SCIENCE SET
OPEN ACCESS PUBLISHERS

Research Article

Science Set Journal of Physics

Dark Matter in Andromeda: A New Alternative Mechanism

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Submitted: 14 February 2025 **Accepted:** 21 February 2025 **Published:** 28 February 2025

Citation: *Danis, F. (2025). Dark matter in andromeda: A new alternative mechanism. Sci Set J of Physics,4(1), 01-07.*

Abstract
Mach's principle links inertia with surrounding masses. After looking at the history of Mach's principle, we will work out a new model of galaxies inspired by the application of Mach's principle. The new model has no ad hoc parameters: all inputs are from observation. Dark matter (DM) models have ad hoc parameters. The new model fits with observations better than current DM models, and could replace DM in Andromeda between 30 and 350kly as will be shown in section A.

With the equivalence principle, only the mass side has been studied with general relativity; the inertial side also gives a force. A previous theoretical paper showed that this force deforms space [1]. This paper is the first application of this force with the corresponding deformation/potential. It could explain the dynamic of Andromeda beyond 350kly as will be shown in section B. This force could also explain other puzzles in the universe.

In the last section, the first two parts will be linked to provide a new formulation of Mach's principle. A test of that new formulation, which can be carried out on Earth, will also be proposed.

Keywords: Inertial Frame of Reference, Mach's Principle, Equivalence Principle, Dark Matter, Dark Energy, Foucault Pendulum, Equatorial Bulge

Introduction

Despite our best efforts, Dark Matter (DM) still hasn't been observed on Earth [2, 3]. DM was first proposed in 1922 as an explanation of galaxies in a cluster which were rotating too fast and should theoretically fly away [4]. Luminiferous aether was still accepted as a possible theory until 1955 Shankland's paper [5]. So, the idea of an invisible substance was around when DM was used to explain the dynamics of a cluster. Has DM effectively blocked the study of other possibilities?

DM has subsequently been used in a galaxy to stop stars flying away as they were rotating too fast to stay within the galaxy [6]. MOND attempt to dispense with DM by MOdifying Newtonian Dynamics (inertia) when gravity is weak, such as at the edge of the visible part of a galaxy [7]. But MOND doesn't solve the puzzle of galaxies clusters.

Instead of modifying inertia as with MOND, section 2 of this paper will modify the inertial reference as suggested by Mach's principle. But in itself, this is not enough and a new force (speed-force) will be used to explain the dynamics of Andromeda beyond 350kly from the galactic centre as presented in section 3 [1]. Together these two ideas could explain most observations concerning Andromeda, without the need for DM.

Part A. Mach's Principle Versus Dark Matter

Nowadays, astrophysicists believe Mach's principle's effect is infinitesimal, so it is simply ignored.

A. a. History of Mach's Principle

Ernst Mach had a big influence on Einstein and he had a problem with the inertial reference. Absolute space as the inertial reference was justified by Newton's famous bucket. Behind his reasoning, Newton wanted to reach 'universality' (of his law of gravity); Newton's bucket experiment fitted very well.

Mach found Newton's absolute inertial reference troubling. An interesting question posed by Mach is this: "Newton's experiment with the rotating vessel of water simply informs us that the relative rotation of the bucket with respect to the sides of the vessel produces no noticeable centrifugal forces, but that such forces produced by its relative rotation with respect to the mass of the Earth and other celestial bodies. No one is competent to say how the experiment would turn out if the sides of the vessel increased in thickness and mass, until they were ultimately several leagues thick” [8]. Mach had a point; Newton’s argument is incomplete and not rigorous.

In 1916, Einstein wrote a 3-page-long article with a strong message about ignoring inertia, a “complacency [which] will appear incomprehensible to a later generation” . In parallel with that warning, in 1918, he wrote: “... in a consistent relativity theory there cannot be inertia relative to “space” but only inertia of masses relative to each other”. He did not expand on this point but it was the birth of Mach’s principle [9]. Later in life, he retracted his support of Mach's principle because he had linked it to general relativity, which aroused criticism.

There is an excellent review of Mach’s principle containing an important quote from James Isenberg: “…the distribution of matter and field energy-momentum everywhere at a particular moment in the Universe determines the inertial frame at each point in the Universe” [10]. That would be the definition of an inertial frame of reference used for this work.

A. b. Application of Mach's Principle to a Galaxy

What does it mean for a galaxy if “… the distribution of matter … determines the inertial reference…”? The stars are the matter and determine the inertial reference. As the stars are rotating, the inertial reference is rotating with the stars. As the inertial reference is rotating, individual stars have little or no relative speed compared to the inertial reference, so no centrifugal force, no reason for the stars to escape the galaxy and no need for DM.

One objective of this paper is to present a model of a galaxy where the inertial reference is rotating. The output of any model of a galaxy should fit two observations: the stellar density and the rotational curve.

A. c. Hubble's Formula and Stellar Density

At the time of Hubble, only the stellar density was available. From many pictures, Hubble had been able to deduce that the stellar distribution was respecting a law. At that time, that law was called Hubble's law. Nowadays, Hubble's law is about the expansion of the universe, so I will call it Hubble's formula. In the case of Andromeda, that formula fits the observation very well as will be seen with Fig.1.

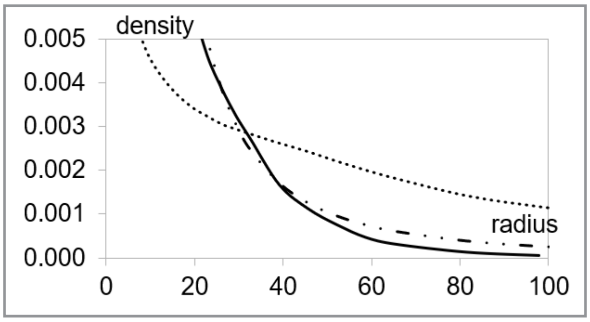

Figure 1: Stellar distribution as a function of the distance from the galactic centre in Andromeda. The density is in (106M□ arcsec-2), the radius is in (kly). The solid line and the dots close together are from Tamm et al and represent respectively the observed distribution of stars and the stel-lar matter output from their best DM model (Einasto’s model). The dash-dot line has been add-ed and is the result of Hubble’s formula [11].

Andromeda is a spiral galaxy. The observation of stellar density doesn't allow us to differentiate between inside the spiral and outside; so the observation at about 60kly is an average which could explain some of the difference with Hubble's formula.

Hubble's formula is about fluid mechanics, and it is an equilibrium between weight and pressure (or energy density) that gives an isothermal density distribution. As soon as the rotational speed of stars around the galaxy was observed, a centrifugal force was expected and the physics behind the formula lost validity, so it was abandoned. With Mach's principle, the formula makes sense and can be justified.

Please notice the poor results produced by the DM model.

A. d. Kepler's Law and Rotational Speed

To recover the observed rotational curve, we will use Kepler’s third law of planetary motion that I refer to as Kepler's law. That law is famous because it recovers the rotational speed of planets around the sun. With a galaxy, Keplerian decrease was expected but not observed; that is one reason for DM. Kepler's law is simply about the rotation of an object in space: rotation around the sun or rotation around the centre of a galaxy.

Kepler's law is using the mass of the sun (or of the galaxy) and the distance of the object rotating, and predicts the rotational speed of that object. Mach's principle suggests a local inertial reference so, instead of applying Kepler's law at a global level (taking the galaxy as a whole), Kepler's law is applied at a local level: from star to star. From a mass and a distance obtained from Hubble’s formula, the result of Kepler's law is a speed. In Kepler's model, Kepler's law is used to calculate a speed between two stars side by side along the radius of the galaxy.

A. e. Kepler's Model

From the stellar density obtained with Hubble's formula and considering a mass equal to the solar mass for all stars, we obtain the theoretical distance between two stars side by side along the radius of the galaxy, so we can apply Kepler's law. The result is a speed of one star compared to the other, which will be interpreted as the difference of rotational speed between those two stars. Moving along the radius by considering the next star etc. we can observe a change of the rotational speed as in Fig.2. Details are presented in appendix A.

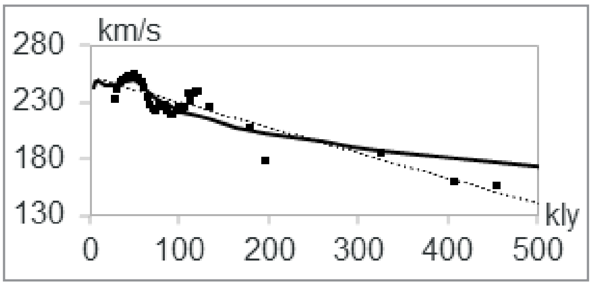

Figure 2: The rotational speed is plotted against the distance from the galactic centre. The dashed line is Kepler's model, the squares are the observed values from Tamm et al. [11]. The plain line is the output of the best DM model presented by Tamm et al.

Astronomers will assert that there are data up to 2,000kly and that Kepler's model doesn't accord with those data. That is correct. In fact, there is a change of mechanism at around 350kly, that we will address in the next section. The result is new and important: it means that with physics alone (Hubble's formula and Kepler's law) there is a model able to fit the observations without DM. The model is simply descriptive and works without Mach's principle. Mach's principle was just the inspiration.

A. f. A Case for Kepler's Model

I will here present some of the very strong arguments against DM models since, as a reason for the rejection of my submission, I have been told that many models do seem to work. DM models don't fit the stellar distribution: look at Fig.1. How can you reconcile the output of DM models with the observation? To improve DM models and get a fit with Fig.1 would mean that there is some kind of repulsion between DM and baryonic matter. Baryonic matter would be pushed towards the centre. This effect is not considered in DM models at the moment probably because it would require justification.

In addition, the distribution of DM is ad hoc to fit the rotational curve. Kepler's model has three parameters [7]. The first two are the initial distance from the centre and its corresponding rotational speed; both are from observation. A linear regression is calculated with the observation and any point on that linear regression provides the two inputs. The third input is the stellar density at the centre of the galaxy. That is an input that cannot be measured accurately enough and, at this point in the paper, is ad hoc. But once Fig.2 is obtained, that same input is the input to obtain Fig.1 and there is only one input for Fig.1. If the fit with Fig.1 were incorrect, it would have implied a mistake. This is what is missing for DM models, the fit with Fig.1.

In principle, the third input could be measured from observation as it would be dictated by the fit to the data of Fig.1. But the uncertainty on the third input obtained with Fig.1 would be too large; indeed Fig.2 is very sensitive to that input as explained in the appendix which is why it is adjusted to fit Fig.2 and is then used for Fig.1. It means that the third input is not ad hoc. The conclusion is that the result of this model is an improvement on DM models for two reasons: no ad hoc parameters are used, and the output is a better fit with Fig.1.

A. g. A case for Mach's Principle

Hubble's formula is reinstated only if there is no centrifugal force; something justified by Mach's principle. Using Kepler's law for motion in space as used in Kepler's model makes sense only if the inertial reference is local, as stated by Mach's principle. The full justification of the use of Kepler's law (for example why stars are not rotating around each other) requires extensive study not important for this paper; the study is in a book [12].

Mach's principle could also explain galaxy clusters: an inertial frame of reference rotating with the cluster. The Foucault pendulum was a strong objection to Mach's principle. But it is a gravitational instrument and not an inertial instrument, as addressed in the last section.

A. h. Discussion About Dark Matter

Another discussion should be about the existence of DM. Stars oscillate around the galactic plane. If DM exists, for a given thickness of a galaxy, the speed of stars perpendicular to the plane should be affected by the additional mass. Near the sun, without DM, that perpendicular speed should be around 15m/s; with 20% of DM the speed should be around 16m/s. Are the observations of speeds and thickness accurate enough to reach a conclusion?

For over 40 years, there have been continual experiments trying to detect DM. Surely a meta study could tell us whether DM is still a credible alternative. DM could have one magical property, a weak repulsion to baryonic matter as suggested to recover a fit of Fig.1. That repulsion would also exist on Earth and would be the reason why we can't detect it. That would make DM an elusive matter impossible to detect with the exceptional property of repulsion. If such a matter can exist, then DM remains a possibility.

Part B. Speed-Force Versus Cold Dark Matter

Section A suggested a different mechanism beyond 350kly. That speed deforms space with the Ehrenfest paradox gave Einstein the idea that space is deformed with gravity [13]. But with Ehrenfest's paradox, it is speed that deforms space, not mass. The rotational speed of a galaxy also deforms space. That deformation has been presented in what is called combined relativity [1].

Combined relativity and speed-force are new concepts that have not yet been validated by the scientific community. If correct, any moving object (like the stars of a galaxy or the disk of Ehrenfest's paradox) would deform space. To that deformation of space corresponds a potential (inertial potential) and therefore a force (speed-force). Those forces and potential will be applied to Andromeda in this section.

B. a. Applying Speed-Force to Andromeda

So, stars rotating around the galaxy would deform space. A deformation of space due to a mass is linked to a force (gravity). So, the deformation of space due to a speed can be linked to a force (speed-force). For an identical deformation of space (due to a change of speed or a mass: equivalence principle), the force should be identical. That is how speed-force in the case of Andromeda has been calculated in Table 1 (see appendix B for more details). Fig.3 corresponds to Table 1.

Table 1: Values of the speed-force. In the upper table, Table 1(a), the speeds are calculated from the linear regression model of the observed speeds from 28 kly up to 500 kly. In the lower table, Table 1(b), the linear regression model is from 198 kly to 1900 kly (see Figure 3). Here, "Δ length" is the length difference with the proper 1m rod from the special relativity effect of speed. The next columns are the gradient of length differences per metre ("Δ Δl/m"), then the gradient transformed into the speed-force ("Spd-force") and the gravitational force due to the galaxy ("Gravity").

Radius	speed	Δ length	Δ Δl/m	Spd-force	Gravity
150	218	2.64E-07			
			5.55E-29	5.43E-12	8.80E-12
200	207	2.38E-07			
			5.26E-29	5.15E-12	5.32E-12
250	196	2.13E-07			
			4.97E-29	4.87E-12	3.56E-12
300	185	1.90E-07			
			4.68E-29	4.58E-12	2.55E-12
350	174	1.67E-07			
300	180	1.79E-07			
			1.21E-29	1.19E-12	2.55E-12
350	177	1.73E-07			
			1.19E-29	1.17E-12	1.92E-12
400	174	1.68E-07			
			1.16E-29	1.14E-12	1.33E-12
500	168	1.57E-07			
			1.04E-29	1.02E-12	4.79E-13
1000	139	1.07E-07			
			7.49E-30	7.34E-13	1.20E-13
2000	81	3.68E-08			
Units: kly	km/s	m/m	m/m /m	N/kg	N/kg

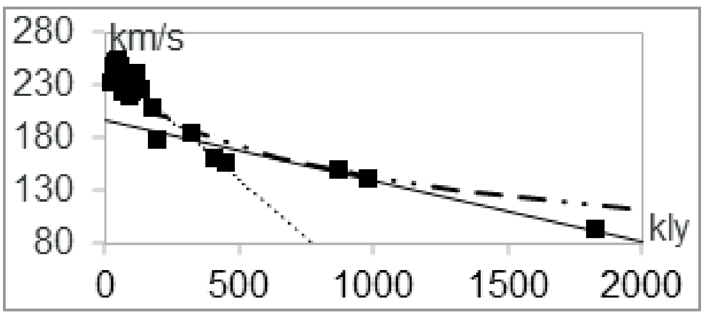

Figure 3: Rotational curve of Andromeda up to 2000 kly. The dash-dotted line represents the output of the DM model. The dashed line is the outcome of Kepler's model. The solid line denotes a linear regression model of the observed data from 196 kly onwards. Data are from Tamm et al. [11].

Table 1a, speed-force equals gravity at around 250kly; in table 1b, equality is reached around 500ly. The two straight lines of Fig.3 cross at 335kly which denotes the change of mechanism. As it all happens in the same area, the exchange of the major force is probably a factor that explains the change of mechanism; but there is no rigorous demonstration behind this idea at the moment.

B. b. Inertial Potential

With speed-force there is a potential called inertial potential. The graph of speed-force with the distance seems linear from 500 to 2000kly, so it is assumed that the speed-force is zero at 3391kly. With this assumption, the potential can be calculated.

As with gravitational potential, a change of inertial potential corresponds to a change of energy. As energy cannot be lost, the change of inertial potential energy is transformed into kinetic energy while moving closer to the galactic centre as presented in Table 2. That additional inertial potential explains the rotational speed of a galaxy without the need for DM.

Table 2: Values for the potential and kinetic energies. Distance ("dist.") from the galactic centre, then the average of the observed rotational speed ("speed"), and the gravitational potential ("grav pot") and Inertial-potential ("Inert pot"). The kinetic energy ("kin en") indicates that when the potential decreases, the kinetic energy increases, and "pot/en" is the ratio of the sum of potentials divided by the negative of the kinetic energy.

dist.	speed	grav pot	Inert pot	kin en	pot/en
2000	81	1.27E+09	3.58E+09	3.28E+09	1.47872
500	168	5.08E+09	1.54E+10	1.41E+10	1.45075
300	185	8.46E+09	1.92E+10	1.71E+10	1.61653
200	207	1.27E+10	2.31E+10	2.14E+10	1.66979
100	229	2.54E+10	2.58E+10	2.62E+10	1.95304
50	240	5.08E+10	2.86E+10	2.88E+10	2.75549
Units: kly	km/s	-J/kg	-J/kg	J/kg	

If matter that forms the galaxy is coming from faraway, the factor (pot/en) should be 1. That factor of 1.5 of the first two lines requires an explanation. One possibility is that matter joins the galaxy from above and below the disk, instead of from the far-end of the galactic plane. But this is an open question.

Then, for distances below 500kly, the factor changes because the mechanism changes, Kepler mechanism of section 2 is dictating the rotational speed and explains the continuous change of the factor.

B. c. A Case for Speed-Force

This inertial potential generates a force which attracts matter towards the centre of the galaxy and should accelerate the formation of galaxy. Cold Dark Matter (CDM) was supposed to generate the same acceleration. Speed-force exists as soon as there is a difference of speed. Integrated into models, that force may well accelerate the formation of galaxies as CDM. There is a similar question about the nucleosynthesis (the ratio hydrogen – deuterium). Models predict more galaxies than observed in galaxy clusters; the difference could also be explained with that force. Speed-force could also explain dark energy.

Another potential of this potential/force is a solution to Ehrenfest's paradox, but in its present state, it is not rigorous enough for a scientific paper [12].

Part C. Combining Sections, A and B.

The two sections are currently incompatible. Because the inertial reference is rotating, there is no kinetic energy in the section A. A change of potential is transformed into kinetic energy in the section B.

C. a. The Equivalence Principle

The incompatibility of the two sections comes with the equivalence principle because with it, there is the automatic assumption that inertial reference equals gravitational reference. The difference between the references existed before Einstein's 1907 paper on the equivalence principle. Wenzel Hofmann had the idea of a difference between gravitational kinetic energy and inertial kinetic energy; such a difference reconciles the two parts [14].

For emphasis, a new formulation of the equivalence principle is presented:

(i) inertial mass = gravitational mass, (ii) inertia uses an inertial reference and gravity utilises Newton's absolute space as the reference (also known as the canonical frame), and (iii) the inertial reference is described in Isenberg's sentence [10]: "…the distribution of matter and field energy-momentum everywhere at a particular moment in the Universe determines the inertial frame at each point in the Universe".

C. b. Application of the New Mach's Principle

The new formulation would mean that the surface of the Earth is the inertial frame of reference. Gravity is the reason for the oscillation of a pendulum; so, the Foucault's pendulum is a gravitational instrument and cannot describe the inertial frame of reference. A gyrocompass could be explained with the inertial frame but Coriolis force (which is also an inertial force) will confuse the explanation. The bulge of the Earth would not be explained by the centrifugal force but by the speed-force. At the equator, the centrifugal force of 3.42 10-2 N/kg would be replaced by the speed-force of 4.35 10-2 N/kg.

Tycho Brahe was a contemporary of Kepler and didn't want to believe that the Earth was rotating on itself, because objects would not fall vertically. He has a point: if the inertial frame of reference is Newton's absolute reference, then an object dropped from a tower would leave the tower with a rotational speed higher than the ground speed, so should not fall vertically. If the inertial frame of reference is rotating with the surface of the Earth, the fall should be vertical.

There is the ZARM drop tower of 122m at the University of Bremen, Germany. The idea above can be tested: a plumb line is exactly vertical and would serve as reference. This test would show if the inertial reference is the surface of the Earth; and because of the equatorial bulge, it would also show whether speed-force is correct or not. If Mach's principle is correct, then Kepler's model has its explanation. Friedlaender's experiment is another possible test; it is admired in and probably deserves to be repeated. The Friedlaender brothers described the surface of the Earth as the inertial reference [10-15].

Conclusion

This work is a call to the scientific community to explain why Kepler's model works and to reconsider Mach's principle. It is a call to consider speed-force/inertial potential as an explanation for some DM and for dark energy. It is a work in progress, in need of validation by further studies and experiments. As predicted by Kuhn, I must expect arguments against this work, but I also hope that some scientists will help defend it [16].

This paper is mostly based on one single observation: Andromeda. More cases need to be studied. A few questions have been presented with each section but there will be more. There is much to digest, and deeper questions will probably be raised.

Acknowledgements

- Professor G. Gilmore told me the story of Hubble's formula.
- Professor Ø. Grøn helped me with the interpretation of "my" solution to Ehrenfest's paradox which was essential in developing combined relativity.
- Professor E. Vishniac helped me understand why astrophysicists might find my ideas difficult to accept.

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Appendices

A: The Maths of Kepler's Model

First, the mass of the sun is assumed to be the mass of all stars mainly because the mass of the sun is a unit (M).The star density ρr (in M ly-2) at radius r is calculated by Hubble's formula (1); the first input ρ0 (adjusted for Fig.2 to 4.34×109 M ly-2) is the density at the galactic centre, and r is the distance from the centre of the galaxy (in ly).

$\rho r = \rho 0/(1+r^2)$ (1)

An expression of Hubble's formula found in ref [17] has been simplified in (1) which is directly used to obtain Fig.1.Andromeda's thickness is assumed to be equal to the thickness of the Milky Way (1000 ly). The factor of 1000 in formula (2) is simply the assumed thickness of Andromeda, and used this way, the density is back to a mass per volume. As we need the distance between two stars, its cubic root is taken.

$r = (1000/\rho r)^{1/3}$ (2)

We saw that the value of ρ0 in (1) is adjusted. Because of how formulae (1) and (2) are written, an eventual thickness error on the

1000 factor would be automatically corrected by adjusting ρ0. So, it is not simply ρ0 which is adjusted but the thickness is adjusted as well. It is a little trick but now you can forget it and consider that only ρ0 is adjusted. The two other inputs for the model, the first r0 (distance from the centre of the galaxy: 10kly) and v0 (rotational speed at r0: 249km/s) are introduced at this point. Those two inputs come from the linear regression of the observed data. Here is the iteration to move from r0 to r1, etc:
rn+1 = rn+δr. (3)

Kepler’s speed vk between a star at rn and a star at rn+δr is calculated using Kepler's law (4); G is the gravitational constant, M is the solar mass, and δr is calculated from ρr at r=rn in formula (2).
$vk = (MG/\delta r)^{1/2}$ (4)

The core of Kepler's model is that the difference in speed between stars is Kepler’s speed, vk. We need vncte, the exact speed of the star sn+1 if there was no difference in speed (therefore the subscript “cte”). If there is no difference in angular speed between sn and sn+1, the linear speed of sn+1 would be vncte which is different to vn because sn+1 is slightly further from the centre and so turns slightly faster.
vncte = vn (rn+δr)/rn (5)

Therefore, vk should be subtracted to the speed vncte. The vncte is just for calculation, and does not correspond to an observation; it is just geometry.The final result vn+1 is the rotational speed in linear value of the stars sn+1, and is calculated with formula (6):
vn+1 = vncte - vk (6)

Then, vn+1 becomes the new vn, and rn+1 becomes the new rn, and we restart from formula (1).

The number of iterations required is large (>10,000). A small error on ρ0 will be added 10,000 times. That is why ρ0 is a very sensitive parameter. As the result is a straight line, it means there is a simple law: the speed at any rn is the linear value (at rn) of the angular speed of the first star (r0) minus the sum of all vk from r0 to rn. It is more complex to mathematically find the straight line because δr is continuously changing but that is the law.

The addition of vncte step seems superfluous but without it, the result is not a straight line. One additional observation: For Kepler's model, the density of stars is needed. Instead of doing interpolation between observed density of stars, it is easier to use Hubble's formula. If the scientific community subsequently were to establish that Mach's principle shouldn't be applied to a galaxy, Hubble's formula wouldn't make sense on physics ground but it fits the observation as on Fig.1. So, Hubble's formula is an easy way to calculate the density of stars. It is used only because its fits; not because it is justified by Mach's principle.

B: Calculation of Speed-Force.
If combined relativity is correct, special relativity is invalid in our macro world, there would be no time-flow dilation due to speed, only length contraction; so, we are interested only in length contraction. The middle term of Schwarzschild solution is showing that a mass generates a length contraction. On Earth, a one-meter rod horizontal will seem longer at some height compared to ground level. Spacetime deformation with GR is given by Schwarzschild solution:
$s^2=(1-rs/r)-1(c\Delta t)^2 - (1-rs/r)(\Delta r)^2 – r^2\Delta\Omega^2$ (7)

rs being Schwarzschild radius: $rs = 2GM/c^2$ (8)

There is a symmetry between length and time-flow with general relativity as with special relativity; (1-rs/r)-1 for time, (1-rs/r) for length. From the middle term of Schwarzschild solution, we can deduce the difference of the length of the rod:

$(l_h-l_{ground}) = gh/c^2\ l_{ground}$ (9)

c is the speed of light.
g is 9.8m/s2 on Earth
h is a height above ground level.
lground is the length of the rod at ground level
lh is the length of the rod at h above ground level.
Equation (9) is the symmetric to the equation given by Rovelli about the change of time-flow with gravity $(t_{ground} - t_{table}) = gh/c^2\ t_{ground}$ [18].

Applying equation (9) at the surface of the Earth, for 1m difference in height (h) the difference of length of the 1m rod is: 1.1 10-16m/m. It is too small to be observed with our current technology but that is the prediction of general relativity. That deformation of space corresponds to a force of 9.8N/kg. So: 1.1 10-16m/m/m ↔ 9.8N/kg. “m” are all metres but m is for the difference in height, m is for the length of the rod and m is for the difference of the length of the rod.

The length contraction of combined relativity is given by γ-1; γ being the Lorentz factor: γ = 1/√(1-v2/c2). This length contraction (in m/m) will change along the radial dimension of the galaxy as the rotational speed will change, giving a gradient in m/m/m. By a simple proportional calculation (1.1 10-16m/m/m ↔ 9.8N/kg) we can know the speed-force of combined relativity along the radius of the galaxy as in Table 1.