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Fermat's Theorem

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Abstract

Dark energy is believed to drive the accelerated expansion of the universe, comprising approximately 71.35% of the total energy density. This work proposes that gravitational potential energy and dark energy are equivalent, and derives expressions predicting both the percentage of dark energy and spatial variation in the speed of light. Results reproduce known light bending near massive bodies and resolve issues such as the black hole time-stop paradox. The approach is based solely on classical mechanics and relativity, without introducing new physical laws.

Keywords: Fermat's Theorem, Pythagorean Triples, Exponentiation, Number Theory, Integer Functions, Proof Techniques.

Introduction

We undertake, in this article, the celebrated Fermat's theorem, with the ambition of presenting a novel and original proof.
We will rely primarily on two implementations:

1 - One of them is widely known, namely the one that allows for the generation of primitive Pythagorean triples.:

$(n^2-m^2, 2nm, n^2+m^2)$

which yields:

$4n^2m^2 = (n^2+m^2)^2 - (n^2-m^2)^2$

Example: $(3, 4, 5) \Longrightarrow 3^2 + 4^2 = 5^2$

which yields:

$3 = 2^2 - 1^2$

$4 = 2 * 2 * 1$

$5 = 2^2 + 1^2$

Note: We will later present in this article another formulation, admittedly less elegant, but possessing highly useful properties.

2 - The second is a novel function and requires close attention from the reader, because despite its apparent simplicity, it possesses numerous properties essential to the problem under investigation:

- For odd integers, one may write:

$$n = 2m+1 \Longrightarrow f(n) = \frac{5n^2-29}{4}$$

- And for the even integers, it will be:

$$n = 2m \Longrightarrow f(n) = \frac{5n^2}{4}$$

Example:

$$3 \Longrightarrow f(n) = \frac{5 * 3^2-29}{4}$$

$$5 \Longrightarrow f(n) = \frac{5 * 5^2-29}{4}$$

$$4 \Longrightarrow f(n) = \frac{5 * 4^2}{4} = 20$$

Explanation:

If we have:

$$3^2 + 4^2 = 5^2$$

then we will obtain:
4 + 20 = 24

Let us note the perfect bijection between the set of integers in Pythagorean triples and the set constituted by their images under **f(n)**. Let us note that in the set consisting of these images, only the two elementary operations—addition and subtraction—can be utilized.

Furthermore, these can only be applied in a restricted manner:

- Subtraction is limited to the integer images of odd integers; the smallest image is subtracted from the largest to yield the integer image corresponding to the even integer.
Example: 24 - 20 = 4
- L'addition ne peut se faire qu'entre le plus petit et le pair.
Example: 20 + 4 = 24

3 - We shall, at first, restrict our study to primitive triplets; composite triplets will be addressed, as a consequence, at the end of this article.
b^2-$a^2$$\square$$c^2$ $\Longrightarrow$$(kb)^2$-$(ka)^2$$\square$$(kc)^2$

I – Pythagorean Triples Put to the Test of f(n)

Odd integers will be transformed as follows:
{{3, 4}, {5, 24}, {7, 54}, {9, 94}, {11, 144}, {13, 204}, {15, 274}, {17, 354}, {19, 444},
{21, 544}}

Even integers will be transformed as follows:
{{2, 5}, {4, 20}, {6, 45}, {8, 80}, {10, 125}, {12, 180}, {14, 245}, {16, 320},
{18, 405}, {20,500}}

First Critical Observation:
- The images under f (n) of the odd integers always terminate in 4. Therefore, their difference will always yield an even integer ending in 0.
- Now, the images under f(n) of even integers end either with 0 or with 5. We must therefore exclude those whose images under f(n) end with 5.

Significant Consequence:
: Only even integers that are multiples of 4 can be elements of Pythagorean
triples.

The following two tables illustrate these properties Let the Pythagorean triple be:
(c , a , b) , such as : b^2 - a^2 $\square$ c^2

With the transformation function f(n), we will write:

$$f\ (c)\ =\ \frac{5\,c^2-29}{4}$$
$$f\ (b)\ =\ \frac{5\,b^2-29}{4}$$
$$f\ (a)\ =\ \frac{5\,a^2}{4}$$

A = f (a) ; B = f (b) ; F = f (c)

```
Table[{c, a, b}, {m, 0, 5}, {k, 1, 5}]
table
Table[{F, A, B}, {m, 0, 5}, {k, 1, 5}]
table

{{{3, 4, 5}, {15, 8, 17}, {35, 12, 37}, {63, 16, 65}, {99, 20, 101}},
 {{5, 12, 13}, {21, 20, 29}, {45, 28, 53}, {77, 36, 85}, {117, 44, 125}},
 {{7, 24, 25}, {27, 36, 45}, {55, 48, 73}, {91, 60, 109}, {135, 72, 153}},
 {{9, 40, 41}, {33, 56, 65}, {65, 72, 97}, {105, 88, 137}, {153, 104, 185}},
 {{11, 60, 61}, {39, 80, 89}, {75, 100, 125}, {119, 120, 169}, {171, 140, 221}},
 {{13, 84, 85}, {45, 108, 117}, {85, 132, 157}, {133, 156, 205}, {189, 180, 261}}}

{{{4, 20, 24}, {274, 80, 354}, {1524, 180, 1704}, {4954, 320, 5274}, {12244, 500, 12744}},
 {{24, 180, 204}, {544, 500, 1044}, {2524, 980, 3504}, {7404, 1620, 9024}, {17104, 2420, 19524}},
 {{54, 720, 774}, {904, 1620, 2524}, {3774, 2880, 6654}, {10344, 4500, 14844},
  {22774, 6480, 29254}}, {{94, 2000, 2094}, {1354, 3920, 5274},
  {5274, 6480, 11754}, {13774, 9680, 23454}, {29254, 13520, 42774}},
 {{144, 4500, 4644}, {1894, 8000, 9894}, {7024, 12500, 19524}, {17694, 18000, 35694},
  {36544, 24500, 61044}}, {{204, 8820, 9024}, {2524, 14580, 17104},
  {9024, 21780, 30804}, {22104, 30420, 52524}, {44644, 40500, 85144}}}
```

And first, let us emphasize this decisive property of Pythagorean triplets: their image under the func- tion f(n) always yields these endings (in one direction or the other):
(274 , 80 , 354) ou (4954 , 320 , 5274)
(1524 , 180 , 1704) ou (7404 , 1620 , 9024)
(12244 , 500 , 12744)
(198994 , 2000 , 200994)
Demonstration:
It suffices to construct a table of all possible combinations between the terminal digits of the integers n and m, since these two integers establish the necessary condition for any Pythagorean triple, as follows:
4 n2 m2 = (n2 + m2)2 - (n2 - m2)2
NB: let us recall that n and m must necessarily be one even and the other odd so that their sum yields an odd integer
Example:
- Termination of n = 2 and termination of m = 1 will yield us, for example:
c = 3 $\Longrightarrow$: 4 - 1
and:
b = 5 $\Longrightarrow$: 4 + 1
Which yields the following triplet: (3 , 4 , 5),
- Termination of n = 3 and termination of m = 2 will yield, for example:
c = 5 $\Longrightarrow$: 9 - 4
and
b = 13 $\Longrightarrow$: 9 + 4
Which yields the following triplet :

(5 , 12 , 13)

- Let us generalize:
n is odd

m is even Tables of Endings of n and of m :

n		n^2
1	⟹	1
3	⟹	9
5	⟹	5
7	⟹	9
9	⟹	1

m		m^2
0	⟹	0
2	⟹	4
4	⟹	6
6	⟹	6
8	⟹	4

Which yields:

Which yields:

n^2+m^2	n^2-m^2	
1 + 0 ⟹ 1	1 - 0 ⟹1	
1 + 4 ⟹5	11 - 4 ⟹7	or: 21 - 4 ⟹7; etc
1 + 6 ⟹7	11 - 6 ⟹5	
9 + 0 ⟹9	9 - 0 ⟹1	
9 + 4 ⟹3	9 - 4 ⟹ 5	
9 + 6 ⟹5	9 - 6 ⟹3	
5 + 0 ⟹5	5 - 0 ⟹5	
5 + 4 ⟹9	5 - 4 ⟹1	
5 + 6 ⟹1	15 - 6 ⟹9	or: 25 - 6 ⟹9; etc

Endings of Pythagorean triples:
Let (c , a , b) be a Pythagorean triple with a as the even integer
The only possible units' digits for c and b are the following, in either order:
(1, 1) ; (5, 7) ; (9, 1) ; (3, 5) ; (5, 5)
Hence the table above :
(274 , 80 , 354) or (4954 , 320 , 5274)
(1524 , 180 , 1704) or (7404 , 1620 , 9024)
(12244 , 500 , 12744)
(198994 , 2000 , 200994)

II – The Fermat's theorem

A - Puissance 4

As stated in the introduction to this article, we assume that the impossibility of Pythagorean triples raised to the power 4 has already been established by others.
Thus, for any natural numbers a, b, and c, it will always hold that:

$a^4 \neq c^4-b^4$

However, we will examine the fourth power through our function f(n), in order to draw conclusions that will assist us in demonstrating the impossibility for exponents greater than 4.

Let us proceed as follows:

(c, a , b) ⟹ a^4 □? c^4-b^4 ⟹ $(c^2)^2$□?$(a^2)^2$-$(b^2)^2$

These "triplets" would therefore exhibit the same structure as Pythagorean triplets raised to the second power.

These "triplets" would therefore exhibit the same structure as Pythagorean triplets raised to the second power.

1 - Let us examine their transformation via the function f(n):

$$f(c) = \frac{5\ (c^2)^2-29}{4}$$

$$f(b) = \frac{5\ (b^2)^2-29}{4}$$

$$f(a) = \frac{5\ (a^2)^2}{4}$$

Which yields the following excerpt:

```
{{{94, 320, 774}, {63274, 5120, 104394}, {18 75774, 25 920, 23 42694},
  {120 074 494, 200 000, 130 075 494}},
 {{774, 25 920, 35 694}, {243094, 200 000, 8840 94}, {5 125 774, 768 320,
  {43 941 294, 2 099 520, 65 250 774}, {234 235 894, 4 685 120, 305 175 774}
 {{2994, 414 720, 488 274}, {664 294, 2 099 520, 5 125 774}, {11 438 274, 6
  {857 18694, 16 200 000, 1 764 47694}, {415 188 274, 33 592 320, 684 976 5
 {{8194, 3 200 000, 3 532 194}, {1 482 394, 12 293 120, 22 313 274}, {22 31
  {151 938 274, 74 961 920, 440 344 194}, {684 976 594, 146 232 320, 1 464 1
```

Let us first note that in the vast majority of cases, these triplets exhibit this configuration (in one orientation or the other):
10k + 94
and 10k + 74
In the same manner as these two cases:
(63274, 5120, 104394)

(19691194 , 81920 , 22313274)
However, no Pythagorean triple yields such a configuration, as demonstrated above (see illustration table).

On the other hand, this problematic situation is encountered here and there, since it also appears in higher-power triples 2 :
(120074494 , 200000 , 130075494)

However, there are only two types of triplets that yield such endings:
- Some triples in which the two odd numbers end with 9 and 1, in either order. But also certain others in which the odd numbers both end with 9 and 9

In any case, neither of these categories concerns us, for the simple reason that the fourth power of an odd integer can never terminate with 9. Indeed, 9 raised to the fourth power always produces an integer whose last digit is 1.

- On the other hand, the odd integers in certain Pythagorean triples end with 1 and 1. More- over, some images under f(n) indeed yield terminal digits of 94
Example:
(231 , 160 , 281)⟹(66694 , 32000 , 98694)

And it is precisely this rare category that we shall examine, in order to demonstrate that it is impossible for both odd elements of such triplets to simultaneously be perfect squares, and thus potentially yield (subject to the parity of the even element) a Pythagorean triple at higher powers 4.

2- General Expression of the Triplet Formula (a, b , c) :

a = (1 + 2 k) *(1 + 2 k + 2 m)
b = 2m *(1+2k+m)
c = (1 + 2 k) *(1 + 2 k + 2 m) + 2 m^2

We obtain exactly the same primitive triples as with the well-known formula cited above.

```
{{{3, 4, 5}, {15, 8, 17}, {35, 12, 37}, {63, 16, 65}, {99, 20, 101}, {143, 24, 145}},
 {{5, 12, 13}, {21, 20, 29}, {45, 28, 53}, {77, 36, 85}, {117, 44, 125},
  {165, 52, 173}},
 {{7, 24, 25}, {27, 36, 45}, {55, 48, 73}, {91, 60, 109}, {135, 72, 153},
  {187, 84, 205}},
 {{9, 40, 41}, {33, 56, 65}, {65, 72, 97}, {105, 88, 137}, {153, 104, 185},
  {209, 120, 241}},
 {{11, 60, 61}, {39, 80, 89}, {75, 100, 125}, {119, 120, 169}, {171, 140, 221},
  {231, 160, 281}}}
```

Note:
Despite its apparent complexity, it is very straightforward to comprehend.

Explanation of its genesis: Step 1:
- Let us consider an example that will serve to illustrate the following steps of our

demonstration.
Let the triplet be:
(11 , 60 , 61)
61 - 11 = 50

We can write:
50 = (11 - 1)*(11 - 1)
2
- Another example: Let the triplet be:
(7 , 24 , 25)
25 - 7 = 18

We can write:
18 = (7 - 1)*(7 - 1)
2

Step 2:
Let us now consider the triplets generated from the root triplet.
(11, 60 , 61)
etc.

We'll have:
(11 , 60 , 61) ; (39 , 80, 89) ; (75 , 100 , 125) ; (119 , 120 , 169) ; (171 , 140 , 221) ;

constitute them.
- Note 1 :
All exhibit an identical difference (50) between the two odd integers that
- Note 2:
The initial elements of these triplets are written as follows: 11 = 1 * 11
39 = 3 * 13
75 = 5 * 15
etc.

The third terms are obtained by adding 50 to the first terms.
11 + 50 = 61
13 + 50 = 89
75 + 50 = 125
etc.

3 - Demonstration of the Impossibility of Fourth Power Triplets

Although this demonstration has already been presented long ago, we cannot resist providing this new proof, if only because it is rendered exceptionally straightforward by means of this new formulation. This approach can be applied to all triplets; however, as evidenced by the function f(n), we have observed that only

those triplets terminating in 1 are relevant to this segment of the demonstra- tion.
The only aspect that differs is the substitution of the first term: $2k + 1$

par

$(1+10k)^2$
The third term will thus be as follows:
$b = 1+20k+300k^2+2000k^3+5000k^4$

As can be readily demonstrated by attempting to extract roots for the term b, it follows that it is impossible for the term b to be a perfect square with k a natural integer.
Therefore, it is impossible to have Pythagorean triples of degree 4.
2 - Study of Triplets with Even Powers Greater than 2

Here again, the transformation function f(n) demonstrates that all powers of the form 4*s yield a scenario identical in every respect to that of the fourth power. This is understood because these exponents have exactly the same endings.

Following the model of this example for the eighth power, in comparison with the power 4

```
F1 = (5*c^4 - 29) / 4;
B1 = (5*b^4 - 29) / 4;
A1 = (5*a^4) / 4;

F2 = (5*c^8 - 29) / 4;
B2 = (5*b^8 - 29) / 4;
A2 = (5*a^8) / 4;

Table[{F1, A1, B1}, {m, 0, 5}, {k, 1, 3}]
table
Table[{F2, A2, B2}, {m, 0, 3}, {k, 1, 3}]
table

]= {{{94, 320, 774}, {63 274, 5120, 104 394}, {1 875 774, 25 920, 2 342 694}},
  {{774, 25 920, 35 694}, {243 094, 200 000, 884 094}, {5 125 774, 768 320, 9 863 094}},
  {{2994, 414 720, 488 274}, {664 294, 2 099 520, 5 125 774}, {11 438 274, 6 635 520, 35 497 794}},
  {{8194, 3 200 000, 3 532 194}, {1 482 394, 12 293 120, 22 313 274}, {22 313 274, 33 592 320, 110 661 594}},
  {{18 294, 16 200 000, 17 307 294}, {2 891 794, 51 200 000, 78 427 794}, {39 550 774, 125 000 000, 305 175 774}},
  {{35 694, 62 233 920, 65 250 774}, {5 125 774, 170 061 120, 234 235 894}, {65 250 774, 379 494 720, 759 466 494}}}

]= {{{8194, 81 920, 488 274}, {3 203 613 274, 20 971 520, 8 719 696 794}, {2 814 844 238 274, 537 477 120, 4 390 599 317 394}},
  {{488 274, 537 477 120, 1 019 663 394}, {47 278 574 194, 32 000 000 000, 625 308 016 194},
   {21 018 906 738 274, 472 252 497 920, 77 824 613 014 194}}, {{7 205 994, 137 594 142 720, 190 734 863 274},
   {353 036 920 594, 3 526 387 384 320, 21 018 906 738 274}, {104 667 422 363 274, 35 224 100 536 320, 1 008 075 114 867 594}},
  {{53 808 394, 8 192 000 000 000, 9 981 156 536 394}, {1 758 010 772 794, 120 896 639 467 520, 398 306 016 113 274},
```

The proof is therefore identical to that for exponent 4, since for any exponent
4*p, we can write: $(n^p)^{4,}$ and thus must necessarily lead to the same outcome:

And thus, it is impossible to have Pythagorean triples raised to that power 4*s

C - study of Pythagorean triples of degree 3
Is it possible to have a triplet (b, a, c) such that?
$a^3 =? c^3 - b^3$
Let us examine the following table, which provides the images under the function
f (n):

```
b = 2*r + 1;
a = 2*s;
Table[{b, (5*b^3 - 29) / 2} , {r, 1, 10}]
table
Table[{a, (5*a^3) / 2}, {s, 1, 20}]
table

Out[156]= {{3, 53}, {5, 298}, {7, 843}, {9, 1808}, {11, 3313}, {13, 5478},
  {15, 8423}, {17, 12 268}, {19, 17 133}, {21, 23 138}}

Out[157]= {{2, 20}, {4, 160}, {6, 540}, {8, 1280},
  {10, 2500}, {12, 4320}, {14, 6860}, {16, 10 240},
  {18, 14 580}, {20, 20 000}, {22, 26 620}, {24, 34 560},
  {26, 43 940}, {28, 54 880}, {30, 67 500}, {32, 81 920},
  {34, 98 260}, {36, 116 640}, {38, 137 180}, {40, 160 000}}
```

Let us first note the following: for any even integer b, the following always holds:
f (b) = 10*t
It would therefore be necessary for the images f(b) and f(c) to both be:
- Either in the form of: 10*r + 3
- Either in the form of : 10*s + 8

This enables us to deduce two interleaved sequences, thereby establishing that all odd integers necessarily belong to one or the other.
Suite n°1 ⟹ f (n) = 3 + 4* k Suite n°2 ⟹ f (n) = 5 + 4* k

Our triplets must necessarily be represented as follows:
(3 + 4* k , 10*t , 3 + 4* k1) (5 + 4* k) , 10*t , 5 + 4* k1)

We shall now demonstrate that, within such a configuration, it is impossible for any triplet to yield
$a^3 = ?? c^3 - b^3$

Warning:
We have emphasized that the images under (f(n)) of all even numbers

raised to the power 3 are expressed as 10*t, and now let us add this very important particu- larity for what follows:
All these images 10*t are multiples of 20 $\Longrightarrow$ which now yields: 10*t

It is therefore necessary that the differences between the images under f(n) of the odd integers also be divisible by 20, if one hopes to find Pythagorean triples in the context of cubes.

This yields the following: in what follows $f(n) = 3 + 4*k$, Odd terms must be spaced apart by 8 or an integer multiple thereof.

Thus, the components of the triplets must be expressed as follows:
c = 3 + 4k
b = 3 + 4k + 8r
Let us therefore compute
$(3 + 4k + 8r))^3 - (3 + 4k)^3 =$
$8r(27 + 72k + 48k^2 + 72r + 96kr + 64r^2)$
8 given that this is already a cube, let us examine the possibility of
cubing the term.
$r(27 + 72k + 48k^2 + 72r + 96kr + 64r^2)$

In order to attempt to identify a unique root that would yield $(x - d)^3$
Let us formulate the equation:
r (27 + 72 k + 48 k2 + 72 r + 96 k r + 64 r2) = 0
We obtain 3 possibles solutions : 1 - r = 0
Which yields the following triplet (excluded by assumption):
(3 + 4k , 0 , 3 + 4k)
2 - Consider these two alternative solutions:

$$r = \frac{1}{16}\left(-9 - 12k + \sqrt{3}\sqrt{-9 - 24k - 16k^2}\right)$$

et

$$r = \frac{-1}{16}\left(-9 - 12k - \sqrt{3}\sqrt{-9 - 24k - 16k^2}\right)$$

And in both cases, the solutions cannot be natural numbers.

Discussed
Note: It is unnecessary to repeat the same process for the second sequence we

Suite n°2 $\Longrightarrow$ f (n) = 5 + 4* k
We arrive at the same impossibility.

Conclusion
Therefore, it is impossible to obtain a Pythagorean triple for the exponent 3.

4- Study of Odd Powers Greater than 3

Let us first observe once again that these exponents yield precisely the same sequences as the exponent 3 (owing to identical terminal digits).
as shown in this table:
n = 2m + 1

```
Table[{n, (5*n^3 - 29) / 2}, {m, 1, 5}]
table
Table[{n, (5*n^5 - 29) / 2}, {m, 1, 5}]
table
Table[{n, (5*n^7 - 29) / 2}, {m, 1, 5}]
table
Table[{n, (5*n^9 - 29) / 2}, {m, 1, 10}]
table
Table[{n, (5*n^11 - 29) / 2}, {m, 1, 5}]
table

{{3, 53}, {5, 298}, {7, 843}, {9, 1808}, {11, 3313}}, etc.

{{3, 593}, {5, 7798}, {7, 42003}, {9, 14 7608}, {11, 40 2613}}, etc.

{{3, 5453}, {5, 19 5298}, {7, 205 8843}, {9, 1 195 7408}, {11, 4 871 7913}}, etc.

{{3, 49193}, {5, 488 2798}, {7, 10 088 4003}, {9, 96 855 1208}, {11, 589 486 9213}, etc.
{{3, 44 2853}, {5, 12 2070298}, {7, 494 331 6843}, {9, 7 845 264 9008}, {11, 71 327 917 6513}},
```

Note: let us bear in mind that:
5 n2 r+1
2
= 10k

And thus, it is necessary that the odd terms possess the same ending. The proof is exactly analogous to that for the power 3, utilizing the same intercalated sequences.
Suite n°1 $\Longrightarrow$ (3, 7, 11, etc.)
Suite n°2 $\Longrightarrow$ (1, 5, 9, etc.)

In conclusion:
It can therefore be concluded that there do not exist any Pythagorean triples with odd exponents.

5- Study of even powers greater than 2, which are expressed as:

$k^2*(2k+1)$

Are there Pythagorean triples that can be represented in this form:
$a^2*(2k+1) = ?? c^2*(2k+1) - b^2*(2k+1)$

follows:

The answer is negative, since none exist, as they can equally be expressed as

Let us define:
$A = a^2$; $B = b^2$; $F = c^2$
We can write:
$A^{(2k+1)} \neq F^{(2k+1)} - B^{(2k+1)}$

General Conclusion

We have not addressed the case of multiple triplets derived from primitive triplets, since, ultimately, the discussion reduces to the primitive ones.

Thus, Fermat's theorem is established by means of this novel approach, which makes exten- sive use of the transformation function f(n), about which I will elaborate in a forthcoming article on a related topic.

To conclude this article, let us dare, nonetheless, a slight smile: if this demonstration proves to be correct, I cannot help but think that perhaps it was this very approach that the great Monsieur Fermat had in mind. And beyond the painstaking nature of this exposition of the celebrated theorem of the illustrious Monsieur Fermat, let us bear in mind that, ulti- mately, it can be presented in such a straightforward manner that one might be tempted to say that, while it could not quite have fit in the margin of a letter, still—it would have required only marginally more space to encapsulate, as we may do ourselves when the appropriate occasion arises...