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The Coupling of Shear Stress and Velocity Profile in Turbulent Flow in a Pipe

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Abstract

The article presents a new solution to the velocity profile of the flow along a pipe. The solution depends only on the Reynolds number, which is based on the average velocity and the radius of the pipe $\left(\frac{U_m \cdot r}{\nu}\right)$. *The solution also considers the laminar sublayer near the wall (although it ignores the buffer layer) and exhibits very good compatibility with experimental data. Also, the first derivative of the velocity along the axis is zero. The solution was compared to the raw experimental velocity profile and showed a very small deviation (see Appendix B).*

Keywords: Big Data Analytics, Data Mining, Predictive Modeling, Business Intelligence, Digital Transformation, Decision-Making, Healthcare Analytics, Data Governance, Machine Learning, Data-Driven Strategy.

Introduction

Being of one-dimensional velocity, the flow in a pipe gives a very convenient tool to understand the turbulence phenomena near walls. Bussinesq [1] was the first one to suggest the eddy viscosity theory. Nikuradse [2] found the velocity profile equation $\frac{u}{U} = \left(\frac{y}{r}\right)^{1/n}$ and the dependence of n according to the Reynolds number in a very wide range. Prandlt [3] published the dependence of the shear stress on the wall equations $\tau_w = \frac{\epsilon}{8} \cdot \rho \cdot {U_m}^2$. The equation for ϵ that known as Prandtl's universal law of friction for smooth pipe, has been verified by Nikuradse experimentally.

Modeling of the Framework

Investigation of the flow profile in a pipe has shown that it can be divided into 3 principal zones. The sublayer laminar inner zone is close to the surface, and the flow is laminar in this zone. At the edge of the laminar sublayer, turbulence starts to affect the flow and is a mixture of both laminar and turbulent. The buffer sublayer is followed by the outer zone, which is completely turbulent.

In the present work, the very thin buffer sublayer is ignored, and the flow is assumed to be converted directly from laminar to turbulent.

Experimental Data

The Turbulence Zone

H. Schlichting [4] summarized the turbulent flow in a circular pipe, a topic that has been extensively explored by many. The empirical results for this case are:

(1) $$\frac{u}{U} = \left(\frac{y}{r}\right)^{\frac{1}{n}}$$

(2) $$\frac{U_m}{U} = \frac{n^2}{(n+1)\cdot(n+0.5)}$$

(3) $$\tau_w = \frac{\epsilon}{8} \cdot \rho \cdot {U_m}^2$$

(4) $$\frac{1}{\sqrt{\epsilon}} = 2 \cdot \log\left(\frac{U_m\, r}{\nu} \cdot \sqrt{\epsilon}\right) - 0.2$$

where r is the radius of the pipe, U_m is the average velocity (flow rate per pipe area), U is the velocity in the axis of the pipe, n is the exponent depending on $\frac{U_m r}{\nu}$, τ_w is the shear stress on the surface, and ϵ is the resistance coefficient.

Table 2.1 presents the relation of n to $\frac{U_m r}{\nu}$ and the conversion of dependence of the shear stress to $\frac{U r}{\nu}$.

Equations (1) to (4) give the experimental shear stress for some cases.

Setting $\frac{U \cdot r}{\nu} = R$, the best approximation is

(5) $\frac{1}{U}\sqrt{\frac{\tau_w}{\rho}} = \frac{0.432}{\ln(R)}$

A comparison between the experimental shear stress to that obtained by equation (5) is provided in Table 1.

Table 1: Shear stress in eq. (5) vs. experimental data for various $\frac{U_m r}{\nu}$ values.

$\frac{U_m r}{\nu}$	$2\cdot10^3$	$1.15\cdot10^4$	$5.5\cdot10^4$	$5.5\cdot10^5$	$1\cdot10^6$
n	6	6.6	7	8.8	10
$\frac{U r}{\nu}$	$2.53\cdot10^3$	$1.43\cdot10^4$	$6.73\cdot10^4$	$6.47\cdot10^5$	$1.16\cdot10^6$
$\frac{1}{U}\sqrt{\frac{\tau_w}{\rho}}$ data	0.056	0.045	0.038	0.032	0.031
$\frac{1}{U}\sqrt{\frac{\tau_w}{\rho}}$ eq.(5)	0.055	0.045	0.039	0.032	0.031

To make the calculation later on easier, the ln(R) will be converted to the form

(6) $\ln(R) = \sqrt{\frac{1}{A}}\cdot R^{0.5\cdot\alpha}$

To make it valid across the range, it has to be

(7) $ln(k\cdot R) = \sqrt{\frac{1}{A}}\cdot(k\cdot R)^{0.5\cdot\alpha}$

Where k is any number in the range $1 \geq k > \frac{1}{R}$
Equations (6) and (7) yield

(8) $\alpha = \frac{2}{\ln(R)}$

And

(9) $A = \frac{\varepsilon}{8}\cdot\left(\frac{U_m}{U}\right)^2\cdot R^{\alpha}$

This form provides an additional advantage: the shear stress is exact.

The laminar sublayer
Ansys [5] provides a very good summary of the laminar sublayer and the turbulent region near the wall.

The laminar flow near the wall is

(10) u+=y+

Where u+= $\frac{u}{\sqrt{\frac{\tau_w}{\rho}}}$ and y+= $\sqrt{\frac{\tau_w}{\rho}}\cdot\frac{y}{\nu}$
And the turbulent flow is

(11) $u^+ = 2.5\cdot ln(y^+) + 5.45$

The converted point from laminar to turbulent is calculated by comparing the laminar velocity to the turbulent velocity.

(12) $y^+_e = 11.57$

Or, in regular terms

(13) $\frac{y_e}{r} = \frac{11.57}{\sqrt{A}\cdot R^{1-0.5\alpha}}$

Derivation of the equations of the flow in the turbulence zone
The shear stress in turbulent flow in a pipe is given by

(14) $\frac{\tau_w}{\rho} = A\cdot R^{-\alpha}\cdot U^2$

Under the assumption that there is a general equation for the shearing stress of turbulent flow over smooth surfaces, we can define:

$$\nu\frac{\partial u}{\partial y} = \frac{\tau_t}{\rho}\cdot\frac{\partial^2 Q}{\partial y^2} = \frac{\tau_w}{\rho}\cdot\left(1-\frac{y}{r}\right)\cdot\frac{\partial^2 Q}{\partial y^2} = A\cdot\left(\frac{\nu}{U\cdot r}\right)^{\alpha}\cdot U^2\cdot\left(1-\frac{y}{r}\right)\cdot\frac{\partial^2 Q}{\partial y^2} \quad (15)$$

Integrating equation (15) from y=0 to y=r and u=0 to u=U yields

(16) $Q = \left(\frac{\nu}{U}\right)^{1-\alpha}\cdot\frac{1}{A}\, r^{1+\alpha}$

Based on equation (16), it can be assumed that

(17) $Q(y) = \left(\frac{\nu}{U}\right)^{1-\alpha}\cdot\frac{1}{A}\, y^{1+\alpha}$

The second derivation of equation (9) yields

$$\frac{\tau_t}{\rho} = \left(\frac{U}{\nu}\right)^{1-\alpha}\cdot\frac{A}{\alpha(1+\alpha)}\cdot y^{1-\alpha}\cdot\nu\cdot\frac{du_t}{dy} = \frac{\tau_w}{\rho}\cdot\left(1-\frac{y}{r}\right) \quad (18)$$

Integration of equation (18) gives

(19) $\frac{u_t}{U} = \left(\frac{y}{r}\right)^{\alpha}\cdot\left(1+\alpha-\alpha\cdot\frac{y}{r}\right)$

Setting $\eta = \frac{y}{r}$ and $f_i = \frac{u_i}{U}$, $i = l, t, e$, equation (19) is rewritten

(20) $f_t = \eta^{\alpha}\cdot(1+\alpha-\alpha\cdot\eta)$

The flow profile in the laminar sublayer

The momentum equation in the laminar sublayer is

$$\gamma \frac{du_l}{y} = \frac{\tau_w}{\rho} = A \cdot R^{-\alpha} \cdot U^2 \quad (21)$$

Converting equation (21) to ηand f, and integrating it yields

$$f_l = A \cdot R^{1-\alpha} \cdot \eta \quad (22)$$

Combined Velocity Profile

Under the assumption of direct transfer from laminar flow to turbulent, the transfer point is calculated by comparing the laminar flow to the turbulent one.

$$\eta_o = \left(\frac{1+\alpha}{A}\right)^{\frac{1}{1-\alpha}} \cdot \frac{1}{R} \quad (23)$$

the combined velocity profile is

$$f(\eta) := \begin{cases} \left(A \cdot R^{1-\alpha} \cdot \eta\right) & \text{if } \eta_o \geq \eta \geq 0 \\ \left[\eta^{\alpha} \cdot (1+\alpha-\alpha \cdot \eta)\right] & \text{otherwise} \end{cases} \quad (24)$$

The velocity profile, for some cases is given in figures 1,2,3,4 - a

The average velocity U_m is calculated by

$$\frac{U_m}{U} = \int_0^1 f \cdot 2 \cdot (1-\eta)\, d\eta \quad (25)$$

It should be noted that the average velocity is not known at the beginning of the calculation. Thus, we have to assume Um/U =0.83, make the calculation, get a new value, and, after a few rounds, get the exact value.

Experimental Equations

The experiment is also divided into laminar flow near the wall, which is converted to turbulent at the point $\eta_e = \frac{11.57}{\sqrt{A} \cdot R^{1-0.5 \cdot \alpha}}$.

The experimental velocity profile near the wall is given by

$$g(\eta) := \begin{cases} \left(A \cdot R^{1-\alpha} \cdot \eta\right) & \text{if } \eta_e \geq \eta \geq 0 \\ \left[\sqrt{\frac{A}{R^{\alpha}}} \cdot \left(2.5 \ln\left(\sqrt{A} \cdot R^{1-0.5 \cdot \alpha} \cdot \eta\right) + 5.45\right)\right] & \text{otherwise} \end{cases}$$

(26)

Table 2: presents some values for 3 cases

$\frac{U_m \cdot r}{\gamma}$	$2 \cdot 10^3$	$5.5 \cdot 10^4$	1·10^6
ϵ	0.04	0.01765	0.01039
U_m/U	0.785	0.865	0.890
R	$2.645 \cdot 10^3$	$6.358 \cdot 10^4$	$1.124 \cdot 10^6$
α	0.255	0.181	0.144
A	0.0228	0.0122	0.0076
η_o	0.085	$4.18 \cdot 10^{-3}$	$3.1 \cdot 19^{-4}$
η_e	0.082	$4.48 \cdot 10^{-3}$	$3.2 \cdot 10^{-4}$

Appendix A provides the calculation order to obtain the velocity profile based on the value of $\frac{U_m \cdot r}{\gamma}$.

The velocity profiles in the entire range and near the wall, compared to the experimental data, are presented in Figures 1, 2, and 3.

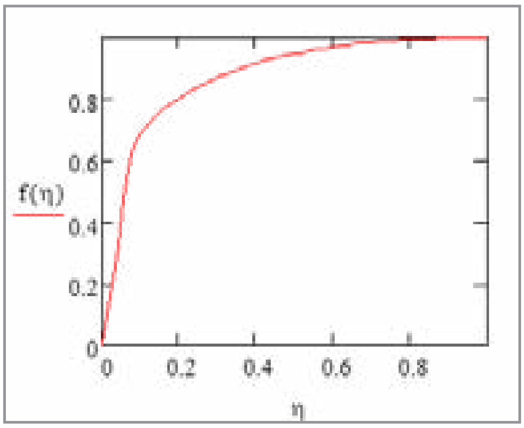

Figure 1: a – velocity profile for $\frac{U_m \cdot r}{\gamma} = 2 \cdot 10^3$.

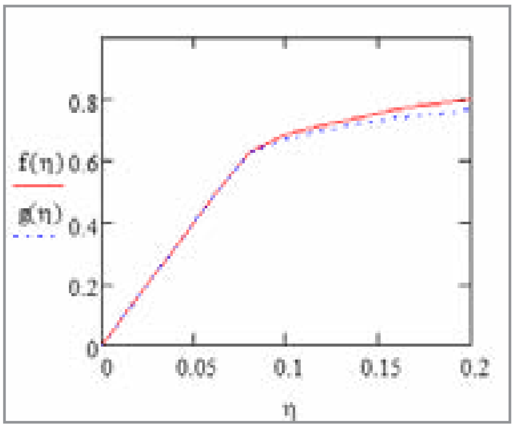

Figure 1: b – calculated velocity vs. experimental

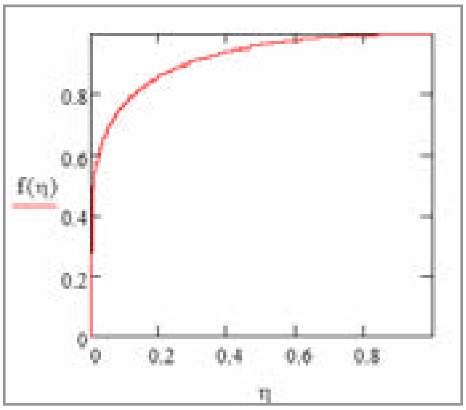

Figure 2a: velocity profile for $\frac{U_m \cdot r}{\gamma} = 5.5 \cdot 10^4$.

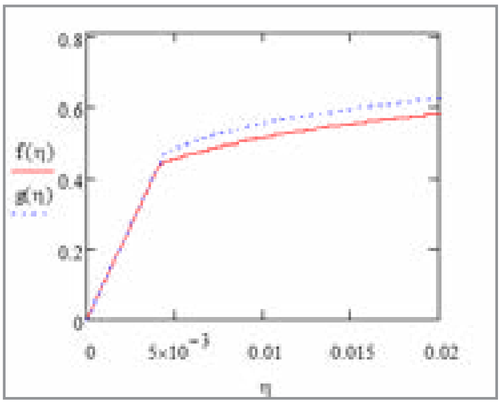

Figure 2b: calculated velocity vs. experimental

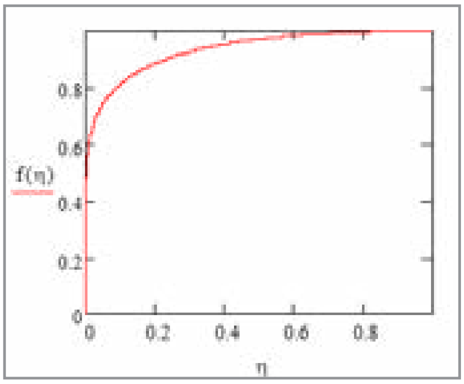

Figure 3a: velocity profile for $\frac{U_m \cdot r}{\gamma} = 1 \cdot 10^6$.

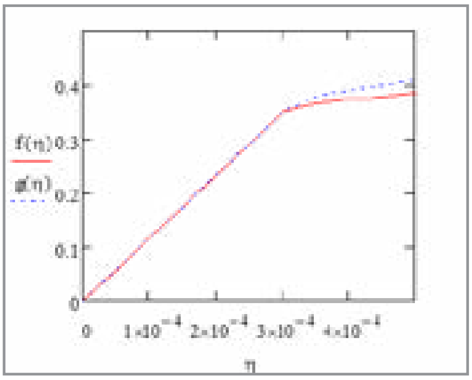

Figure 3b: calculated velocity vs. experimental

Sergio de Campus Junior [6] presents a raw velocity profile with $\frac{U_m \cdot 2r}{\gamma} = 85061$. This profile was compared to a calculated one, and the result is very good compatibility (see Appendix B).

Conclusions

The experimental data that are presented in Table 1 were used to calculate a general equation for the eddy viscosity in the pipe. The eddy viscosity enables the calculation of the velocity profile in the turbulence zone, while the shear stress on the surface is used to calculate the velocity profile in the laminar sublayer. These 2 profiles give the complete velocity profile in the pipe. The velocity profile near the wall was compared to the experimental one with very good compatibility. The whole velocity profile, as compared to the profile that is presented by Sergio de Campus Junior, is also competes.

Least of the Symbols

y- distance from the surface
τ-shear stress
ϵ - turbulent resistance coefficient
U_m - the average velocity in the pipe
U – velocity in the pipe axis
u – velocity at y
ρ - density
r - radius of the pipe
γ-kinematics viscosity
η - y/r
f(η) - the relative velocity u/U
g(η) – the relative experimental velocity $\frac{u_e}{U}$
R - $\left(\frac{U \cdot r}{\gamma}\right)$

Subscript

w – refers to the surface
l - refers to the laminar sublayer
t – refers to the turbulent zone
e – refers to experimental data

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Appendix A- Calculation order to get the velocity profile

Stage	Value	Equation
1	$\frac{U_m \cdot r}{\nu}$	given
2	ϵ	(4)
3	$\frac{U_m}{U}$	(initial 0.83) (25)
4	R	$\frac{U_m \cdot r}{\nu} \cdot \frac{U}{U_m}$
5	α	(8)
6	A	(9)
7	f_t	(20)
8	f_l	(22)
9	η_o	(23)
10	f	(24)
11	$\frac{U_m}{U}$	(25)
12		Go to stage 3

Appendix B- A Comparison of the calculated velocity to the raw experimental one

Sergio de Campus Junior (6) presents a raw experimental velocity in a pipe (see Figure B1 below). The radius of the pipe is 0.42m, the velocity along the axis is 38.25 m/s, and $R_e = \frac{U_m \cdot 2r}{\nu} = 35061$.

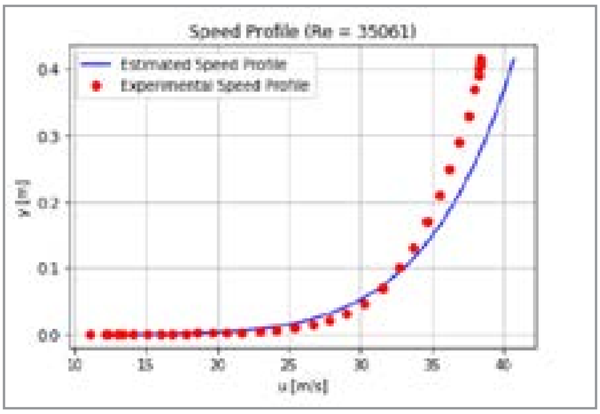

Figure B1: The experimental velocity in the pipe.
(Note: Sergio de Campus Junior drew the estimated speed profile.)

The velocity for this case was calculated. The entire profile is illustrated in Figure B2. 4 specific points are presented in Table B1.

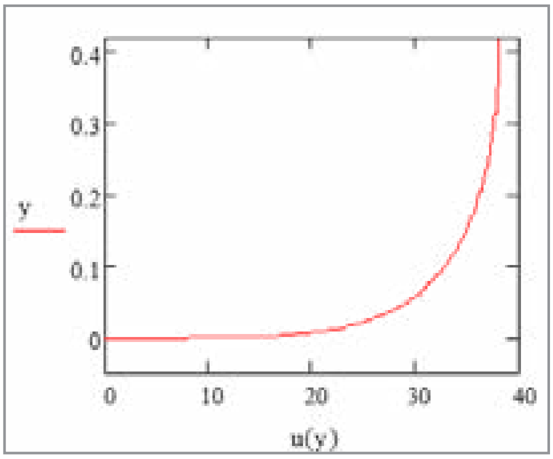

Figure B2: velocity profile (Re=35061)

Table B1-points on the velocity profile

Y(m)	u(m/s)	u[illegible](m/s)
0.05	29.4	30.4
0.10	33.0	32.8
0.20	36.4	35.5
0.30	37.8	37.0