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Study Report on Applications of Complex Number and Conformal Mapping

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Abstract
The main goal of this chapter is to introduce some basic concept of complex analysis, by its real-life applications, for motivating the students to work on complex number by understanding the real role of these in real life.

Basic Concepts of Complex Analysis

Introduction

One of the oldest mathematical concepts, complex analysis has roots in the 19th century. Prior to the 16th century, Italian mathematicians made significant contributions that helped to develop complicated analysis. In the 20th century, significant mathematicians involved in complex analysis included Euler, Gauss, Riemann, Cauchy, and many others.

Numerous texts say that the first use of complex numbers was in the context of "quadratic equations," but this is un true. But when he thought about it, Bombelli was the one who was presented. considered the case, $\frac{c^2}{4}+\frac{b^2}{27}>0$

Grean Bernoulli demonstrated the relationship between. tan−I x and the imaginary number's logarithm in 1702 in order to introduce fictitious numbers to high mathematics. Once more, a British mathematician named Costs demonstrated that (cosø+sinø)=iø which is now known as Euler's formula.
iθ =cosø+isinø

And Demoiver's relation (cosø+isinø) 2=cosnø+isinnø the use of i instead of √−1due to Eulers. Cauchy has suggested that the name conjugate for (a+ib), (a-ib) and the name for modules for =√a2+b2. the absolute value Modulus is a+ib and it can be writes it as modulus of a-ib.

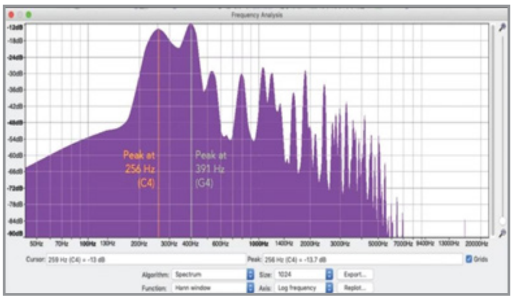
The area is known as "complex analysis,it is known as the theory of functions of a complex variable.It helpful in variety of areas of mathematics, such as number theory, applied mathematics, and physics. Additionally, it includes complex numbers in the forms of conjugates, moduli, pictorial representations, and geometric representations of complex numbers. polar representation of complex numbers, the complex integrals formula, and instances of complex integrals. Additionally, it requires a number of intricate applications [1-5].

Applications

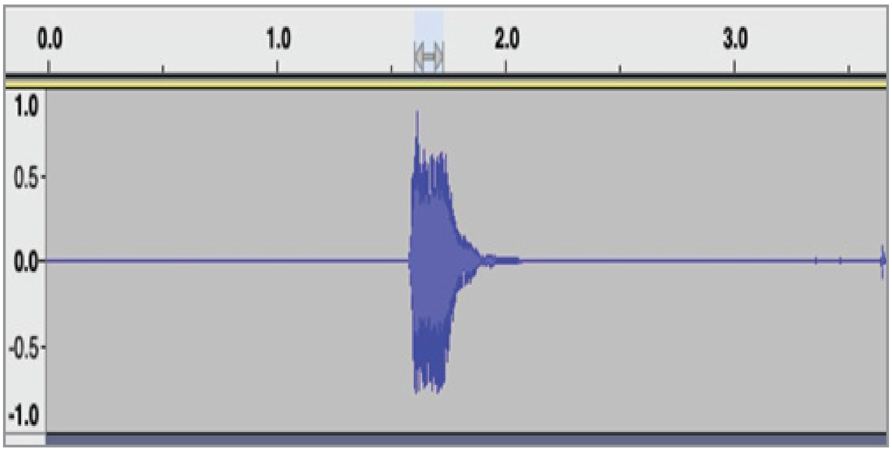
Real life Application of Complex Numbers

Sigal Processing

Consider a pianist recording in a music studio. He asks you to play a game where you have to guess the notes he plays on the piano without looking at it. If you don't have perfect pitch—the ability to recognise a musical note only by hearing it how would you win this game?



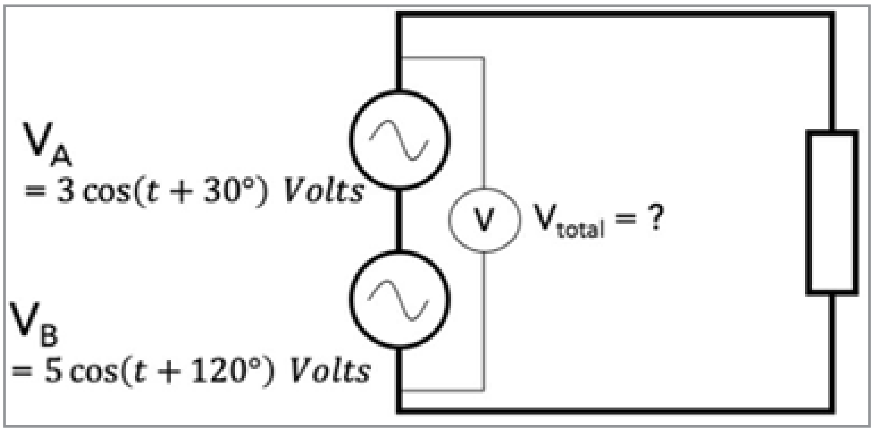
The waveform signal can then be subjected to Fourier Transform to reveal the frequencies that are most commonly heard in the recording. Determine the "peaks" in the frequency distribution that results from using the Fourier Transform to show this. Given the clear peaks at 256 Hz and 391 Hz, we can assume that the pianist performed the piano notes C and G. (which correspond to C4 and G4, respectively).



For audio editors and music producers, peak locations provide important information. In addition to determining the origin any background noise, they can Additionally, use its frequency as a guide to eradicate it using equalisation (EQ).

AC Circuit Analysis

Complex numbers can also be used to In AC circuits, determine the current, voltage, or resistance (AC stands for Alternating Current, which is a current that changes magnitude and direction over time). Calculating the potential difference between two AC power supply with respect to time typically involves complex numbers, more precisely Euler's formula. A calculation of this kind is demonstrated by the example on the right.



Simply combining VA and VB together will not provide the total potential difference. However, we may represent both voltages as the Real Part of a complex number (represented by the x-coordinate on the Argand Diagram).

Let Z_A=3(cos(t+30^0)+i sin(t+30^0);

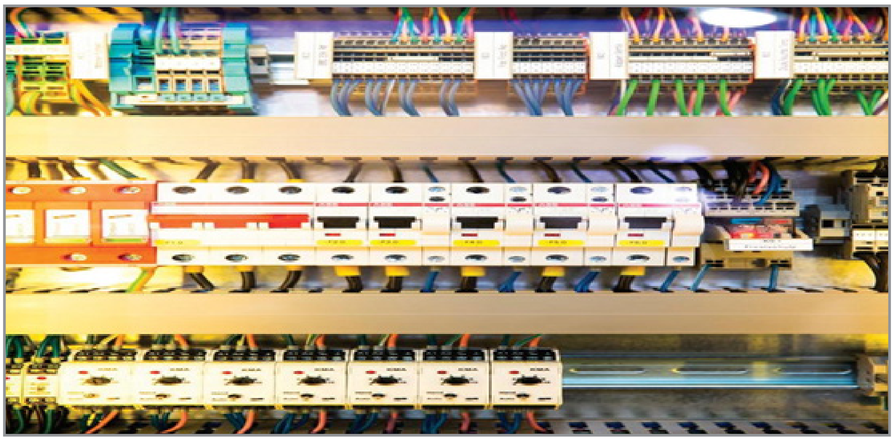
Hence $V_A = R_e|Z_A|$

And let Z_B=5(cos(t+120^0)+I sin(t+120^0)

So that $V_B = R_e|Z_B|$

Note that $Z_A = 3e^{jt\ 30^0}$ and $Z_B = 5e^{jt\ 120^0}$.

In order to avoid misunderstanding with current, it is customary to use j instead of I to indicate imaginary values in circuit analysis (which its symbol is i or I).



We can add the complex numbers and factories:
In an AC circuit, the magnitude and phase of impedance are also expressed using complex numbers. Impedance slows down the electrons in the circuit, much like resistance does. The difference is that resistance does not result in a phase change of the electrical current, whereas impedance does. A complex number representation is essential because impedance occurs in typical electrical components like inductors and capacitors. Complex numbers are typically used to indicate phase, which is crucial for understanding AC circuit analysis [6].

Applications of Complex Numbers in Electronics
Complex numbers have fundamental applications in electronics. A potential value, such as +10 volts or -10 volts, is used to describe the voltage of a battery. However, a home's "AC" voltage requires two inputs.

The first potential is, like 120 volts, and the second is a phase, which is an angle. Voltage has two dimensions, and formal representations of two-dimensional objects include vectors and complex numbers.

The rectangular coordinates are designated as X and Y in the vector representation. Complex. Complex numbers are represented using both real and imaginary components.

The voltage is totally real, has a potential of 120 volts, and has a phase of 90 degrees when a complex number has only imaginary components, such as a real part of 0 and an imaginary part of 120.

The Fourier transform is used in electrical engineering to examine changing voltages and currents. By introducing fictitious resistors that are frequency-dependent and merging them into a single complex value known as the impedance, it is possible to unify the treatment of resistors, capacitors, and inductors.

This use extends to, which and restore image, auto, or video signals using digital variants of Fourier analysis. Because it is often used to represent variable currents, electrical engineers use the letter j for the fictitious unit.

4.Complex functions are used in fluid dynamics to explain possible two-dimensional flow.

5. It is also used in computer science.

Applications Complex Analysis
Signal processing and control theory are two of the major subject areas that use complex analysis.

Signal Processing
Together, complex analysis and Fourier analysis are used to analyse signals in signal processing. In communication systems (such your internet, Wi-Fi, satellite communication, image/video/audio compression, signal filtering/repair/reconstruction, etc.), for example, this alone has a tonne of uses. If we look for applications of signal processing, those are the applications that are indirectly the applications of complex analysis, and I have found that this is quite helpful in going beyond just using the Fourier transform, etc.

Control Theory
Control theory is the second application field, specifically in the analysis of system stability and controller design. In this context, the word "system" has a more general meaning and is not always used to describe a system of electricity. For instance, one could use it to monitor changes in the stock market or chemical processes. Furthermore, complex analysis and control theory are also widely applied in robotics.

I should also point out that, contrary to popular belief, very few "real-world applications" include complicated analysis in its "pure" form as it is taught in math classes. One use is the use of complex analysis to assist in the solution of differential equations, which are used to simulate various fascinating phenomena, such as cellular activities in system biology.

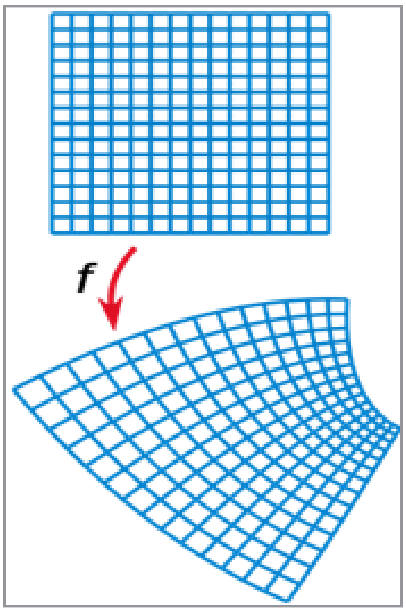
For understanding how a wave propagates and how oscillation occurs and the analysis of waves need a strong knowledge of complex analysis.

It is also use in computer science (image processing)

Conformal Mapping its Types and Applications

Introduction

In mathematics, A function that preserves local direction and angles is the conformal mapping. For instance, the transformation w=f(z) is regarded as conformal if it preserves the size and direction of the angle between oriented curves. From a above observation, it follows that if f is analytical in a domain D and z0D with f'(z0)0, f is conformal at z0. An extended complex plane map onto itself is only regarded as conformal when it undergoes a Mobius transformation. for example, a transformation that results in form f(z)= (az+b)/(cz+d) In this instance, angles are maintained but orientation is held back. Because it only preserves the conformal mapping is a function that maintains local direction and angles. For instance, if the transformation w=f(z) maintains the magnitude and direction of the angle between oriented curves, it is deemed to be conformal [7].

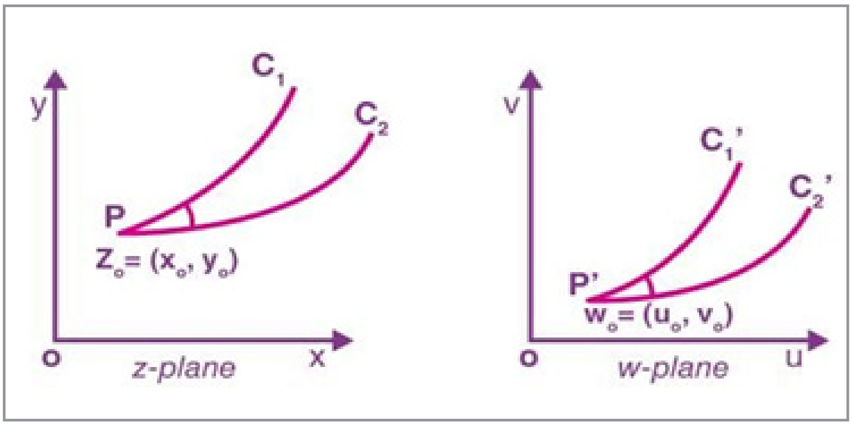


Definitions

Conformal Mapping

An angle has same magnitude and same direction. Or Consider the transformations u = u (x, y) and v = v(x, y), which maps a point P(xo, yo) in z-plane to a point P(uo, vo) in w-plane. Let curves C1 and C2 intersect at point zo = (xo, yo) in z-plane is mapped into curves C1' and C2' in w-plane intersecting at wo = (uo, vo).

If the transformation is such that the angle between C1 and C2 at zois equivalent to angle between in both sense and magnitude. the curve C1' and C2' at wo, it is known as conformal mapping at zo = (xo, yo).



Isogonal Mapping

An Angle have same magnitude but opposite direction then it is called Isogonal mapping.

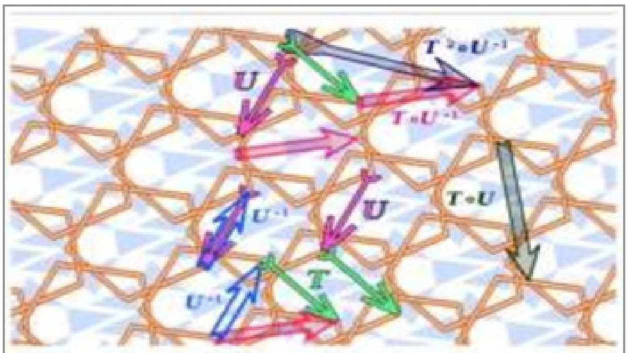
Critical Point

The point z0 is referred to as the critical point function f. f(z0) is analytical at z0D & f'(z0) =0. Critical points are separated because they are zeroes of the analytic function f' [8].

Invariant

A property of an object that does not change when a certain type of transformation is done to it is called an invariant property.

For instance, a triangle's area is invariant with regard to the isometric Euclidean plane.



A wallpaper is invariant under an infinite number of translations, members of a group, of which the operation denoted by ∘ is the function composition.

Bilinear Transformation

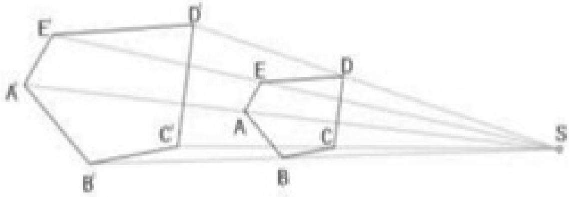
The transformation of the form W=(az+b)/(cz+d)is called a bilinear transformation or fractional or Mobius transformation

Where a,b,c and d are complex contents and ad-bc≠0.

Types of Conformal Mapping

Homothetic Transformation

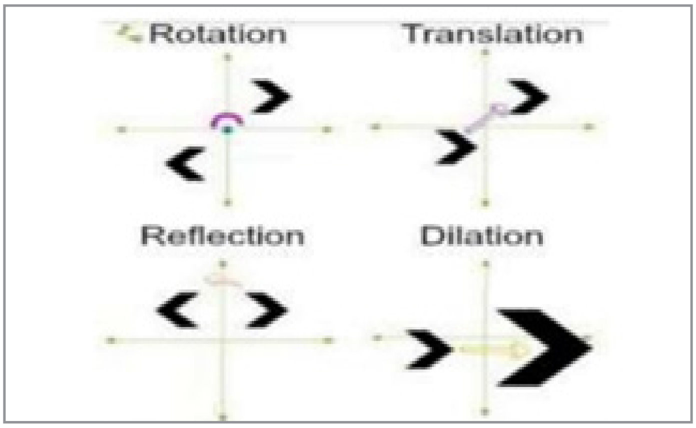
The pointwise invariant line is broken at infinity by this kind of transformation, which is a similarity transformation.



Two similar geometric figures related by a homothetic transformation with respect to a homothetic center S. The angles at corresponding points are the same and have the same sense; for example, the angles ABC and A'B'C' are both clockwise and equal in magnitude.

Isometric Transformation

It is a transition that keeps the original shape in either space or a plane. The initial shape is maintained while the plane or space transitions. It consists of reflection, rotation, translation, and combos of them like Glide, which combines a reflection with a translation.



Special Conformal Transformation

A fractional linear transformation known as "Special Conformal" a transformation that is not affine. This kind of transformation uses multiplicative inversion, a generator of linear fractional transformations. The unique conformal transformation of a sphere consists of an inversion and reflection.

Problems

Find the bilinear transformation that transforms points 1, 0, and -1 of the Z-plane into points i, ∞,1of the W-plane, respectively.

$$\left.\begin{matrix} Z1 = 1 \\ Z2 = 0 \\ Z3 = -1 \end{matrix}\right\} \rightarrow \text{W1=i, W2=∞, W3=1}$$

Here using Cross ratio method

$$\frac{(W-W_1)(W_2-W_3)}{(W_1-W_2)(W_3-W)} = \frac{(Z-Z_1)(Z_2-Z_3)}{(Z_1-Z)(Z_3-Z)}$$

$$= \frac{(W-W_1)W_2(1-\frac{W_3}{W_2})}{W_2(\frac{W_1}{W_2}-1)(W_3-W)} = \frac{(Z-Z_1)(Z_2-Z_3)}{(Z_1-Z)(Z_3-Z)}$$

$$= \frac{(W-i)(1-\frac{1}{\infty})}{(\frac{i}{\infty}-1)(1-W)} = \frac{(Z-1)(0+1)}{(1-0)(-1-Z)}$$

$$= \frac{(W-i)}{(-1)(1-W)} = \frac{(Z-1)}{-(Z+1)}$$

$$= \frac{(W-i)}{(1-W)} = \frac{(Z-1)}{(Z+1)}$$

(W-i) (Z+1)=(Z-1)(1-W)

WZ+W-iZ-i = Z-WZ-1+ZW

2WZ= Z-1+iZ+i

2WZ= (1+i) Z+(i-1)

$$W = \frac{(1+i)Z+(i-1)}{2Z}$$

let f(z) be a bilinear transformation such that f(∞)=1, f(i)=I and f(-i)=-i. find the image of the unit disk {Z E C:|Z|<1} under f(z).

W=f(z)

f(∞)=1→ Z1=∞,W1=1

f(i)=i → Z2=i,W2=i

f(-i)=(-i)→ $Z3 = -i, W3 = -i$

cross ratio method

$$\frac{(W-W_1)(W_2-W_3)}{(W_1-W_2)(W_3-W)} = \frac{(Z-Z_1)(Z_2-Z_3)}{(Z_1-Z)(Z_3-Z)}$$

$$\frac{(W-1)(i+i)}{(1-i)(-i-W)} = \frac{z_1(\frac{z}{Z_1}-1)(Z_2-Z_3)}{z_1(1-\frac{z}{z_1})(Z_3-Z)}$$

$$= \frac{(W-1)}{-(W+i)}\frac{2i}{(1-i)} = \frac{(\frac{z}{\infty}-1)(i+i)}{(1-\frac{i}{\infty})(-i-Z)}$$

$$= \frac{(W-1)}{-(W+i)}\frac{1}{(1-i)} = \frac{-1}{-(Z+i)}$$

$$= \frac{(W-1)}{(W+i)(i-1)} = \frac{1}{(Z+i)}$$

$= (W-1)\ (Z+i)=(W+i)((i-1)$

=(W-1)Z+i(W-1) = iW-W+i^2-i

= (W-1)Z+iW-I = iW-W-1-i

= (W-1)Z = -(W+1)

$Z=-\left(\frac{W+1}{W-1}\right)$

$Z=\frac{1+W}{1-W}$

|Z|< 1

$=\left|\frac{1+W}{1-W}\right| <1$

$= \frac{|1+W|}{|1-W|} <1$

= |1+W|<|1-W|

=|1+u+iv|<|1-u-iv|

$= \sqrt{(1+u)^2+v^2} < \sqrt{(1-u)^2+v^2}$

$= (1+u)^2+v^2 < (1-u)^2+v^2$

= 1+u^2+2u< 1+u^2-2u

= 4u<0

= u< 0

The Given transformation w = f(z) = z^2, whic

by the axes and circles |z| = a and |z| = b, he

check whether it is a conformal mapping [9]

The Given transformation w = f(z) = z2, which lies in the area in the z-first plane's quarter, enclosed by the axes and circles |z| = a and |z| = b, here (a > b > 0). Describe transformation in w-plane and check whether it is a conformal mapping [9].

Solution

Here given transformation is w = z^2

Let z =$re^{i\theta}$ and w = $Re^{i\emptyset}$. Then we have

$Re^{i\emptyset} = r^2e^{i2\theta}$ that is,

R = r^2 and ϕ = 2θ

Now, the circle |z| = r = a, 0 ≤ θ ≤ π/2 The modified shape is a semicircle in the z-plane.

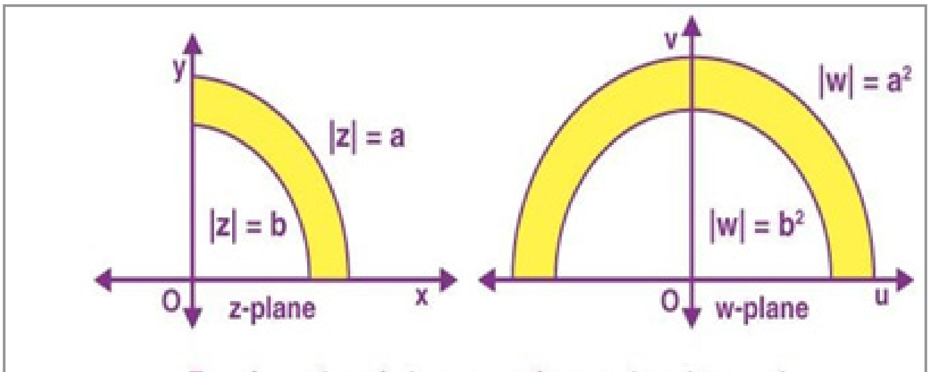
|w| = R = a^2, 0 ≤ ϕ ≤ π (since R = r^2 and ϕ = 2θ) in the w-plane.

Similarly, the quadrant |z| = r = b, 0 ≤ θ ≤ π/2 is turned into a semicircle in the z-plane.

|w| = R = b^2, 0 ≤ ϕ ≤ π (since R = r^2 and ϕ = 2θ) in w-plane.

Similarly, the quadrant |z| = r = b, 0 ≤ θ ≤ π/2 in the z-plane is transformed into a semicircle

|w| = R = b^2, 0 ≤ ϕ ≤ π (since R = r^2 and ϕ = 2θ) in w-plane



Transformation of given curve from z-plane into w-plane

This demonstrates how, as shown in the picture, the annular the annular region between |w| = a2 and |w| = b2 in the upper half-plane of the w-plane changes into the region between the circles |z| = a and |z| = b in the first quadrant of the z-plane.

Let's verify the given transformation's conformity.

Differentiating both sides of w = z2 w.r.t z,

Then
$dw/dz = 2z \neq 0$ For every value of z in the specified region
Thus, the given transformation is a conformal mapping.

Applications of Conformal Mapping
Conformal mappings are a crucial tool for complex analysis and have numerous uses in a variety of physical contexts. Conformal mappings have the advantage that shape distortion can be reduced to the required level by keeping the map's circumference short enough, making them extremely useful for solving difficulties in stereographic projection and cartography.

In engineering and physics, conformal mappings which have awkward geometries but can be modelled as functions of a conformal variable are frequently used. By choosing an analyst can convert the troublesome geometry into one that is substantially more useful by using the right mapping.

When a conformal map transforms a function into a different plane domain and the function is harmonic, that is, when it satisfies Laplace's equation 2f, the transformation is also harmonic. Any genus have zero surface may be mapped conformally into a sphere and local region on a disc, making the application of conformal mapping for brain surface mapping viable [10-17]


Reconstruction of brain onto sphere

Conclusion
In this paper I covered all basic concepts complex analysis and complex numbers. it is a wide area with more applications to real life which helps the researchers to get more ideas to manage the problems in the rea life situation. It has many applications in physics and engineering. In conformal mapping Conformal mapping Despite the fact that the fundamentals There are many practical uses for conformal mapping since it preserves the local angle and shape and keeps harmonic potential mappings harmonic.

Because of these characteristics, conformal mapping is useful in challenging scenarios like genus zero conformal mapping challenges. conformal mapping approach, however, the limited to problems that can be reduced to two dimensions and those with a lot of symmetry. When the symmetry is disturbed, using this method is frequently difficult.

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