ISSN: 3066-8131
Research Article

Science Set Journal of Physics

# Heuristic Interpretation of Bell's Inequalities for Photons (enlarged version)

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**Submitted:** 01 Apr 2023 **Accepted:** 04 Apr 2023 **Published:** 10 Apr 2023

https://doi.org/10.63620/MKSSJP.2023.1014

***Citation:*** *Schuller, F. (2023) Heuristic Interpretation of Bell's Inequalities for Photons (enlarged version). Sci Set J of Physics 2(2), 01-02.*

***Abstract***
*We show in a heuristic way that recovering the validity of Bell's inequalities implies conditions incompatible with photon entanglement*

Introduction

A discussion of Bell's inequalities for photons has been presented in ref. in the case of conventional quantum optics, where these inequalities are violated [1]. Here we extend these calculations and show how in a heuristic way the validity of these equations can be recovered. An additional example from ref. is presented in the appendix [2].

The Conventional Calculation

In ref.one considers pairs of photons originating simultaneously from a common source, their number being designated as the number of coincidences [1]. The two photons of a pair go respectively trough two polarizers, which are set to certain angles with respect to the vertical. For the number of coincidences, one then derives Bell's inequality in the form

(1)

The probability of going through the polarizer being characterized by the cosine squared of its angle with the vertical, we have the relations

(2)

The inegality of eq.(1) then becomes

50% × 75% ≤ 50% × 25% + 50% × 25% (3a)

or

(3b)

(3c)

showing that the inequality is violated.

It is important to note that in this derivation the value of each term depends only on the difference of the two angles involved. In particular the second term on the rhs of eq. (2) is equal to the first one, according to

(4)

This is due to the fact that the outcome of a measurement on one photon is determined by the outcome of the measurement on the other. This link between the measurements constitutes the EPR paradox. As is well known, experiment confirms this result. (Clauser, Aspect, Zeilinger).

The Modified Calculation

In this note we go one step further and consider the case of local reality. We then assume that in this case there is no such link, so that the two measurements at −30° and 30° are independent of each other. We then write instead of eq's (3a) and (3b)

50% × 75% ≤ 50% × 25% + 50% × 75% × 75% (5)

(5a)

(5b)

which satisfies Bell's inequality, as it should if local reality is assumed [3].

Discussion

Clearly, in terms of quantum entanglement, the outcome of measurements involving the two polarizers can only depend on their mutual angle, thus excluding the assumption which validates Bell's inequalities. In this way the violation of Bell's inequalities becomes a trivial fact.

Conclusion

In this context Bell's inequalities appear as somewhat overrated, since the conclusions drawn from their violation can be inferred directly from basic principles of quantum theory.

Quoting Gell-Mann

« Of course, it was the same old quantum mechanics. Nothing was new except its confirmation and the subsequent flurry of flapdoodle ».

We believe however, that only the present and possibly future experiments, e.g. on electrons, can decide against Einstein's

## References


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## Appendix

**Adopting the notations and definitions of ref [2] we introduce the following polarizer angles of observer 1 and 2 respectively:**

$$\Theta_1 = 0 \quad \Theta_2 = \frac{3\pi}{8} \quad \Theta_1' = -\frac{\pi}{4} \quad \Theta_2' = \frac{\pi}{8}$$
$$= 0 \quad = 67{,}5° \quad = -45° \quad = 22{,}5° \qquad (1)$$

In terms of the transmission probabilities $P$ Bell's inequality then takes the form

$$P(\Theta_1, \Theta_2) - P(\Theta_1, \Theta_2') + (\Theta_1', \Theta_2) + P(\Theta_1', \Theta_2') - 1 \leqslant 0 \qquad (2)$$

or

$$P\left(0, \frac{3\pi}{8}\right) - P\left(0, -\frac{\pi}{4}\right) + \left(-\frac{\pi}{4}, \frac{3\pi}{8}\right) + P\left(-\frac{\pi}{4}, \frac{\pi}{8}\right) - 1 \leqslant 0 \qquad (3)$$

Case 1

In the quantum mechanical approach we then have, with the transmission probabilities through the polrizers given by the sin squared of their angular differencies

$$\frac{1}{2}sin^2(\Theta_1 - \Theta_2) - \frac{1}{2}sin^2(\Theta_1 - \Theta_2') + \frac{1}{2}sin^2(\Theta_1' - \Theta_2) + \frac{1}{2}sin^2(\Theta_1' - \Theta_2') - 1 \leqslant 0 \qquad (4)$$

yielding, with the values of the angles given in eq.(1), the relation

$$\frac{1}{2}sin^2\frac{3\pi}{8} - \frac{1}{2}sin^2\frac{\pi}{8} + \frac{1}{2}sin^2\frac{5\pi}{8} + \frac{1}{2}sin^2\frac{3\pi}{8} - 1 \leq 0 \qquad (5)$$

Using the relations

$$sin^2\frac{\pi}{8} = cos^2\frac{3\pi}{8}\,;\, sin^2\frac{3\pi}{8} - sin^2\frac{\pi}{8} = 2sin^2\frac{3\pi}{8} - 1$$

and $sin^2\frac{5\pi}{8} = sin^2\frac{3\pi}{8}$

we obtain from eq.(5) (6)

$$2sin^2\frac{3\pi}{8} - \frac{3}{2} \leq 0$$

With the explicit values

$$sin^2\frac{3\pi}{8} = \frac{1-cos\frac{3\pi}{4}}{2} = \frac{2+\sqrt{2}}{4} \qquad (7)$$

eq.(6) yields the result

$$\frac{\sqrt{2}-1}{2} = 0.207 \leq 0 \qquad (8)$$

So that again Bell's inequality is violated.

Case 2

We now make assumptions similar to those of the previous example and write

$$\frac{1}{2}sin^2\frac{3\pi}{8} - \frac{1}{2}sin^2\frac{\pi}{8} + \frac{1}{2}sin^2\frac{\pi}{4}sin^2\frac{3\pi}{8} + \frac{1}{2}sin^2\frac{\pi}{4}sin^2\frac{\pi}{8} - 1 \leq 0 \qquad (9)$$

or

$$\frac{1}{2}\left(sin^2\frac{3\pi}{8} - sin^2\frac{\pi}{8}\right) + \frac{1}{2}\,sin^2\frac{\pi}{4}\left(sin^2\frac{3\pi}{8} + sin^2\frac{\pi}{8}\right) - 1 \leq 0 \qquad (10)$$

yieldind with the relations of eq.(6)

$$\frac{1}{2}\left(2sin^2\frac{3\pi}{8} - 1\right) + \frac{1}{2}\,sin^2\frac{\pi}{4} - 1 \leq 0 \qquad (11)$$

With the explicit vaues

$$sin^2\frac{3\pi}{8} = \frac{2+\sqrt{2}}{4}\,;\, sin^2\frac{\pi}{4} = \frac{1}{2}$$

we then arrive at the final result (12)

$$\frac{\sqrt{2}-3}{4} = -0.396 \leq 0$$

thus validating Bell's inequalities as in the previous example.