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Is the Observational Dark Energy Universe Completely a Coincidence?

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Abstract

In this article, we propose a new cosmological model called 'Fractal Cosmology' based on two pos- tulates that gives a 'not really' answer to the question in the title: At any epoch of the universe, for an arbitrary local observer living well below the scale of Hubble horizon, the observational universe centered on this observer appears to be accelerated expanding. The anthropic principle is thus un- necessary for our current observation of an accelerated expanding universe. We will argue how such a story is qualitatively compatible with the CMB and low-redshift observations on the expansion history. Moreover, Fractal Cosmology implies four characteristic signals that could substantially distinguish it from the standard ΛCDM cosmology: 1) Unlike the prediction in ΛCDM, in Fractal Cosmology, the local Hubble rate will be positively correlated with regional matter overdensities. 2) In a conventional expansion history data analysis of modern cosmology, effectively, dynamical dark energy will show phantom behavior. 3) Over-aged high-redshift astronomical objects/events will generally exist in the observation samples, where 'over-aged' specifically means that the astronomi- cally (local physics) derived event age is longer than the ΛCDM predicted universe age at the event redshift. 4) Astronomical events with a characteristic time, for example the type Ia supernovae light curves, are subject to a growing characteristic time scattering (variance) with their redshifts, even after being modulated by the (1 + z) factor expected in standard cosmology; On the contrary, in for example ΛCDM, no known effect would lead to such a redshift-dependent trend of the characteristic time variance of the same type of events. Each of those four signals has either inconclusively shown some hints in recent observation or is feasible to be tested with current and near-future available data.

Keywords: Geopolymer Concrete, Coconut Fiber, Marble Powder, Mechanical Properties, Durability, Alternative Aggregates

Introduction

When asked why we happen to be living in an epoch of the universe where the negative pressure dark energy is taking the majority, 70% of the cosmic fluid, the answer is usually the anthropic principle [1], i.e. a civilized observer needs to be born in an environment where the dense structure of the universe is diluted by dark energy. However, many find this explanation unsatisfactory- tory due to its arbitrariness and lack of further testable implications. In this article, we will provide an alternative solution to this question, which appears to be less human-centric, and point to observational implications that can be tested in the near future.

The article is organized as follows. In Section II, we will re-explain the Einstein equation as a conservation law of the 4D spacetime substance volume, which naturally incorporates a positive metric term. In Section III, we will try to connect the obtained positive metric term to the observational 'dark energy', or to be more specific, the accelerated expansion reality of our universe. We will also discuss how both low and high redshift observed expansion history of our universe can be accommodated in the theorems proposed in this article. In section IV, we will compare the 'Fractal Cosmology' proposed in this article with other theoretical candidates of beyond standard cosmology, and point out four intriguing implications of the Fractal Cosmology that can be tested against current and near-future astrophysical/cosmological observations.

In this article, we will use the (—+++) signature. We will adopt differential manifold notations mainly from [2], and Raychaudhuri's equation derivation and results from [3].

Einstein Equation with Positive Metric Term

In the original Einstein's equation Λ was an extra term added with no good explanation within classical general relativity, and thus has no prediction on its value. Here, we will reinterpret the Einstein equation as a result of two postulates, which naturally lead to a positive metric term resembling the role of the cosmological constant, but different in ways that will be discussed by the end of this section.

An overview of the story is as follows: we will show how Einstein's equation can be explained as a differential version of the 4D spacetime substance volume conservation law. This conservation rule is applied to the flow along a vector field $F \in X(P)$, where $P = (-\epsilon, \epsilon) \times\Sigma$ is a 4D submanifold in the spacetime, i.e. a spacetime/cosmological patch. The variations on the volume and vector fields is defined in terms of the pullbacks of the diffeomorphism given by the flow of F.

On the other hand, to define the distance on a pseudo-Riemannian manifold with signature (− + ++), we need a metric g: TpM ×TpM → R, which in Cartan's coframe formalism can be expressed as $g = -\alpha t \wedge \alpha t + P\alpha i \wedge \alpha i$, and α are 1-forms. Denoting the base vector field dual to the coframes as Xt, $Xi \in X(P)$, we have them orthonormal with each other, and locally spanning TpP. However, they could all be non-commutative with F, namely, the Lie derivative L_F in the most general case is nonvanishing in every direction.

The above rather mathematical description can be understood in the following physics interpretation: a patch of the universe as a 4D spacetime (sub)manifold is like a fluid cylinder with the 3D space as the cross-sectional area, 1D time duration as the thickness of the fluid cylinder, and the vector field F the flow velocity (we will see later what makes X_t a little bit more special, by choosing a synchronous gauge). See Figure 1 for a schematic illustration.

Imagine that a bug, or a human, flowing in the fluid, or spacetime substance, is trying to construct a metric so that it can measure the physical size of the things around it. It is natural for the bug to take the flow direction as special and build a metric anchored to the flow velocity. However, when flowing in the fluid without any reference outside the flow, it is impossible for the bug to know the true flow velocity, and thus to construct the exactly 'right' metric that has one of the coframes commuting with the flow vector field, the one that does not need any recalibration as it drifts along the flow.

When enforcing the 4D volume conservation law for the variation of scalars and tensors along the vector field F flow, where the 4D volume form is defined conventionally as $\alpha_t \wedge\alpha_1 \wedge\alpha_2 \wedge\alpha_3$, such a conservation law puts constraints on the metric. We will see that locally they take the form of Einstein's equation with a metric term, and the coefficient for the metric term is always positive, but could be a general scalar function instead of a constant.

Just like a fluid cylinder could be stretched or compressed along the direction of flow, so could the scale of time of a patch of the universe be stretched or compressed. This metric term coefficient is an outcome, thus reflecting how much a spacetime patch is stretched or compressed along its flow, i.e. the time direction.

To begin with, we start from a 4D pseudo-Riemannian manifold M, and a vector field $F \in X(M)$. The physical meaning of them is the spacetime manifold our physical world resides in and the flow of the substance of the spacetime 1. Now we take a hypersurface $\Sigma \subset M$, which is compact, integral, well-behaved and nowhere tangent to F, i.e. $Fp \notin Tp\Sigma$ for all $p \in \Sigma$.

The restriction of F on Σ, F|Σ can specify a local coordinate chart that $\partial \partial\tau \sim F$. Denote the 4D manifold $P = (-\epsilon, \epsilon) \times \Sigma$ constructed by vector field straightening that is diffeomorphic (smooth and invertible mapped) to a submanifold $S \in M$ as a patch.

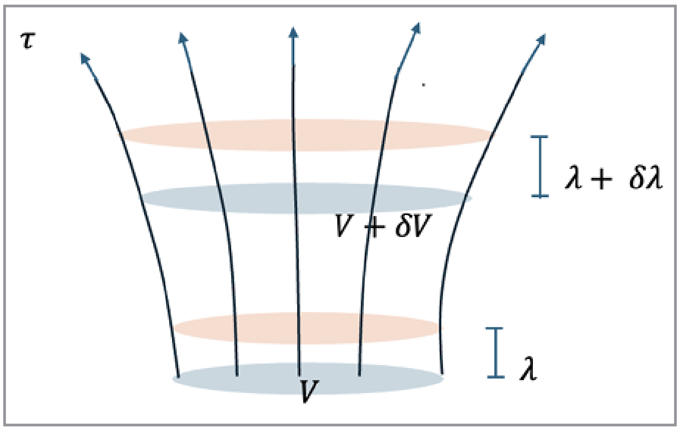

Figure 1: An illustration of time-like congruence. A 4D cylinder confined by red and blue hyperfurfaces λ away from each other has volume U = V λ, and the discussion in section II is focused on how the evolution along a congruence vary this 4D volume $\delta U = V \delta\lambda + \lambda\delta V$.

Now as a standard next step to do any physics in volving distances and volumes on this spacetime patch, we need a metric g: TpP × TpP → R. In general, it can be decomposed into a Cartan's coframe expression: $g = -\alpha t\wedge\alpha t + \Sigma\, \alpha i\wedge\alpha i$, with dual frame vector fields Xt,Xi defined by $\langle\alpha a, X_b\rangle = \delta_{ab}$. In the most general case, $[\partial/\partial\tau, Xt]$ and $[\partial/\partial\tau, Xi]$ could be all non-vanishing. However, we have the freedom to choose synchronous gauge by requiring $[\partial/\partial\tau, Xt] \propto \partial/\partial\tau$, or even requiring $Xt \propto \partial/\partial\tau$. Note that there is nothing physical happening in this step, simply a gauge choice– by rotating the dual frame we are guaranteed to find such a X_a among the four, and we just need to call the coframe dual to it as αt. Namely, the physical spacetime substance flow direction determines the time coordinate of our metric, but only the direction of it. There is no theory that can guarantee the point-to-point identification between the spacetime substance flow vector field and the time-like frame of the metric.

A volume form corresponding to the metric is $\Gamma = \alpha_t \wedge_{\alpha 1} \wedge_{\alpha 2} \wedge_{\alpha 3}$. Let us look into the variation of the element volume of submanifold U ∈ P along the flow of $\frac{\partial}{\partial \tau}$. Lie derivative of the volume form Γ has the property:

$$L_{\frac{\partial}{\partial\tau}}\Gamma = (L_{\frac{\partial}{\partial\tau}}\alpha_t) \wedge \alpha_V + \alpha_t \wedge (L_{\frac{\partial}{\partial\tau}}\alpha_V) \tag{1}$$

where $\alpha_V = \alpha_1 \wedge \alpha_2 \wedge \alpha_3$ is a 3-form, the volume form of the spacial hypersurface.
Now we gradually switch from the differential mani fold language to the languages more familiar to the general relativistic physicists, so that we can use some of the well-established geometrical results and facilitate physical interpretation towards the end. We will denote the Lie derivative $L_{\frac{\partial}{\partial\tau}}$ as variation $\frac{\delta}{\delta\tau}$. We take the 4D volume of an element of the spacetime substance as U = λV, where λ,V are element volumes on $\frac{\delta}{\delta\tau}$ direction 1D sub manifold and on 3D submanifold Σ.

Based on the Lie derivative property acting on the volume forms in equation (1), and the notation introduced above, we can write the variation of the 4D volume along the flow of $\frac{\delta}{\delta\tau}$ as:

$$\delta U = \lambda\delta V + V\delta\lambda \tag{2}$$

Suppose that the 4D volume of the spacetime substance is conserved along its flow:

$$\delta U + \delta Q = 0, \tag{3}$$

where δQ is the in/output of spacetime substance along the flow to the element volume we are studying:

$$\delta Q = T_{ab}\, \xi a \xi b V \lambda \delta\tau \tag{4}$$

T^{ab} is a current tensor of the spacetime substance, defined by equation (4), and ξa is the covariant notation of the Cartan's frame X_t. We hereby denote the other frames X_i as $\eta_a^{(i)}$.

It is well known that the expansion θ of the cross-sectional area of a flow can be obtained by Raychaudhuri's equation [4]. In our scenario, the variation of expansion gives us the variation of the cross-sectional area of the spacetime substance flow, i.e., the 3D hypersurface volume by δV = Vδθ. We need to find the similar result for δλ.

Since the volume forms are defined based on the choice of frames $\eta_a^{(i)}$ and ξ_a, following the same arguments in Wald's 9.2 [3], $B_{ab} = \nabla_a \xi_b$ contains the information of the expansion in time and space directions. In the text book, it is assumed that the flow is along the geodesics, because in the conventional discussion, Einstein's equation is a theory given beforehand; thus, the metric and geodesics determined by Einstein's equation can be used in the investigation of Raychaudhuri's equation. How ever, in the setup here, we do not presume Einstein's equation; the metric is an unknown geometrical property to be solved, following the spacetime substance volume conservation rule. So the spacetime substance flow in the derivation here has to be general enough to be any time-like vector field. We will see, towards the end of this section, how the geodesic congruence always saturates to the spacetime substance flow after long enough time, flow-related τ or time-frame-related t.

In the most general case, assuming that a synchronous gauge can be adopted, we can decompose Bab into the textbook expression with one extra time-like term:

$$B_{ab} = -\alpha\xi_a\xi_b + \frac{1}{3}\theta h_{ab} + \sigma_{ab} + \omega_{ab} \tag{5}$$

where $h_{ab} = g_{ab}+\xi_a\xi_b$ is the spacial metric, characterized by h^{ab} ξa = 0 for the time direction frame ξa.

Contracting the above equation with ξa, we get $\xi^a \nabla_a \xi_b = {}_\alpha\xi_b$. Further contraction gives us the result $\xi^a\nabla_a(\xi^b\xi_b) = 2\alpha$.

This result implies that the connection ∇_agbc does not vanish on arbitrary vector fields in our setup. gab, thus the normalization of the time frame, is drifting along the flow. When requiring α to be perturbative order, it will only introduce effects in the next-to-the-leading or der term in the investigation on the variation along the $\frac{\partial}{\partial\tau}$ flow. In the physical sense, it represents the extent to which a bug in the flowing fluid described previously fails to calibrate its metric instantaneously to the flow velocity to cancel the stretching/compression of the scale along the flow.

Because in our initial setup **P** is obtained by vector field straightening, the foliation structure is guaranteed to be available. We can thus further require the flow direction $\frac{\partial}{\partial\tau}$ to be orthogonal to Σ, and this will put con straints on the metric (and connections) through Frobe nius's theorem[3], that the antisymmetric term $\omega_{ab} = 0$ vanishes.
Recall that we chose the gauge $X_t \propto \frac{\partial}{\partial\tau}$. Thus, the variation along the spacetime substance flow $F \sim \frac{\partial}{\partial t}$ is proportional to

$$\frac{\delta}{\delta\tau}B_{ab} \propto \xi^c\nabla_c B_{ab} = \xi^c\nabla_c\nabla_a\xi_b \tag{6}$$

$$= \xi^c\nabla_a\nabla_c\xi_b + R_{cab}{}^d\xi^c\xi_d \tag{7}$$

$$= \nabla_a(\xi^c\nabla_c\xi_b) - (\nabla_a\xi^c)(\nabla_c\xi_b) + R_{cab}{}^d\xi^c\xi_d \tag{8}$$

$$= \xi_b\nabla_a\alpha + \alpha B_{ab} - B_a{}^c B_{cb} + R_{cab}{}^d\xi^c\xi_d \tag{9}$$

Contracting equation (6, 9) with h^{ab}, we can get the famous Raychaudhuri's equation:

$$\frac{\delta\theta}{\delta\tau} \propto \xi^a\nabla_a\theta = \alpha\theta - \frac{1}{3}\theta^2 - \sigma^{ab}\sigma_{ab} - R_{ab}\xi^a\xi^b + R_{cabd}\xi^c\xi^d\xi^a\xi^b \tag{10}$$

Because R_{cabd} has antisymmetry, the last term goes to zero.
Contracting equation (6) with ξaξb,

$$\xi^a\xi^b\xi^c\nabla_c\nabla_a\xi_b = -\xi^a\xi^c(\nabla_c\xi^b)(\nabla_a\xi_b) = -\xi^a\xi^c B_c{}^b B_{ab} \tag{11}$$

which cancels the B2 term in equation (9), thus

$$\xi^a\xi^b\xi_b\nabla_a\alpha = -\alpha B_{ab}\xi^a\xi^b \tag{12}$$

$$\frac{\delta\alpha}{\delta\tau} \propto \xi^a\nabla_a\alpha = -\alpha^2 \tag{13}$$

Equation (10) and (13) are proportional to $\frac{\delta\theta}{\delta\tau}$ and $\frac{\delta\alpha}{\delta\tau}$ with the same factor $\frac{\delta t}{\delta\tau}$, and as the variation of the expansion factor on the frame, they are related to the 3D spacial and 1D time-dimension volume variations through δV = Vδθ and δλ = λδα by the fractional expansion of the corresponding frame.

δα =−α2δτ is easy to see from equation (13). For δθ, the first term on the right-hand side of equation (10) gives an exponentially diverging or decaying mode. Dropping the second-order terms, we get:

$$\delta\theta = -R_{ab}\xi^a\xi^b\delta\tau \tag{14}$$

Those dropped terms contribute partially to the 'back reaction' term in Buchert's gauge [5]. A cosmological model, Timescape Cosmology, based on Buchert's discussion on the backreaction term, resembles Fractal Cosmology, based on the 4D spacetime substance volume conservation theorem in this article, in many aspects. We will discuss them in the section IV.

Substituting those variations of the volumes back into equation (3), we get:

$$-\lambda V R_{ab}\xi^a\xi^b\delta\tau - \lambda V\alpha^2\delta\tau + T^{ab}\xi_a\xi_b V\lambda\delta\tau = 0 \tag{15}$$

We can expand the second term into the contraction of a covariant tensor with time direction frames using $-1 = g^{ab}\xi_a\xi_b$. Although we have noticed that this nor malization varies along the $\frac{\partial}{\partial\tau}$ flow, at leading order we can still approximately adopt this relationship. Thus, for an arbitrary time-like frame vector ξa, the scalar equation (15) gives rise to the covariant tensor equation:

$$T^{ab} = R^{ab} - \alpha^2 g^{ab} \tag{16}$$

It seems like the Einstein equation that we are familiar with, but not exactly. We have made no statement about the spacetime substance current tensor Tab by far, and it needs a dressing on its formalism to be related to the energy-momentum tensor. As a derivation from the skew-symmetry property of general volume forms, Bianchi's identity holds for the Ricci curvature in equation (16) regardless of the slight drifting of the metric $\nabla_a g_{bc} \neq 0$ mentioned before. According to the Bianchi identity, Tab is not conserved on U:

$$\nabla_a T^a{}_b = \nabla a R^a{}_b - 2\alpha\nabla b^\alpha \tag{17}$$
$$= \nabla_b R - 2\alpha\nabla_b\alpha \tag{18}$$

However, rewriting the T^{ab} in equation (16) into T^{ab} , where

$$\tilde{T}^{ab} \equiv T^{ab} - \frac{1}{2}Tg^{ab}, \tag{19}$$

we get

$$R^{ab} - \frac{1}{2}Rg^{ab} + \alpha^2 g^{ab} = \tilde{T}^{ab} \tag{20}$$

When α is constant throughout P, this equation takes the exact form of Einstein's equation with a positive cosmological constant. By far, we have 'derived' the Ein stein's equation, at least something taking its form, from only two postulates.

1. Our physics lives on a 4D pseudo-Riemannian manifold. The flow of 4D space time substance can be described by an arbitrary vector field living on this manifold.
2. The 4D spacetime substance volume is con served along its flow.

The metric, thus the corresponding volume's definition, has the basic features in the differential manifold context, to ensure the general assumptions of a well behaved physics system, such as smoothness and local Euclidean. Some of the conventional thoughts in vanilla general relativity, such as the absolutely non-drifting metric and geodesics taken for granted before-hand, have to be loosened. Lastly, the perturbative expansions in the above derivation assume the variation of the volumes, shears, distortions, and curvature to be small on a well behaved spacetime manifold patch that we described at the beginning, with no singularities or other unnatural, sophisticated structures.

Now we proceed to interpret the physics implied by equation (20) whose derivation so far has been highly geometrical. By comparing equation (20) with the original Einstein's equation,

$$R^{ab} - \frac{1}{2}Rg^{ab} + \Lambda g^{ab} = \tilde{T}^{ab}, \tag{21}$$

it seems that ˜Tab takes the role of an energy-momentum tensor. Taking the dual of $\tilde{T}^{ab}$, it goes back to $T^{ab} = \tilde{T}^{ab} - \frac{1}{2}\tilde{T}g^{ab}$. Recall that the current tensor T^{ab} was originally introduced in this article in equation (4), to describe the in/output of the spacetime substance to the element volume we are studying. Combining all those intriguing hints, we can conclude a new perspetive on the concept of 'matter' that has been standing in the center of physics research in the past thousands of years:
The concept 'Matter' in the physics world is the current of spacetime substance that we sub consciously identify with the energy conservation. Our early infancy (3-5 months) cognitive devel opment of the 'object permanence' [6] automated this process.

One important reason why we can make such a statement is that equation (16) to (20) only took a rewrite to separate a part of the degrees of freedom, and the two equations are mathematically equivalent. Through the perspective above, we are suggesting that the reason why the original Einstein's equation has come forth before equation 16 is because we tend to explore the physical world on a permanent matter (conserved energy) based view. Moreover, the reason that such a cognitive strategy is developed so early and so widely among different species (for example, cats [7]) is probably because equation (20) with vanishing δα applies in almost every earthly scenario, thus becoming a ubiquitous feature trained out from evolution of the neural systems.

To see why δα is negligible in any earthly scenarios but could play a non-negligible cosmological role, we can use the earlier introduced analogy of the spacetime substance flow and a regular fluid flow, for example, the water flowing in a riverbed. The α term regulates the rescaling of the whole frame, thus the metric, due to the stretching/compressing of the fluid cylinder thickness along the flow. Under the volume conservation law, this effect is only significant when the cross-sectional area variation is significant. In the flowing river case, that corresponds to flowing from a branch to a mainstream or the inverse. In the flowing spacetime substance case, that corresponds to the scenario where the variation of the expansion θ, which on the first order is proportional to the Ricci curvature R^{ab}, becomes significant; or the scenario where we are studying the variation of α in an extremely large area, for example a Hubble-sized patch or even larger. The natural physics environment on the earth is known to be extremely gravitationally weak, in contrast to the strong (in natural units) gravity case that only becomes relevant in astronomical and cosmological discussions, and of course, is very small-scale compared to the cosmological scale.

Back to equation (20), one intriguing feature we can see is that the coefficient $\alpha2(x)$ of the metric term is always positive. In the expression here, we explicitly point out that $\alpha(x)$ could be a function of the 4D spacetime coordinate, as up to this point, all our discussion hap pens on an element volume on a patch P on the space time manifold M. We will discuss the space and phase space average on it, which is more relevant for a practical observational universe scenario, in the next section. In any case, one can see the similarity and difference between $\alpha^2(x)g^{ab}$ term and the Einstein's constant term Λgab: the coefficient for the former is a scalar function, while the latter is a constant; Thus, the former could spoil the conservation of energy-momentum tensor by $\nabla a\ \tilde{}Tab = 2\alpha\nabla b\alpha$, while the latter nicely respects the conservation of energy-momentum tensor $\nabla a\ \tilde{}Tab\ = 0$ one of the reasons for it to be introduced by Einstein originally; Lastly, the former coefficient $\alpha2(x)$ is always positive, while the latter Λ could be positive, zero, or negative, corresponding to de Sitter, flat, and Anti de Sitter universe.

In the past decades, it has been almost certain that our observational universe is de Sitter, i.e., when testing expansion history data in the framework of the cosmological constant, we need a positive Λ. Considering this observational reality, the automatic positivity of the metric term coefficient in equation (20) is quite encouraging. How ever, the breakdown of energy-momentum conservation is not so welcoming, although we briefly discussed before how conservation of energy-momentum tensor could originate from cognitive adaptation in the earth environment instead of a more fundamental physics rule. In the theoretical framework in this article, the conservation of energy momentum tensor is only a special case secondary result of equation (20) when α is approximately constant throughout the physics system in question, and a consequence of the contracted Bianchi identity. Thus, any physical consequence of the breakdown of energy momentum conservation is only expected to show up in scenarios like the expansion history of the universe or a strong gravity environment such as near a black hole horizon, where a non-negligible gradient of α is present on either extremely large-scale or strongly curved spacetime patches. The violation of energy-momentum conservation in such regimes is irrelevant to, and thus does not ruin, the gravitational dynamics of Newtonian systems.

In an idealized case, let us imagine what will happen if a spacetime patch P is not 'small' but can flow to large τ, even asymptotic infinity, along the $\partial\ \partial\tau$ direction. When approximating $\delta t\ \delta\tau$ with a constant, the integral of equation (13) tells us $\alpha \sim 1\ \tau$, which goes to zero as $\tau \to 0$. Even taking the variation of $\delta t\ \delta\tau$ into account, as long as it does not change sign, the trend of vanish ing α with $\tau \to \infty$ would still apply. So it seems that after long enough time, the congruence of geodesics con verges to the spacetime substance flow, as we expected. On the other hand, if we regard the integrated τ as the lifetime of a patch of observational universe, under the approximately constant assumption of $\delta t\ \delta\tau$, equation (20) together with $\alpha \sim 1\ \tau$ suggests that $\sqrt{\Lambda} \sim |\alpha| \sim 1\ \tau$. This is exactly the case that we have found out about our own observational universe: the content of the cosmic fluid is mainly cosmological constant or something behaving like it at low redshift, and as a result of this fact, the lifetime of our universe is the same order of the inverse of the cosmological constant square root.

Indeed, if one has not noticed this, the widely cited values of cosmological constant Λ, Hubble constant, universe lifetime, and many other ways of reformulating the first (zeroth) order expansion rate of the universe, are all roughly the same degree of freedom extracted from redshifts and distances data. All attempts to jump out of this framework, giving an alternative quantitative de scription of the above physics, for example, the calculation of Λ as the vacuum energy in quantum field theory, have been a failure. The argument in this section is another bold and rare endeavor in the literature to reason why $\Lambda \sim H0$ might not be a coincidence. We will also give an independent estimator of the expansion rate, or $\alpha2(x)$, or approximately Λ, from the time domain astronomical observation in Section IV.

Before we conclude this section, it is worth noting that the theorem based on the two postulates here is much motivated by the thermodynamics explanation of the Ein stein's equation by Ted Jacobson [8]. Instead of looking into the black hole case where one of the space dimensions is highly compressed, here we are studying the less special, well-behaved 4D spacetime submanifolds, and the conservation of energy $dQ = TdS$ in [8] is substituted by the conservation of 4D volume proposition $dU + dQ = 0$. The fundamental arguments are similar, that the Einstein's equation can be understood as how spacetime distortion is driven by the flow of thermal energy/4D volume current tensor, under the constraint of energy/volume conservation law.

Cosmological Effect

Now that we have Einstein's equation with a positive metric term, we want to see if we can connect it to the observational 'dark energy'.

In the field of observational cosmology, dark energy has been a placeholder for the unexplained fact that we measure an acceleratingly expanding universe around us. The astrophysical objects at distances far enough to be in the 'Hubble flow' run away from us with 'increasing speed'. Such an accelerated expansion reality is fairly homogeneous, and the negative pressure portion of the energy density of the cosmic fluid has an equation of state very close to $w \equiv p\ \rho \sim -1$ [9–11]. Those are about the uncontroversial facts about what we know of the observational dark energy so far. Dark energy has no observed perturbative effects so far.

Hence, most of the time, it is only discussed in the background level cosmology, through its effects on the Friedmann equations, although some recent research has already put efforts into modeling the perturbation of the dark energy fluid and its effect on the large-scale structure. In the spirit of focusing on explaining what has already been confirmed or hinted by the real data in the discussion, we will concentrate on the background level cosmology in this article. Besides, we lack sufficient theoretical tools to investigate the perturbative level cosmology, for example, the large-scale structure evolution, in the current premature status of the theorems proposed in this article.

We denote the average over the space as $\bar{x}$ and the expectation value over the full phase space as $\langle x \rangle$. The two Friedmann's equations are the time and space com ponents of the Einstein's equation in a space-averaged gauge:

$$\langle \bar{R} \rangle^{ab} - \frac{1}{2}\langle R \rangle g^{ab} + \langle \bar{\alpha}^2 \rangle g^{ab} = \langle \bar{\bar{T}} \rangle^{ab} \quad (22)$$

Assuming that our observable universe patch has evolved 'long enough' time, that the geodesics are sat urated to the averaged mainstream of the spacetime sub stance on the patch, the leading background order $\langle \bar{\alpha} \rangle$ vanishes. This could be regarded as a default calibra tion for any small, much below Hubble-scale observer in the universe, that their frames have always been tuned to the spacetime substance flow on their cosmological, Hubble-scale patch.

Thus, the metric term in equation (22) $\langle \bar{\alpha}^2 \rangle g_{ab}$ is effectively $\sigma^2(\bar{\alpha}) g_{ab}$, where the variance of α, $\sigma^2(\bar{\alpha}) = \langle \bar{\alpha}^2 \rangle - \langle \bar{\alpha} \rangle^2$. Comparing equation (22) with the original Einstein's equation with a cosmological constant (21), we see that $\sigma^2(\bar{\alpha}) g_{ab} \sim \Lambda g_{ab}$, but with a possibly spacetime dependent coeffecient, thus breakdown of energy momentum conservation in the cases discussed before: at cosmological scale or extremely strong gravity. Hence, it could potentially have implications for the baryogenesis problem.

Here, let us focus on how to accommodate the theoretical story so far into the observational reality of the relatively stable scaling of $\sigma^2(\bar{\alpha})$ with the scale factor a and the homogeneity of an accelerated expanding universe.

We provide an educated guess for the zeroth-order scaling of the variance of spacetime substance flow speed, matching the background-order expansion history of the universe derived from physical dimension analysis as follows. In natural units, we have the dimensions of energy, spatial distance, and time as $[\epsilon] = 1, [d] = 1, [t] = 1$. The physical dimension of Einstein's equation is −2, so is the cosmological constant $[\Lambda] = -2$, and they are consistent with $[\alpha] = -1$ from its definition in Section II as the time derivative of a dimensionless geometric property.

Recall that the physical meaning of α is the stretching of the time frame along the spacetime substance flow.

We discussed how this effect should be roughly on the order of the spacetime curvature. Thus, in a matter dominated universe, without resolving the details of the dynamics happening at smaller scales, from physical dimension analysis, we deduce that:

$$\sigma^2(\bar{\alpha}) \approx d_F \bar{\rho}_m \quad (23)$$

The dimension of the above equation is −2, where d_F should be some dimensionless universal constant that does not care about the detailed spacetime substance dynamics happening on a patch, as we have already operated the phase-space average in the calculation of σ(α).

A perfect candidate for the d_F coefficient is the fractal dimension of the Poission-like distribution of matter in our universe, which has a measured value of 2.4 as demonstrated in [12]. The fractal dimension is dimensionless, insensitive to local dynamics, and reflects how an isotropic physical quantity is equally partitioned into equivalent physical dimensions.

Next, let us investigate whether this guess from the physical dimension analysis works quantitatively. In our local universe, we have the measured values in $\Omega_\Lambda \approx 0.7$, $\Omega_m \approx 0.3$ and $d_F \approx 2.4$. It seems that these numbers perfectly fit the equation (23), with $0.7 \approx 2.4 \times 0.3$. 2.

Equation (23) seems fine for the low-redshift area $z <\sim 1$ in the sense that it does not drastically disobey any observational facts. However, the problem arises when we consider not only the low redshift, but also the high redshift expansion history. If equation (23) applies to the spatial averaged matter density regardless of the scale, deep into high redshifts, then we would not obtain the right expansion history confirmed by the current data, specifically, it would spoil the nice fitting of the CMB power spectrum to the ΛCDM predictions.

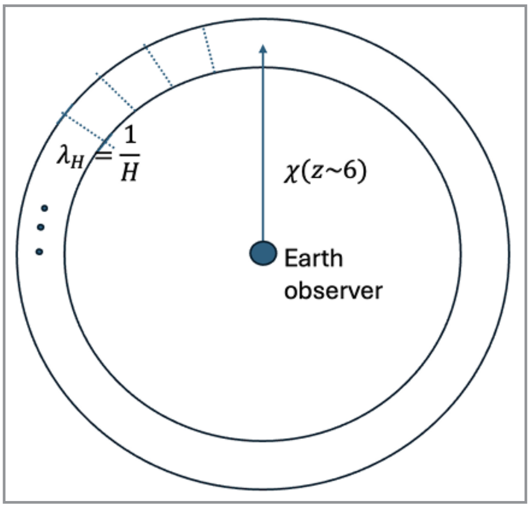

Figure 2: All the Currently Observable Galaxies Around Certain Redshift, say $z \sim 6$, Reside on a Shell with comoving distance χ away from us. A shell at χ(a) accommodates many Hubble horizon $\lambda_H(a) = 1/H(a)$ sized patches, namely Hubble bubbles

The remedy here is to add constraints on the scale to which the proportionality between averaged density of matter and $\sigma^2(\bar{\alpha})$ is maintained. The variance $\sigma^2(\bar{\alpha})$ will only trace the averaged matter density up to the Hubble scale, whichever value it is at the corresponding redshift, because the physics beyond this scale is causally disconnected.

Assuming that the distribution of α is not correlated beyond

Hubble scale, then we can regard the calculation of the variance of ¯α in a large region consisting of several Hubble-sized patches as carrying out redraws on the same distribution, thus subject to a suppression by the factor 1/N, where N being the number of Hubble-sized patches in the whole region. For example in figure 2, on a shell of comoving distance χ(a) away from us, there are number of N =Vshell/VHubble causally disconnected patches that follow roughly the same distribution of ¯α in their local Hubble volume 3. Hence, the variance of ¯α on the whole shell, corresponding to a specific redshift σ2(¯α)(χ(z)) is suppressed by 1/N.
In the regime of χ(a) >> 1/H(a) where the approx imations discussed above can be adopted, we can write the variance σ2(¯α) as a function of the scale factor or redshift, Λ(a):

$$\Lambda(a) \equiv \sigma^2(\bar{\alpha}) \approx \frac{\sigma^2(\alpha)}{N} \tag{24}$$

$$\approx \frac{d_F \rho_m(a)}{V_{\text{shell}}(\chi(a))/V_{\text{Hubble}}(a)} \tag{25}$$

$$= \frac{d_F \rho_m^0 a^{-3}}{4\pi\chi^2\lambda_H/(4\pi/3\lambda_H^3)} \tag{26}$$

$$= \frac{d_F \rho_m^0 a^{-3}}{3\chi^2/\lambda_H^2} \tag{27}$$

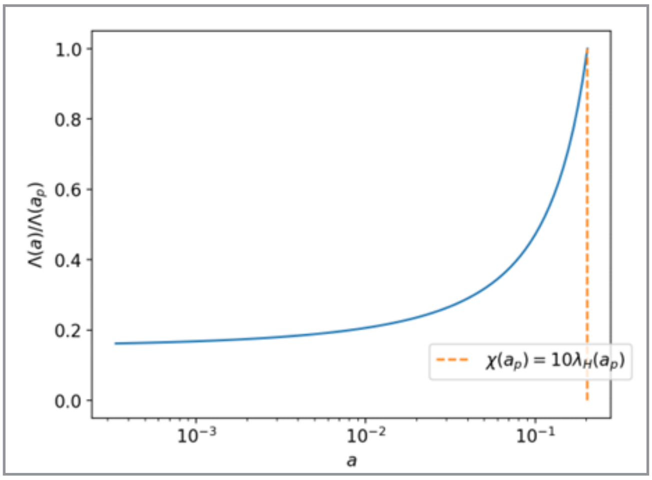

Figure 3: The scale factor dependent shape of Λ(a) ≡ σ2(α¯) as a function of scale factor a. In the regime where far-field approximation holds, χ >> λH , the evolution of Λ(a) is sup- pressed by 1/N factor towards high redshift, as required by the CMB observation.

We can solve for the shape of Λ(a) by taking deriva tive of the integral equation (27) in the region where Nχ/λH = 10 is large enough to apply the 1/N suppres sion approximation, corresponding to a = 0.2, or z = 4, regardless of the value of H0. Denoting $X(a) = \frac{\Lambda(a)}{\Lambda_0}$, where Λ0 is the value of Λ(a) at redshift 0, from equa tion (27) we get:

$$X'(a) = 2\sqrt{3}\frac{X^{3/2}(a)}{a^{1/2}} - 3\frac{X(a)}{a} - 2E'(a)E^{-1}(a)X(a) \tag{28}$$

where $E(a) = H(a)/H_0 = \sqrt{\Omega_\Lambda(a) + \Omega_m a^{-3}}$, , and we have used the relationships $\chi = \int_a^1 \frac{1}{a'^2 H(a)} da'$ and λH = 1/H.

On the other hand, in the regime χ(a) << λH, roughly z < z< 4.0 needs more dedicated modeling, which we leave for future work.

Figure 3 shows the scale factor dependency of Λ(a) ≡ σ2(¯α) in the far field z > 4 by solving the ordinary differential equation (28). It is not scaling up as a−3 with the matter density, instead decreasing to a smaller platform, thus agreeing with the observation that the dark energy was subdominant in the early universe. The decreasing rate varies with the trial boundary value of Λ(ap), but the trend remains stable with reasonable trial values that confine X(ap) between 0 and 1. In a sentence, regard less of the initial/boundary fraction of dark energy at far-field redshift ap, a general conclusion is that the 1/N suppression dominates, thus diluting out the metric term σ2(¯α)gab when we look out toward decreasing scale factor a.

Discussion

Fractal Cosmology and Comparison with Other Cosmological Models

One implication of the explanation using the space time substance flow variance for the 'negative pressure' domination of the regional cosmic fluid is that our 'currently' accelerated expanding universe is not special in space and time of the universe. Anthropic principle is not needed in the picture implied by the theorems proposed in Section II, which we hereby name as 'Fractal Cosmology'. Because at any redshift, an observer living well below the Hubble horizon scale would see an accelerated expanding universe around them. Inside an arbitrary Hubble patch, or bubble, at a higher redshift, resides another accelerated expanding universe. When the residents in that Hubble bubble look outward from a universe centered on themselves, they would see a simi lar chronicle of the evolving universe like ours: inflation, primordial plasma, recombination, reionization and large-scale structure formation, and 'currently' accelerated expansion.

When we received signals from their cosmological patch, we only took random snapshots of what had happened and what would happen on that patch of spacetime with no chronicle. Despite all the seemingly wild theorems in this article, one should think twice about the statement of 'we are reconstructing the evolution history of our universe by tracing down to those high-redshift objects', because those objects that we are observing now are light-like connected to us in spacetime, not timelike. This fact suggests that none of those astronomical objects we are observing today would evolve to the current same-time hypersurface in any frame transformation, so they do not have to be the precursors of a similar type of astronomical objects in our patch of the observational universe. An absolute chronicle of the universe since 'The Big Bang' loses its meaning in this picture, when radially tracing far enough along light-like curves, every observer reaches their own 'Big Bangs', and the scale factor could play the role of time coordinate for any local (below Hubble size) observer in the universe.

Forward-time, or the flow direction of the spacetime substance, might be a concept as trivial as the downward direction of the universe. When zooming out to large enough scale, the universe manifests itself as a series of indefinitely unfolding self-similar structures at hierarchical scales, only pivoted at a local observer's scale when a chronological story needs to be told. The statement that our universe is currently 14 billion years old gives the Earth a special position in the time dimension of the universe. If the Earth is not special in the space of the universe, why should it be in time?

The idea of such a Fractal Cosmology conceptually inherits some genes from the Steady State theory of cosmology. They both imply that the universe does not have a global beginning or ending, and looks quite the same at any epoch. However, unlike the Steady State theory, which stresses the unchanging of the universe over time, the Fractal Cosmology stresses the self-similarity of the universe over spacetime, which is a more radical application of the postulates of relativity.

On the other hand, one might have noticed that Fractal Cosmology echoes many aspects of the Conformal Cyclic Cosmology (CCC) [13, 14], especially the smooth joint of the conformal infinity of one patch and the Big Bang of another. The additional spice in Fractal Cosmology compared to the CCC is its suggestion of patch-wise dis synchronization of the Hubble-scale regions, which leads to potentially more characteristic observational signals that could be tested with recent and near-future data. We will discuss those signals in Section IVB.

The property of $\sigma 2(\bar{}\alpha) \sim \Lambda$ tracking the value of back ground average matter density in equation (23) is similar to the proposal in ever-present Lambda models [15, 16]. However, as we have discussed, such a positive correlation between these two physical quantities can only be valid up to a limited scale. Otherwise, such an effect will heavily violate the expansion history suggested by real data at high redshift. We provided a possible remedy to this problem in section III.

Lastly, Timescape Cosmology [17] might be the cosmological model that shares the most ideas with Fractal Cosmology in the current literature. It explicitly introduced the concept of 'volume-averaged time' and 'lapse', which is the multiplicative difference between the voids and walls area time frames. In the current development of this model, it seems that the observational tests still focus on its implications on the background-level expansion history. Recently, a supernova Hubble diagram data analysis gave a concrete constraint on a major parameter in the Timescape Cosmology, the void fraction [18]. In the last subsection IVB4 of this article, we will discuss what kind of time-domain signals could provide another venue to probe the physics that distinguishes this type of cosmology from other candidates.

We also want to note that the Buchert's average gauge [5], the theoretical basis of the timescape cosmology, resembles the patch average view in this article in many ways, and their backreaction term might correspond to the dropped-off higher order geometrical variations in Section II.

Observational Tests

So far, most of the extended cosmological models focus on their observational implications in the background level expansion history and perturbative fields of the matter density and photon temperatures. Corresponding data used to constrain those theoretical predictions are typically the Supernovae light curves, baryonic acoustic oscillations, CMB power spectrum, galaxy clustering's, and weakening's, etc. Due to the complexity of analytical and semi-analytical calculations on the perturbative level cosmology, most of the extended cosmological models are only tested against the expansion history. The danger of doing so is that people might be playing with too few degrees of freedom in the data, with too wide theoretical possibilities.

Specifically, in the heated discussions on the Hubble tension recently, many seemingly different models are actually degenerate on the background-level cosmology. The same amount of information contained in the Hubble diagram is translated into different fancy-named theoretical parameters, which observers cannot distinguish at current or realistically near-future precision. A bigger problem is that, in the maze of transforming the same set of numbers back and forth using different languages in different subfields, we could lose track of the real input and output information of a theory, and make circular argument like the one recently spotted out for the 'Hubble cut-off' in holographic dark energy model [19].

Hence, we will present a discussion incorporating different, independent aspects in observations to scrutinize what kind of signals could be implied by the Fractal Cosmology model proposed in this article. In general, Fractal Cosmology unavoidably introduces variation in the Hubble rate correlated with regional matter overdensity and redshift. Other than that, the most characteristic observational implication of Fractal Cosmology could be the dissynchronization of the observer-dependent cosmological time between different Hubble-sized patches.

The Positive Correlation Between Hubble Rate Variation and the Matter Overdensity

In ΛCDM, or any cosmology with standard Einstein's gravity, it is expected that the variation of the Hubble rate is negatively correlated with the variation of matter overdensity. It can be physically understood as a result of the standard gravitational theory, in the way that the mass particles in a void would be sucked away from the void center by the growing matter density towards the outbound. As a result, the observational Hubble rate $H = \langle v \rangle \langle d \rangle$ measured from the center of the void exceeds the overall average. There are multiple ways to semi analytically derive this negative correlation relationship at the linear perturbation level with or without a cosmological constant [20]. Moreover, this theoretically predicted negative correlation has been validated by multiple N-body simulations carried out by different groups in the literature [21].

Hence, it is important to stress an unusual implication of equation (23), that the metric term, thus the main contributor to the accelerated expansion of the current universe, would be positively correlated with the regionally averaged matter density in Fractal Cosmology. It is substantially different from the standard cosmology prediction described in the first paragraph.

Even though the negative correlation between the Hubble rate and the matter overdensity has been a consensus among cosmologists, especially the N-body simulation experts, the observational confirmation of this pre diction in our real universe has been a blank space. It is by no means an easy task to obtain trustworthy measurements on the Hubble rate centered on a distant location, not to mention the reconstructions of the matter density field at a required precision to test this relationship. The Hubble rate centered on ourselves, the earth, has only come to a precision ∼ 10% not so long ago.

Recently, a real data analysis on the Hubble rate variation and regional matter density variation has been carried out for the first time in the literature, using the density field reconstruction from BOSS DR12 and Supernovae Hubble rate measurement from Pantheon [22]. They have surprisingly found a positive correlation between the local Hubble rate and the reconstructed mat ter density field. Such a counterintuitive result naturally faces many questions from the community and awaits cross-validation analysis carried out by other independent groups. However, the model proposed in this article provides one of the possible physical explanations for such an unexpected result and suggests that such a phenomenon is not strictly forbidden when the accelerated expansion of a cosmological scale patch originates from certain local physics in an extended gravitational theory.

Redshift Evolution of the Dark Energy-like Term

In a sense, the expansion history data analysis involving any tracer and the combination of the reforms of their redshifts and distances is mainly probing the density redshift dependence of a cosmic fluid. Because the theory predictions in those analysis are based on the first Friedmann equation, namely the background-level expansion of that component of Einstein's equation. The constraint on the equation of state w of the fluid is obtained from the assumption of energy-momentum conservation (fluid Euler equation), which we have discussed why it could be fairly safely loosened in the story presented in this article. Under these considerations, we can directly borrow the discussion on the metric term, thus its tt component in Section III, to deduce three features of the Fractal Cosmology when considering the expansion history implied by it under the framework of dynamical dark energy:

1. At low redshift, the metric term behaves much like 10 the cosmological constant term, and takes about the right amount (70%) of the total cosmic fluid given fractal dimension dF ≈ 2.4.
2. When going to higher redshift, at some point, the curve shown in figure 3 suggests that the phan tom point (wde = −1) will be crossed. Here, by 'phantom behavior', we mean that the metric term tt component that can be effectively regarded as 'dark energy' density will appear to be increasing with scale factor, which, under the conservation of energy-momentum assumption in the dynamical dark energy framework, is equivalent to w < −1.
3. Combining the first two implications and assuming a smooth transition of Λ(a) at any redshift, negative wa is likely preferred when analyzing Fractal Cosmology with a w0wa cosmology pipeline.

The above perspectives are compatible with the recent results from DESI [23], which implies that a phantom dark energy is slightly preferred using their latest spectroscopic baryonic acoustic oscillation data.

High-redshift Astronomical Objects with a History Longer Than Our Universe Lifetime

An unusual and characteristic implication by Fractal Cosmology is that the lifetime of the observational universe centered on a civilization living in a galaxy at, say, z = 6, could be longer than 1Gyr, which is the number of the 'universe lifetime' at z = 6 calculated in our time frame assuming a ΛCDM cosmology. As mentioned in Subsection IVA, the light signals that we receive nowadays from high-redshift objects are likely non chronological snapshots drawn from the whole history of their Hubble bubble. The history of such a high redshift astronomical event could be longer than the universe life time as calculated in our time frame, because those astronomical events governed by baryonic physics follow the proper time of their local atomic clocks. As a result, this dissynchronization between different Hubble patches gives longer accretion time for those high redshift supermassive black holes, whose overabundance and overweight have been a concerning confusion in recent high-redshift observations [24]. The discovery of more-than-expected > 109M⊙ supermassive black holes (SMBH) above redshift z > 6 is forcing astrophysicists to look for exotic mechanisms to allow super-Eddington accretion of the black holes, where the Eddington limit is the accretion rate at which the radiation pressure force cancels the gravity. Even with a relatively heavy black hole seed ∼ 100M⊙, the Eddington limit accretion needs at least ∼ 0.8 Gyr to form a SMBH ∼ 109M⊙, and the universe lifetime at redshift 6 based on Big Bang theory is just about enough. Many cosmological approaches to the problem rearrange the expansion history of ΛCDM. In the Fractal Cosmology picture, an observer-dependent proper lifetime of the universe, synchronized to the time frame determined by a local atomic clock, could be an alternative cure.

Similarly, astrophysicists might find that some of the high-redshift galaxies behave older than theory predictions. In recent and upcoming high-redshift astrophysical surveys, represented by JWST [25, 26], these kinds of puzzling early-universe, highly evolved galaxies have already been found, though with arguably inconclusive significance. How galaxies could have formed their stars and quenched the star formation at a stage so early in the universe has already triggered a wide discussion [27–29].

Although currently still troubled by systematics and selection effects, the high-redshift galaxy and SMBH properties, especially the charts on their ages as theoretically predicted by astronomical and baryonic physics, will be crucial for testing the most characteristic implication of Fractal Cosmology, patch-wise cosmological time discynchronization.

What is more, if the observation of a high-redshfit Hubble bubble could contain non-chronological snapshot signals of the full history of their expanding universe, then futuristic astronomical events could also be observed at high redshift as implied by Fractal Cosmology. In the high-redshift observations, we could potentially uncover the past and future of our patch of the observational universe.

Astronomical Event Time Duration Variance Introduced by Dissynchronized Cosmological Clocks

An important new physics that distinguishes Fractal Cosmology from standard cosmology is the dissynchronization of the cosmological time between Hubble-scale patches, or Hubble bubbles. The standard cosmology implicitly assumes a global time frame regardless of the scales or coordinates in spacetime. However, in the picture of Fractal Cosmology, we have discussed how a Hubble-sized patch at any position in the universe could harbor its different but homomorphic cosmological scale evolution history of the local universe. Intuitively, we would expect that the flow of time of the astronomical events happening on a cosmological patch would be anchored to the cosmological time on that patch. Again, one can think of a galaxy or a similarly-sized astronomical object in the spacetime substance bulk of a Hubble sized patch as a leaf flowing in a river. Those smaller sized objects could have peculiar spacetime substance flow, but on the leading order, they should follow their local Hubble patch spacetime substance flow.

In practical observations, we are already equipped with the instruments that can study objects at high redshifts with $z > 1$, objects deep into the Hubble flow, and on other Hubble patches. If the dissynchronization between Hubble patches exists, then it should be reflected in the time-domain signal of those high-redshift astronomical events.

For example, let us consider the light curve of a su pernova. Recently, time dilation has been observed in the supernovae light curves [30]. In standard cosmology, where the cosmological clock is synchronized throughout the whole universe, the time duration of the super nova light curve is predicted to have a time dilation of 1 +z = T(z) T0 , based on a similar argument for the red shifts in textbooks. This prediction is roughly confirmed by the analysis in [30].

There are caveats when one draws parallels between the redshift of photons and the time dilation of a macro scopic event. The former does not probe the local time frame, as the emission of a photon, considering its quan tum nature, could be seen as instantaneous. The red shift of a photon can be derived from the spatial scale factor growth, thus the stretching of the photon wave length without any information needed on the local time frame. On the other hand, the beginning and end of a supernovae event are time-like separated events and are sensitive to the dissynchronization of the local time frame under any gauge.

Let us start our discussion from the standard cosmol ogy case. 1 + z = T(z) T0 holds exactly when the clock of us, the observers, ticks at the same speed as the proper time of the source astronomical object. Taking into ac count the random fluctuation of the time duration of su pernovae light curves due to different environments and other unknown astronomical reasons, measurements on a supernovae light curve time duration at redshift z could be denoted by:

$$T(z) = (1+z)(\bar{T}_0 + \sigma_{\text{int}}) \tag{29}$$

where ¯T0 is the averaged pivot value of the supernovae light curve time duration at z = 0, and σint denotes the uncertainty due to any intrinsic scattering.

If there is any dissynchronization between our, the observer's Hubble patch, and the source galaxy's Hubble patch that needs to be modeled, we can use a factor γ ≡ dτo dτs called lapse to quantify it. Here we borrowed the name lapse from the timescape cosmology [17], which was originally introduced to describe the multiplicative factor between volume-averaged clocks of walls and voids. Fol lowing the argument on how the clock of an astronomical event should primarily anchor to the clock of the cosmological patch it resides on in the first paragraph of this section, the measurement on the time duration will be dressed by this lapse factor T(z)− > γT(z).

We have no means to measure the absolute value of time lapse from observation for a single event. It would be degenerate with the intrinsic scattering of the time duration.

We do, however, have the possibility to statistically test if such a non-trivial (non-unity) time lapse exists or not. The scattering of the time duration of a specific type of astronomical events around the center character istic value determined by its physical process, and estimated by the z = 0 averaged measurement ¯T0, could be decomposed into two uncorrelated uncertainties:

$$T(z) = (1+z)(\bar{T}_0 + \sqrt{\sigma_{\text{int}}^2 + \bar{T}_0\sigma^2(\gamma)}) \tag{30}$$

namely, the uncertainty due to intrinsic scattering and due to patch-wise dissynchronization. Here σ(γ) is the variance of γ, and we used the fact that the characteristic time for an astronomical event ¯T0 on cosmological scale is a small time duration that can substitute dts in the definition of lapse $\gamma \equiv \frac{d\tau_o}{d\tau_s}$.

Now we investigate the modeling of σ(γ). Recall one of the most important results in Section II of this article is Λ(a) ∼ σ2(¯α), and when the saturation of geodesic congruence to the spacetime substance flow happens, α can be effectively interpreted as the acceleration of a patch. Assuming a constant acceleration motion, thus a parabolic trajectory of any distant Hubble patch with respect to our Hubble patch, the relationship between the acceleration and the time lapse of a distant patch becomes

$$\gamma_s = \chi_s \alpha_s \tag{31}$$

When writing down this relationship, we are putting the patch in which a distant galaxy resides in Rindler coordinates, and treating the Earth observer as stationary. 4 Then the time transformation between a Rindler proper time and the stationary observer time gives the above re sult, and χs is the comoving distance of the source patch. Thus,

$$\sigma(\bar{\gamma}_s) = \chi_s \sigma(\bar{\alpha}) \tag{32}$$

And the variance of the supernovae light curve dura tion time modulated by the (1 + z) factor and its local average pivot value, denoted by $W \equiv \frac{T(z)}{\bar{T}_0(1+z)}$, is:

$$\sigma^2 (W) = \tilde{\sigma}_{\text{int}}^2 + \chi^2(z)\sigma^2(\bar{\alpha}) \tag{33}$$

where $\tilde{\sigma}_{\text{int}} = \sigma_{\text{int}}/\bar{T}_0$ (33) is the fractional intrinsic scattering. Equation (33) shows an obvious difference between the signal predicted by Fractal Cosmology and standard cosmology: The variance of the observable W would have redshift dependence in Fractal Cosmology, as a re sult of the cosmological clock dissynchronization, while in standard cosmology σ(W) will only have a redshift independent intrinsic scattering term.

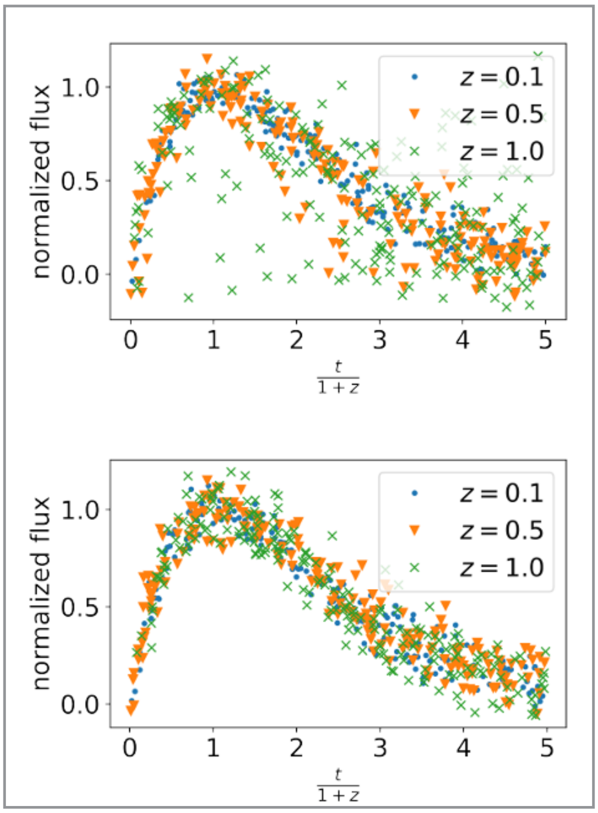

Figure 4: Fake supernovae light curves behavior predicted in Fractal Cosmology (upper panel) and standard cosmology (lower panel). In the upper panel, the lifetimes of the ten fake light curves in a redshift bin are subject to a growing uncertainty described in equation (33), while the lower panel only has a linearly growing uncertainty on the normalized flux mimicking the growing uncertainty on fainter objects and a redshift-independent scattering of the light curve lifetime.

A fake data illustration of the difference between the standard cosmology and the Fractal Cosmology predictions on the overplotted supernova light curves grouped by redshift is presented in figure 4. This figure cannot be read quantitatively, as it is only designed to schematically show what kind of mode could potentially distinguish the two models: the light curves will spread in a wider range on the time axis with growing redshift in Fractal Cosmology, while this effect is not expected to be as drastic in the standard cosmology.

Most intriguingly, equation (33) provides an observable estimation of the metric term coefficient σ2(¯α), thus a completely new quantitative prediction of the accelerated expansion rate of the observational universe. Specifically, it suggests that if Fractal Cosmology is a successful cosmological model, at medium redshift (0.1< z <2), the variance of the dimensionless characteristic time W ≡ T(z) ¯ T0(1+z) of a type of astronomical events could be linearly fitted by the comoving distance square χ2. The slope is predicted to take approximately the value of the cosmological constant Λ as defined in the ΛCDM model, and the constant term accounts for the intrinsic scattering.

It is possible that the intrinsic scattering term ˜σ2 int is subject to unknown redshift drift due to certain environ mental evolution of a type of astronomical events. Even if such an extra redshift-dependent mode does exist in σ(W), it is not very likely to have a perfect degeneracy with the χ2(z) mode with a slope ∼ σ2(¯α). However, it could be a major contributor to the systematics on the σ2(¯α) estimator proposed here.

This article hereby raises the analysis described in this section on the characteristic time variance of any type of astronomical events as a challenge to groups working on time-domain astronomy.

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