic solution of thermal boundary layer over a shrinking surface with mass transfer [21]. The impact of buoyancy on MHD stagnation point flow and heat transfer of a nano-fluid past a convectively heated stretching/shrinking surface discussed by Makinde [22]. When Wang cogitated the flow character of liquid film over an unsteady stretching surface, he explained the flow nearby shrinking area [23].

The created vorticity is impossible to be enclosed inside the boundary layer as investigated by Miklavcic and Wang [3]. Different researchers treated the impact of MHD on boundary layer flow on shrinking surface. Hayat presented an analytic solution of MHD flow of a second grade fluid over a shrinking area [24]. Merkin and Kumaran carefully considered time dependent MHD boundary layer flow on a shrinking surface [25]. MHD viscous flow and heat transfer caused by a penetrable shrinking area with prearranged heat flux was discussed by Ali [26]. Muhaimin flaunted consequence of heat and mass transfer on MHD boundary layer flow over a shrinking surface having suction [27]. Fang and Zhang secured a closed form analytical solution for time independent MHD flow on a penetrable shrinking plane subjected to mass suction [28].

More over a number of studies have been conducted on Newtonian fluid over shrinking surface with the various parameters and their impact. For instance, K. Bhattacharyya studied the boundary Layer flow and heat relocation on an exponentially shrinking sheet [29]. The impact of heat source/sink over MHD flow and heat transfer on a shrinking surface through mass suction observed by K. Bhattacharyya [30]. This problem extended by P. R. Babu perceiving the radiation influence on MHD heat and mass transfer flow on a shrinking surface through mass suction [31]. The MHD boundary layer flow passing over a shrinking surface in the presence of heat transfer and mass suction examined by Jhankal and Manoj Kumar [32]. Then, S. Balakrishna investigated about the viscous dissipation and heat relocation

in boundary layer flow of an electrically conducting Newtonian fluid on a shrinking surface through mass suction [33]. Viscous dissipation and heat transfer influences on MHD boundary layer flow passing over a wedge of nano-fluid surrounded in permeable media inspected by N. Amar [34]. H. Waqas attempted the MHD boundary layer flow of micropolar fluids owing to penetrable shrinking surface by viscous dissipation and radiation [35]. The effect of radiation and MHD on the heat relocation of uninterrupted stretching/shrinking surface by mass transportation of the flat boundary inspected by L.T. Benos [36]. M. Ijaz Khan and Faris Alzahrani elaborated the entropy-optimized dissipative flow of Carreau–Yasuda fluid with radiative heat flux and chemical reaction [37].

Motivated by all of the above works, the researchers aimed to scrutinize the impact of heat source, viscous dissipation and thermal radiation on the boundary layer flow of an electrically conducting Newtonian fluid over linearly shrinking surface with mass suction. And solved the obtained mathematical problem computationally, which has been coded on Python software. Hence, having greatest authors' information, the numerical explanation of this problem has not been reported yet. The velocity and temperature fields are sketched for different non-dimensional parameters and analyzed. In addition, the physical quantities which are important in engineering point of view, namely, local skin friction coefficient and heat transfer rate are computed, plotted and compared with earlier literature for trivial case.

Mathematical Formulation and Method of Solution

A viscous two dimensional electrically conducting steady Newtonian boundary layer flow passing on a permeable linearly shrinking surface on the occurrence of mass suction and thermal radiation is considered. Figure 1 shows the physical appearance of the problem Where.

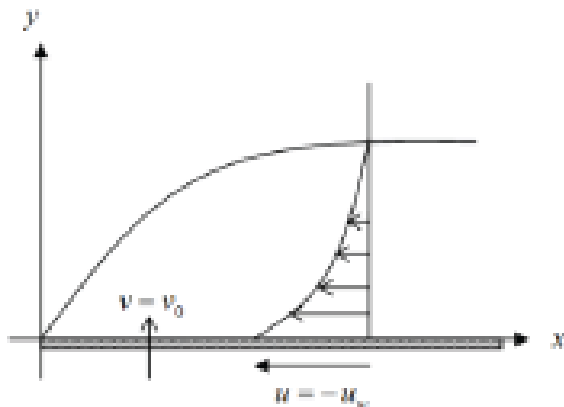


Figure 1: Physical configuration and Co-ordinate system.

x and y-axis are denoted along and perpendicular to the shrinking sheet, respectively and the applied magnetic interaction B_0 is at right angle to the x-axis. By taking small value of the fundamental dimensionless magnetic Reynolds number, the induced magnetic interaction could be disregarded as [33]. In addition the effects of viscous dissipation, heat source and thermal radiation included under energy equation. The fundamental governing equations of continuity, momentum and energy for assumed time independent incompressible laminar flow through the boundary layer approximations are reduced to the following

form as with the effect of thermal radiation [33].

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma_e B_0^2 u}{\rho} \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho c_p} (T - T_\infty) + \frac{\mu}{\rho c_p} \left(\frac{\partial u}{\partial y} \right)^2 - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} \quad (3)$$

Where u: velocity component along horizontal way, v: velocity component along vertical direction, ρ : the fluid density, σ_e : electrical conductivity of the fluid, μ : dynamic viscosity, ν : kinematic viscosity, T : fluid temperature, T_∞ : free stream temperature, k : fluid thermal conductivity, Q_0 : volumetric rate of the heat generation or absorption and c_p : specific heat at constant pressure.

By using the Rosseland approximation for radiation, the heat flux q_r is given by [38] as

$$q_r = -\frac{4\sigma_s}{3k_e} \frac{\partial T^4}{\partial y} \quad (4)$$

Where σ_s : Stefan-Boltzmann constant and k_e is mean absorption coefficient.

Applying Taylor series expansion of T^4 about T_∞ , the term T^4 can be expanded as follow.

$$T^4 = T_\infty^4 + 4(T - T_\infty)T_\infty^3 + 6(T - T_\infty)^2 T_\infty^2 + 4(T - T_\infty)^3 T_\infty + (T - T_\infty)^4 \quad (5)$$

Ignoring the higher order of Equation (5), we obtain

$T^4 = 4T_\infty^3 T - 3T_\infty^4$. Then, the radiation heat flux q_r becomes

$$q_r = -\frac{4\sigma_s}{3k_e} \frac{\partial T^4}{\partial y} = -\frac{4\sigma_s}{3k_e} \frac{\partial (4T_\infty^3 T - 3T_\infty^4)}{\partial y} = -\frac{16\sigma_s T_\infty^3}{3k_e} \frac{\partial T}{\partial y}$$

Thus, Equation (3) reduced in to the form

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho c_p} (T - T_\infty) + \frac{\mu}{\rho c_p} \left(\frac{\partial u}{\partial y} \right)^2 + \frac{16\sigma_s T_\infty^3}{3\rho c_p k_e} \frac{\partial^2 T}{\partial y^2} \quad (6)$$

The appropriate boundary conditions are given in the following equation for the velocity components and temperature.

$$u = -u_w = -Cx, \quad v = v_w, \quad T = T_w \quad \text{when } y \rightarrow 0 \\ \text{And } u \rightarrow 0, \quad T \rightarrow T_\infty \quad \text{as } y \rightarrow \infty \quad (7)$$

$C > 0$ and T_w denoted as shrinking constant and temperature of the surface wall respectively. By introducing stream function $\psi(x, y)$, the similarity transformation variables η and ψ expressed as [33]

$$\eta = y\left(\sqrt{\frac{C}{\nu}}\right), \psi = \sqrt{C\nu}xf(\eta), \theta(\eta) = \frac{T - T_{\infty}}{T_w - T_{\infty}} \quad (8)$$

Here η denoted the independent similarity variable. Therefore, the velocity components u , v and temperature T are given below in terms of similarity variable as:

$$u = \frac{\partial\psi}{\partial y} = Cxf'(\eta), \quad v = -\frac{\partial\psi}{\partial x} = -\sqrt{C\nu}f(\eta) \quad \text{and} \quad T = \theta(\eta)(T_w - T_{\infty}) + T_{\infty} \quad (9)$$

Equation of continuity (1) is possibly gratified by inserting u and v from Equation (9). But substituting the expression for u , v and T from Equation (9) into Equations (2) and (6), brings the transformed non-linear set of ordinary differential equations in the following form.

$$f'''' + ff'' - Mf' - (f')^2 = 0 \quad (10)$$

$$\{1 + R\} \theta'' + Pr \{f\theta' + \lambda \theta\} + Pr Ec (f')^2 = 0 \quad (11)$$

In this, differentiation with respect to η symbolized by prime. The initial and boundary circumstances in Equation (7) also reformed using the similarity transformation variables defined in Equation (8) as follows.

$$f(0) = S, \quad f'(0) = -1, \quad \theta(0) = 1, \quad f'(\eta \rightarrow \infty) \rightarrow 0, \quad \theta(\eta \rightarrow \infty) \rightarrow 0 \quad (12)$$

Where constant mass transfer parameter represented by S and for suction $S > 0$ and for injection $S < 0$.

The non-dimensional parameters obtained above defined as follows.

$$M = \frac{\sigma_e B_0^2}{\rho \nu} \text{ is Magnetic interaction parameter, } Pr = \frac{\mu c_p}{k} \text{ is Prandtl number,} \\ \lambda = \frac{Q_0}{\rho \nu c_p} \text{ is heat source parameter, } S = -\frac{v_w}{\sqrt{C\nu}} \text{ is mass suction parameter,} \\ Ec = \frac{(u_w)^2}{(T_w - T_{\infty})c_p} \text{ is Eckert number, and } R = \frac{16\sigma_s T_{\infty}^3}{3\alpha \rho c_p k_e} \text{ is thermal radiation parameter, } \alpha = \frac{k}{\rho c_p}$$

Interested physical non-dimensional quantities are coefficient of local skin friction (C_f) and Nusselt number (N_{ux} or local heat transfer rate) which are considered in this investigation. They are defined as follows [39], respectively.

$$C_f = \frac{\tau_w}{\rho u_{\infty}^2} \quad \text{and} \quad N_{ux} = \frac{\alpha q_w}{k(T_w - T_{\infty})}$$

We have also wall shear stress τ_w and local heat flux q_x defined as follows.

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0} \quad \text{and} \quad q_w = -k \left(\frac{\partial T}{\partial y} \right)_{y=0}$$

respectively, with μ become dynamic viscosity and k being thermal conductivity. By means of similarity variable on Equation (8), we can write $-f''(0)$ and $-\theta'(0)$ in terms of Reynold number (Re_x), coefficient of skin friction (C_f) and rate of heat transfer (N_{ux}) in the following way.

The hi $f''(0) = C_f \sqrt{Re_x}$ and $-\theta'(0) = \frac{N_{ux}}{\sqrt{Re_x}}$ and (11) are ini difficult

to solve analytically. Hence, these equations have been solved numerically. To apply numerical method, the higher order differential equations are simplified to system of first order differential equations. The resulting system of first order non-linear differential equations are solved by Runge-Kutta method with shooting scheme which is coded in the Python software based on system of symbols uploaded by Tamirat T. Dufera [40]. For the validation of these numerical outcome, we compared our results with reported by S. Balakrishna (2020) on table 1 and 2 for some cases.

Results and Discussion

The numerical answers show the heat source, viscous dissipation and thermal radiation influences over the boundary layer flow of an electrically conducting Newtonian fluid on a linearly shrinking surface in the presence of mass suction. Impact of different dimensionless parameters on the fluid velocity and temperature have been revealed through graphs plotted. Numerous computations have been carried out on the viable ranges of the coming up parameters, such as M , Pr , λ , S , Ec , and R involved in this study. The graphs considered along the y-axis are distribution of velocity ($f'(\eta)$) and temperature ($\theta(\eta)$), coefficient of skin friction ($-f''(0)$) and rate of heat transfer ($-\theta'(0)$) against the variable η , S and $\lambda > 0$ represented horizontally on the x-axis. Computed results have been plotted from Figure 2 to 14 which aids for identifying the effect of applicable parameters on velocity and temperature field, coefficient of skin friction and local rate of heat transfer.

Figure 2 and 3 explain the outcome of magnetic interaction parameter (M) on velocity and temperature profiles. Raising the value of M , decelerates the flow and the magnitude of $f'(\eta)$ decreases against M as noticed in Figure 2. Accordingly, thickness of the velocity boundary layer deescalate. This occurs because of the increment on resistive force, called Lorentz force which comes from the interaction between electromagnet fields and movement of an electrically conducting fluid. In another way, Figure 3 indicates that temperature $\theta(\eta)$ falls when the values of M increase. It is known that temperature measures the average kinetic energy and, further the fluid moves slowly as M step-up. The magnitude of velocity and there upon kinetic energy abate under the circumstances of intensifying M . This engenders dropping down dimensionless temperature profiles.

The impact of mass suction (S) over the velocity and temperature profiles depicted on graph 4 and 5. Figure 4 delineates that the effect of S decreases the magnitude of velocity profile and also the momentum boundary layer fatness reduces. Dimensionless temperature field is demonstrated in Figure 5 for several values of S . By increasing the value of wall mass suction S , non dimensional temperature $\theta(\eta)$ decreases. Consequently, the thermal boundary layer thickness reduces.

Impact of heat source ($\lambda > 0$) on temperature profile is exhibited on Figure 6. It is evident that the non dimensional temperature $\theta(\eta)$ rises directly with the heat source strength. As a result, thickness of thermal boundary layer augments when heat source being increased.

Figure 7 describes dimensionless temperature profile by taking different numbers of Prandtl Pr. The plot elucidates that enhancing the values of Pr, causes to decline the temperature profiles and the thermal boundary layer depth becomes smaller. This phenomena arise, since thermal diffusivity and energy transfer ability decrease, as Pr increases. The nondimensional number Pr defined as

$$Pr = \frac{\text{velocity boundary layer thickness}}{\text{thermal boundary layer thickness}}$$

Therefore, lower value in the fluid thermal conductivity implies higher value in Pr which leads to a decrement in temperature.

Mechanical energy rehabilitated into heat energy due to the relation between Eckert number E_c and thermal dissipation. The higher input of E_c raises the temperature $\theta(\eta)$ profile as exhibited in the Figure 8, Since Eckert number measures the kinetic energy of the flow relative to the entropy across thermal boundary layer. This is the reason for boosting the values of temperature by emphasizing the Eckert number.

Figure 9 illustrates, temperature profile $\theta(\eta)$ expands through increment of thermal radiation parameter R . This is due to the enhancement of radiative mode of heat transportation when the thermal radiation parameter increases. As conservation of momentum equation (10) is independent of θ for this problem, no effect of Pr, E_c , λ and R on the velocity field is inspected.

The results on coefficient of skin friction C_f and Nusslet number N_{ux} are presented from Figure 10 to 14 for different dimensionless physical parameters.

From Figure 10, coefficient of skin friction increases by uprising values of magnetic field interaction and mass suction parameters.

Figure 11 addresses the local heat transfer rate (N_{ux}) for dissimilar numbers of Eckert E_c against the continuous change of mass suction parameter S . Thus, the local heat transfer rate extends for large numbers of E_c . In another way, as the mass suction accumulates continuously, the rate of heat transfer (N_{ux}) diminishes. This occurs due to the reduction of temperature when mass suction roll up.

The local heat transfer rate (N_{ux}) for distinct numbers of Prandtl Pr against uninterrupted variation of mass suction S observed in Figure 12. Figure 12 shows that the Nusslt number (N_{ux}) decreases with increasing Pr. Moreover, the N_{ux} reduces as the mass suction increases continuously.

Figure 13 indicates the local non-dimensional rate of heat transfer $-\theta'(0)$ against heat source λ with different value of E_c . It appears that local heat transfer rate, accelerates with higher number of E_c and for the continuous increment of heat source.

The local dimensionless rate of heat transfer $-\theta'(0)$ against heat source $\lambda > 0$ with different value of Pr is portrayed in Figure 14. From this plot, the Nusslet number ($-\theta'(0)$) reduces when the input value of Pr is higher. However, it accelerates with the continuous increment of heat source.

Further, the approximated value for coefficient of skin friction (C_f) and Nusslet number (N_{ux}) are presented in Table 1 and 2

respectively. As a result, by fixing all parameters, Table 1 reveals that C_f escalate by rising magnetic interaction parameters M and mass suction parameter S . The negative and positive values of $\theta'(0)$ interpreted as heat transfer and absorption respectively. Hence, based on the Table 2, rate of heat transfer spreads by enlarging M , E_c and $\lambda > 0$, where as it declines by enhancing suction parameter and Pr. Moreover, the result of table 1 and 2 layout the similarity between present study and S. Balakrishna et al (2020) outcome. Thus, the numerical solutions of this problem have been validated.

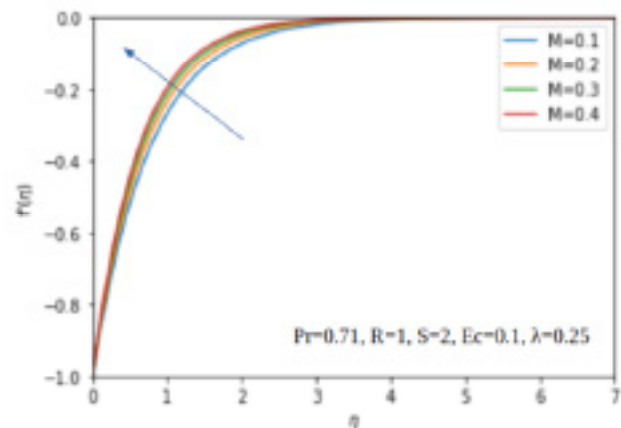


Figure 2: Impact of magnetic interaction parameter (M) on velocity field.

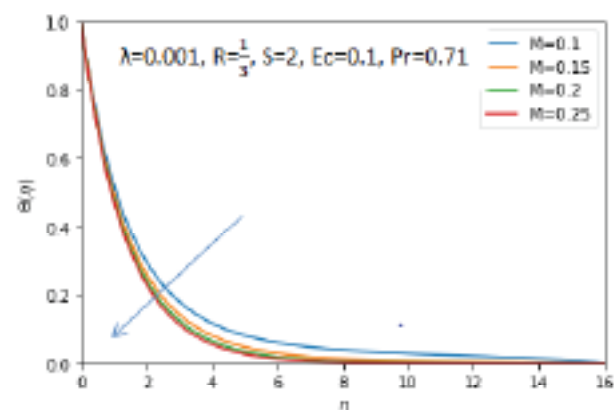


Figure 3: Impact of magnetic interaction parameter (M) on temperature field.

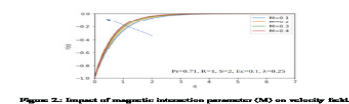


Figure 2: Impact of magnetic interaction parameter (M) on velocity field.

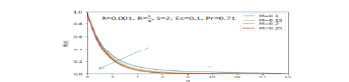


Figure 3: Impact of magnetic interaction parameter (M) on temperature field.

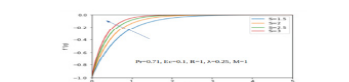


Figure 4: Impact of mass suction parameter (S) on velocity field.

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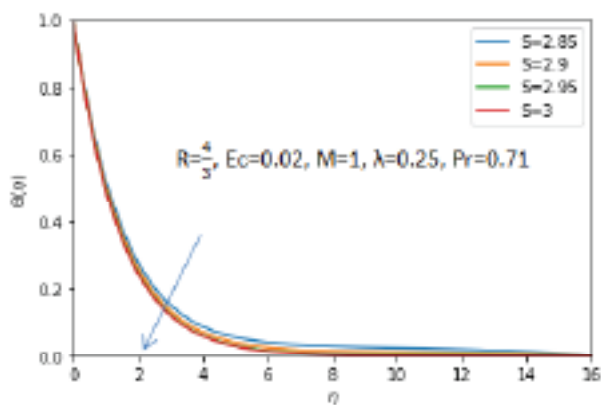


Figure 5: Impact of mass Suction parameter (S) on the temperature field.

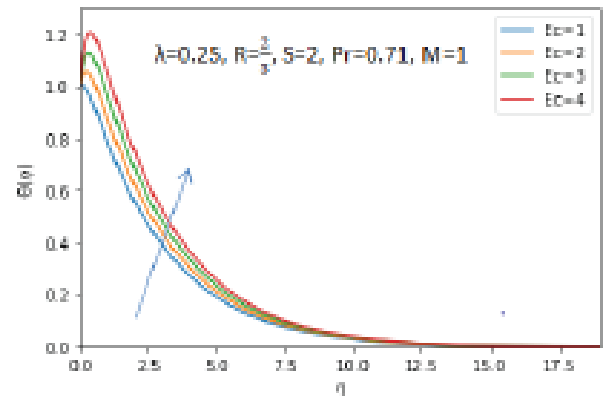


Figure 8: Impact of Eckert number (Ec) on the temperature field.

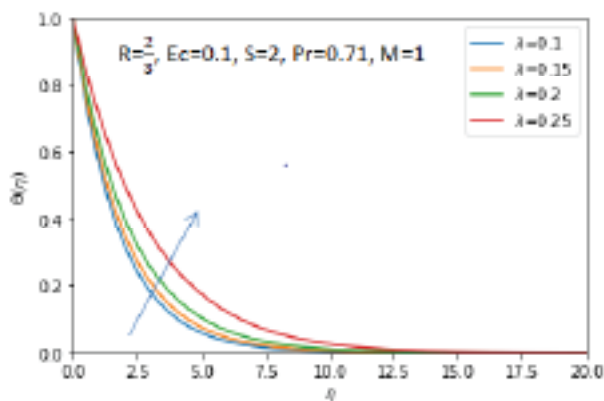


Figure 6: Impact of heat source parameter ($\lambda > 0$) on the temperature field.

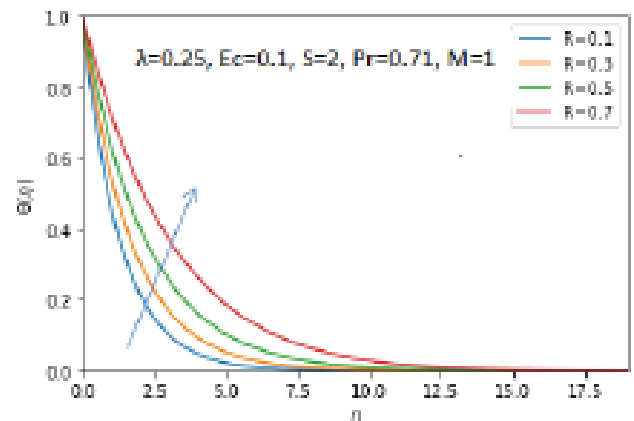


Figure 9: Impact of thermal radiation parameter (R) on the temperature field.

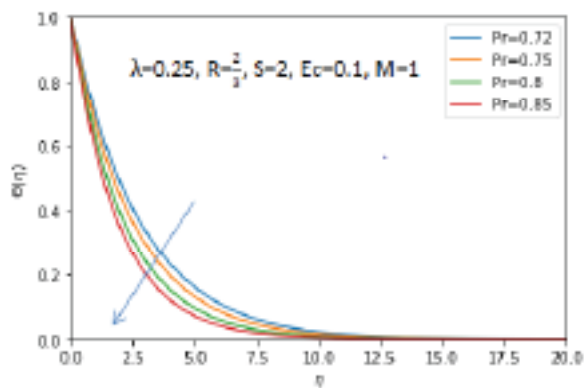


Figure 7: Impact of Prandtl number (Pr) on the temperature field.

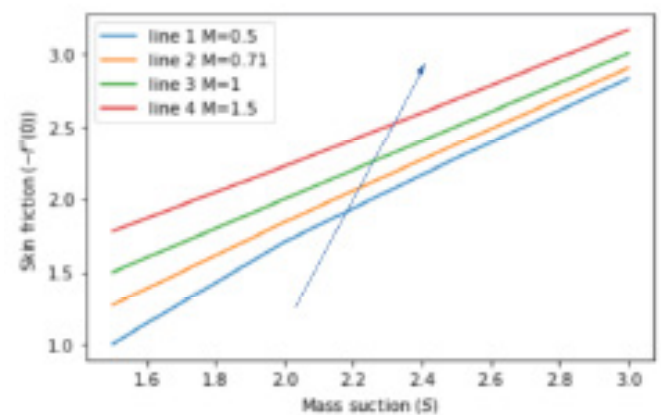


Figure 10: Impact of Magnetic interaction M and mass suction parameter S on skin friction coefficient.

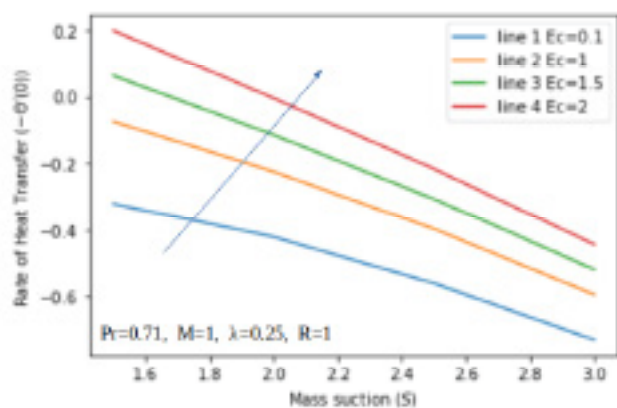


Figure 11: Impact of Eckert number Ec and mass suction S on the local heat transfer rate.

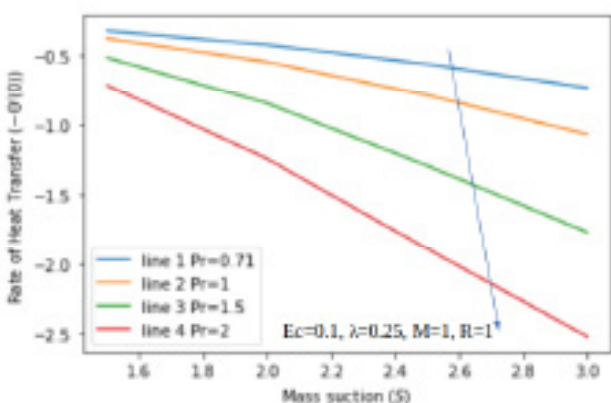


Figure 12: Impact of Prandtl number Pr and mass suction S on the local heat transfer rate.

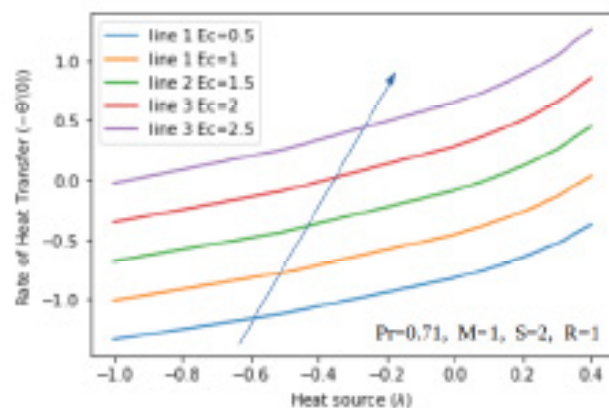


Figure 13: Impact of Eckert number Ec and heat source λ on the local heat transfer rate.

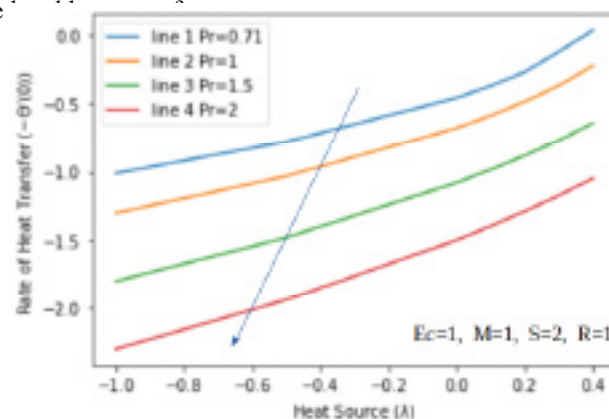


Figure 14: Impact of Prandtl number Pr and heat source λ on the local heat transfer rate.

Table 1: Skin friction ($-f''(0)$)

| M | Pr | λ | S | Ec | S. Balakrishna [25] $-f''(0)$ | Present results $-f''(0)$ |
|------|------|-----------|------|----|----------------------------------|------------------------------|
| 1 | 0.71 | 0.25 | 0.25 | 1 | 0.27222 | 0.272262 |
| | | | 0.5 | | 0.51181 | 0.511821 |
| | | | 1 | | 1.00335 | 1.003360 |
| | | | 1.25 | | 1.25170 | 1.251701 |
| | | | 1.5 | | 1.50081 | 1.50083 |
| 0.5 | 0.71 | 0.25 | 1.5 | 1 | 1.02417 | 1.02429 |
| | | | 1.75 | | 1.39472 | 1.39480 |
| | | | 2 | | 1.70843 | 1.70842 |
| 0.25 | 0.71 | 0.25 | 2 | 1 | 1.50531 | 1.50537 |
| 0.50 | | | | | 1.70842 | 1.708442 |
| 0.75 | | | | | 1.86642 | 1.86648 |
| 1 | | | | | 2.00183 | 2.00018 |
| 1.25 | | | | | 2.11811 | 2.118115 |
| 0.25 | 0.71 | 0.25 | 3 | 1 | 2.72492 | 2.72480 |
| 0.5 | | | | | 2.82289 | 2.822903 |
| 0.75 | | | | | 2.91426 | 2.91422 |
| 1 | | | | | 3.00007 | 3.000007 |

Table 2: Variation of Nusselt Numbe ($-\theta'(0)$)

| | | | | | S. Balakrishna [25] | Present results |
|------|------|-----------|---|----|---------------------|-----------------|
| M | Pr | λ | S | Ec | $-\theta'(0)$ | $-\theta'(0)$ |
| 0.5 | 0.71 | 0.25 | 2 | 1 | -0.33488 | -0.33554 |
| 0.75 | | | | | -0.29963 | -0.300017 |
| 1 | | | | | -0.26788 | -0.267959 |
| 0.5 | 7.0 | 0.25 | 2 | 1 | -6.44042 | -6.44844 |
| 1 | | | | | -5.48401 | -5.485484 |
| 0.5 | 0.71 | 0.25 | 2 | 1 | -0.33488 | -0.3355 |
| | | 0.5 | | | -0.02364 | -0.024244 |
| | | 1 | | | 2.07592 | 2.060975 |
| 0.5 | 0.71 | 0.25 | 2 | 1 | -0.33488 | -0.3355 |
| | | | | 4 | 1.66812 | 1.6670 |
| 0.5 | 0.71 | 0.25 | 3 | | -0.84401 | -0.844182 |
| | | | 4 | | -1.26364 | -1.26364 |

Conclusion

The effect of viscous dissipation and thermal radiation on boundary layer flow of an electrically conducting Newtonian fluid past linearly shrinking surface through mass suction have been reflected. In this, the non-dimensional physical quantities namely fluid velocity $f'(\eta)$, temperature $\theta(\eta)$, coefficient of skin friction, and local heat transfer rate have been computed numerically for different dimensionless parameters included in this study prototype by applying the approach of Runge-Kutta along with shooting scheme and coded on Python software. Outcomes of this analysis presented through summarized graphs and tables. The obtained results of skin friction coefficient and heat transfer rate values in tables 1 and 2 are very close to that of S. Balakrishna for a specific set of parameters to compare [33]. This confirms that our computational procedure (algorithm) works well. In general, the results obtained from this research are condensed as follows:

- The investigation reveals that due to increasing magnetic interaction (M) and mass suction (S) parameters, the velocity decreases in magnitude and consequently the momentum boundary layer thickness reduces.
- Temperature flow over the shrinking surface decreases by increasing the magnetic interaction and mass suction parameters.
- The non dimensional temperature profile and depth of thermal boundary layer becoming lower as Pr becomes higher.
- When physical parameters such as heat source ($\lambda > 0$), Eckert number (Ec) and thermal radiation (R) increase, the temperature profile also rises.
- The coefficient of skin friction ($f''(0)$) accumulates about 17.05% and 96% by doubling the number of M and S respectively.
- Heat transfer rate (Nusslet number or $-\theta'(0)$) is expanding with increasing M , Ec and $\lambda > 0$.
- Finally, from this study, the researchers noticed that the Nusselt number declines when the suction parameter (S) and Pr becoming progressively greater.

The presented article, encompassed by considering the parametric variation data to assist the findings for this investigation.

Conflicts of Interest

The writers announce that they have no discord concerning publication on the presented article.

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Data Availability

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