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G-Processes with Engineering Applications (Quality & Reliability) through Reliability Integral Theory (RIT)

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Abstract

The stochastic processes [HMP (Homogeneous Markov), NHMP (Non-Homogeneous Markov), SMP (Semi-Markov), RP (Renewal), A&RP (Age and Repair)] used for reliability analyses (to the author knowledge) are particular cases of the G-Process. We present the basics of RIT (Reliability Integral Theory) a theory able to deal with the G-processes. It can be applied to Reliability, Availability, Maintenance and Statistical applications (Control Charts and Time Between Events Control Charts); its power allows the readers to prove that the T Charts and the reliability computations for repairable sys-tems (e.g. the Duane method), used in Minitab 21 are wrong: various cases are considered, from pub-lished papers. due to lack of knowledge of RIT); moreover, with RIT anybody can prove that the T Charts and the reliability computations for repairable systems (e.g. the Duane method), used in Minitab 21 are wrong. We introduce the Stochastic G-Processes, via the Integral Equations, which rule the rela-tionships between the reliabilities Ri(t|s) related to the system states. We show the advantages of using RIT for Quality decisions (economics and business).

Keywords: Reliability Integral Theory, G-Processes, Exponential Distribution, T Charts, Minitab, JMP, Costs by Wrong Ideas.

Introduction

In 1999 the author met some managers of a "certified" company developing a new engine; he saw them using the "Duane Method" for predicting the in-service MTBF by elaborating the test reliability data. In 2022-25 he read various papers about Control Charts with wrong Control Limits. Between 1999 and 2023, the author read a lot of papers, of documents of Masters in RAMS (Reliability, Availability, Maintenance, Safety) and books on quality, reliability, fatigue tests, maintainability, maintenance, statistical tests for decision, Control charts, Six Sigma, Taguchi methods, …, and unfortunately, he found many doubtful ideas on the basics of Quality, Probability and Statistics (QPS).

Due to that, he decided to show his views about such points

These subjects can be dealt by Stochastic Processes: there are several documents about them; a sample is in the references [1-6]. Quality Methods, applied in industries, depends on Stochastic Processes, information provided and on Probability and Statistics [7-16]. Their knowledge is fundamental for taking sound decisions: we will see the many wrong decisions taken by lack of knowledge. Since we consider Engineering Applications of Stochastic Processes applied to physical systems in relation to the analysis of their Quality and Reliability/Availability and state of Control, we model the systems, by an engineering point of view, with a finite number of states (finite state space) and continuous time [17-23].

We begin by presenting it here through quite a simple example, a stand-by repairable system of two units, A and B, with reliabilities $R_A(t)$ and $R_B(t)$ for the "mission time" (interval) 0-----t. The system can be depicted as a three-state (fig. 1) process (representing the system with states 0 (unit A is working and B is in stand-by), 1 (unit A fails at some instant s<t and B starts working), 2 (both units are failed). The state space is denoted by S={0, 1, 2}; it is partitioned into disjoint sets, the set of the Up-states S1={0, 1} and S2={2}, the set of the Down-states (only one in the figure 1, yellow coloured). Forward transitions are related to failures of the units while backward transitions are related to repair of the units; the system fails if it enters the set S2; any transition from S2 to S1 restores the system to a working

condition; we did not depict the "inner transitions" j→j, showing that the system remains in the same state j. The system is reliable as soon it makes transitions within S1: for each Up-state j ∈ S1 we define a reliability function $R_j(t)$ which is the probability that the system does not fail (i.e. does not enter the set S2), for the whole "mission time" (interval) 0-----t, while for each state j ∈ S we define an availability function Aj(t) which is the probability that the system is not failed (i.e. does not enter the set S_2), at the instant t.

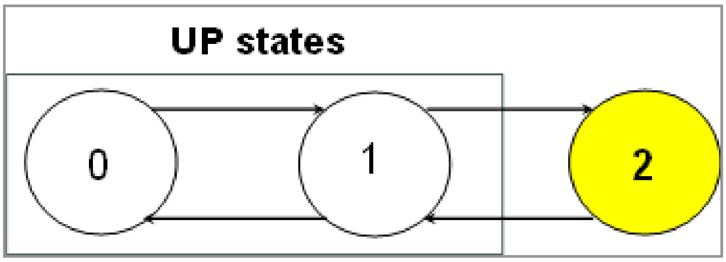


Figure 1: Transitions of a Stochastic Process.

Excerpt 1: Some Documents with Several Drawbacks
For a single unit we define the "failure rate" h(t)=f(t)/R(t) which generally depends on t. IF and only IF h(t)=*, a constant not depending on t, then MTTF=1/*, and *=1/MTTF and R(t)=-exp(-*t): exponential reliability, the item is always "as good as new". In any other case the failure rate is h(t)*1/MTTF, and hence MTTF*1/h(t). For the Weibull distribution (* characteristic life, * shape parameter) R(t)=exp[-(t/*)*], the failure rate h(t)=(*/*)(t/*)*-1, and hence MTTF*1/h(t): many professionals do not know that.

Consider what we found on an EJTAS paper December 2023 "A New Approach for Effective Reliability Management of Biomedical Equipment" (3 Indian authors): there "Reliability is defined as the probability that an equipment or process performs its intended function adequately for a specified period of in a defined environment without failure." What is wrong with the definition? The specified period is not the interval 0----t. They add, later, "where 'μ' is mean of time between failure (MTBF), 'σ' is standard deviation of MTBF and 'x' is breakdown time". This is misleading because they confuse MTBF with MTTF [a constant value equal to the area below R(t)] and confuse the "standard deviation of the RV T" with the standard deviation of MTBF. We will see some their other problems later. They are in good company… See the Excerpt 1.

To be more general we define the interval reliability R(t|r) as the probability that the system does not fail in the interval r----t, given that it did not fail before r.

For reliability analysis we have to consider (in fig. 1) R0(t|r) and R1(t|r) where r is the instant of entrance into the states 0 and 1, respectively.

For availability analysis we have to consider (in fig. 1) A0(t|r), A1(t|r) and A2(t|r) where r is the instant of entrance into the states 0, 1 and 2, respectively: Aj(t|r) is the probability that the system is not failed at the instant t.

The functions Rj(t|r) and Aj(t|r) depend on the probabilities of the various transitions (failures or repair of the units). Letting S(t) the state occupied by the system at time t, we have that S(t), at time t, is a Random Variable taking the values in the state space S=S1∪S2={0, 1, …, nU, nU+1, …, N}: N+1 is the number of states. The ("real") variable t is a parameter indexing the Stochastic Process S(t).

Safety, Reliability, Maintainability, Conformity, Durability, Service, Process Control, Testing, are some of the most important dimensions of Quality; they must be taken into account during Product Development. To make Quality of products and services, knowledge of Quality ideas and Quality tools for achieving Quality are absolutely needed, for any person involved in any Company management (Universities, as well …). To find and use the Quality tools for Quality achievement, education of Managers on Quality is essential. Unfortunately, too many managers [and not only managers ...] do not know much about Quality ideas and Methods; see Deming statements (in his exceptionally good books) [24, 25].

Excerpt 2. Some statements of Deming about Knowledge and Theory (Deming 1986, 1997)

In the author's opinion, the first step to Quality achievement, through problem prevention, is to define logically what Quality is. It is very important defining correctly what Quality means, because Quality is a serious and difficult business; to provide a practical and managerial definition, since 1985 F. Galetto was proposing the following one: Quality is the set of characteristics of a system that makes it able to satisfy the NEEDS of the Customer, of the User and of the Society. This definition highlights the importance of the needs of the three actors: the Customer, the User and the Society. Prevention is the fundamental idea present in this definition: you possibly satisfy the needs only by preventing the occurrence of any problem against the needs.

To measure and analyse the «Characteristics» of Quality during the total life of a product, from its design until its use in the field we NEED Probability Theory: for this reason, we decided to write this paper on the Stochastic Processes.

Materials and Methods

Reliability Integral Theory (RIT)

Let's now show how RIT manages the (Stochastic Processes) for reliability analysis of physical systems. We use the figure 2 (it is like the fig. 1, without the transition 2→1): if the system fails, enters the state 2, it remains there forever. In the model the transitions are ruled by some functions bi,j(s|r) named kernels, related to the interval reliability Runit(t|r) of the units and by the probability of repair of the failed units. The instant "r" is the time of entrance into a state, while "s" is the time instant of leaving a state.

Rj(t|r) [reliability associated to the state j] is the probability that the system is working, at time t, i.e. [S(t)=j]∈S1={0, 1, …, nU}, when it entered the state j at time r of the "mission time", r∈0----t. The functions bi,j(s|r) ds are the instantaneous probability of transition from state i to state j and W‾_i (t | r) are the probabilities of staying in the state i for the whole interval r----t.

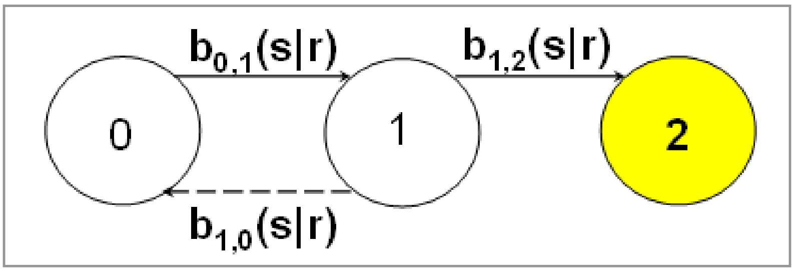


Figure 2: Transitions of a Reliability Stochastic Process.

Applying the probability theory, we can write the two general equations (1) [related to the model in fig. 2]

$$R_0(t|r) = \bar{W}_0(t|r) + \int_r^t b_{0,1}(s|r)\, R_1(t|s)ds$$

$$R_1(t|r) = \bar{W}_1(t|r) + \int_r^t b_{1,0}(s|r)\, R_0(t|s)ds \qquad (1)$$

The two equations (1) are Integral Equations with unknown functions Rj(t|r) [j=0. 1]; we name the previous equations the fundamental system of the Reliability Integral Theory (RIT).
We name G-Processes the stochastic processes ruled by the Integral Equations (1).
For any type of system, we write

$$R(t|r) = \bar{W}(t|r) + \int_r^t B(s|r)\, R(t-s)ds \qquad (2)$$

where R(t|r) is the column vector of the reliabilities Rj(t|r), j∈S1={0, 1, …, nU}, B(s|r) is the kernel matrix and W‾(t|r), j∈S1={0, 1, …, nU}, is the diagonal matrix of the waiting functions in the up-states before any transition.
It is the fundamental system of the Reliability Integral Theory.

The unknown reliabilities Rj(t|r) depends on the kernels bi,j(s|r), related to the failure rate and the repair rate of the units; if they assume some particular form then the G-Processes become known processes (see fig. 3): Homogeneous Markov Processes (HMP), Non-Homogeneous Markov Processes (NHMP), Semi-Markov Processes (SMP), Poisson Processes (PP), Wiener Processes (WP), Branching Processes (BP), Birth and Death Processes (BDP), …

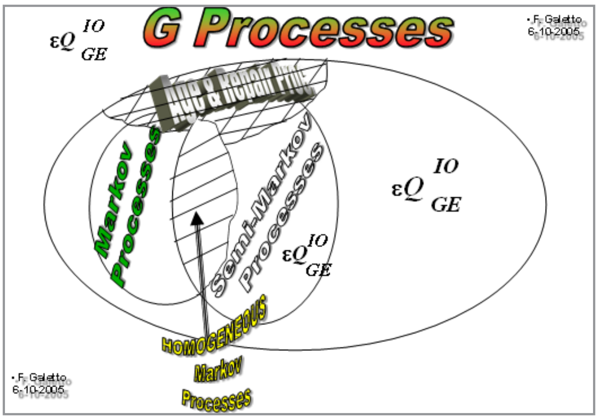


Figure 3: G-Processes comprise several Stochastic Processes (depending on the kernels).

To the author knowledge, there is no Theory (but RIT) able to deal the Age& Repair (A&R) processes, where the forward transitions depend on the age of the system, i.e. bi,j(s|r)=bi,j(s), and the repair (backward transitions) depend on the time interval r----s from the entrance r into a state, i.e. bi,j(s|r)=bi,j(s-r). The transition rates are as in the figure 4 (an example of a "parallel system of 2 identical units" with Weibull reliability and Erlang repair of the failed unit)

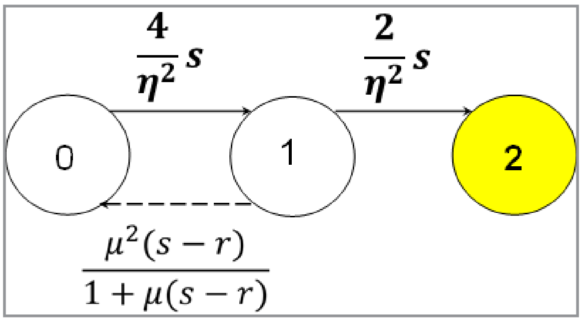


Figure 4: Transitions of an Age & Repair Stochastic Process (transition rates).

Generally, we are interested to the interval 0-----t (mission time) and then we compute the two functions R0(t)=R0(t|0) and R1(t)=R1(t|0).

If both the kernels are exponential (no aging behaviour) we can draw a flow graph with the transition rates λ (failure rate) and μ (repair rate) and write the matrix equation where R(t) is the column vector with entries R0(t) and R1(t) and A the "transition" matrix (see fig. 4 for the parallel, where there is no age)

$$(t) = u + \int_0^t A\, R(s)ds \quad with \quad A = \begin{bmatrix} -2\lambda & 2\lambda \\ \mu & -(\lambda+\mu) \end{bmatrix} \qquad (3)$$

(3) is the model of a Homogeneous Markov Processes (HMP).
For a renewable system we write

$$(t) = \overline{W}(t) + \int_0^t B(s)\, R(t-s)ds \qquad (4)$$

It is the fundamental system of the Reliability Integral Theory, for SEMI-Markov processes (SMP).
From the reliabilities we compute the two Mean Time To (system) Failure MTTF0 and MTTF1: (MTTF not MTBF…)

$$MTTF_0 = \int_0^\infty R_0(t)dt \qquad MTTF_1 = \int_0^\infty R_1(t)dt \qquad (5)$$

For HMP and SMP we can get the MTTFs without actually computing R0(t) and R1(t), as follows

$$MTTF_0 = m_0 + p_{0,1}MTTF_1 \; MTTF_1 = m_1 + p_{1,0}MTTF_0 \qquad (6a)$$

where m0 and m1 are the mean holding time in the up-states 0 and 1, and p0,1 and p1,0 are the steady state transition probabilities from 0 to 1 and from 1 to 0.

It is very useful (figure 5) to see the difference of the various reliabilities R0(t) and R1(t) dealt with the three stochastic processes: Homogeneous (red curves), Non-Homogeneous (blue curves), Age&Repair (green curves). Notice that the reliabilities generated by the NHMP (with linear failure and repair rates) are the highest curves; that does not mean that they are the best curves: the linear repair rate is such that it depends on the age of the system (the older the system, the higher the repair rate: absurd!): this causes huge costs, due to wrong analyses and decisions.

It is clear that when the failure rate is increasing (due to wear out) we can benefit from "Preventive Maintenance": the units are replaced before they fail. Optimized Maintenance Actions (based on reliability, costs of repairs and cost of Preventive Maintenance, and Spare parts Availability) improve the earning of Systems.

To do that we need suitable Methods.
By integrating from 0 to ∞ the column vector R(t) in the formula (4) we find the column vector of the system MTTFj, j∈S1={0, 1, …, nU},

$$MTTF = \int_0^\infty R(t)dt = M + P * MTTF \qquad (6)$$

where P is the matrix of the steady transition probabilities between the Up-states and M the diagonal matrix of the "Mean Holding Time mj" (the length of time in the state j, before transition).

So, we see that we can compute the MTTF, without actually computing the column vector R(t); we need only the matrices M and P

$$MTTF = [I - P]^{-1}M \qquad (7)$$

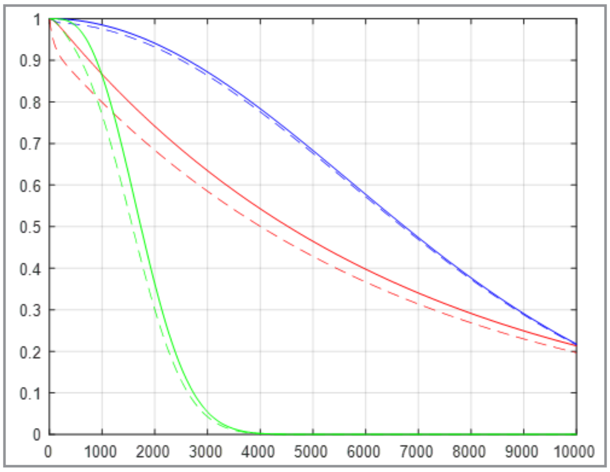


Figure 5: R0(t) and R1(t) for the HMP (red curves), for the NHMP (blue curves) and for the A&RP (green curves).

The matrix [I-P]-¹ provides the "Mean Number of Transitions Between the Up-states, before the system failure". Notice that we can use the formula (7) only for the Semi-Markov Processes; hence in the figure 5 we must compute by numerical methods the area below the blue and green curves.

Availability Integral Theory (AIT)
If we (fig. 1) allow that the failed system, in the state 2, be repaired [transition from 2 to 1, with kernel $b_{2,1}$(s|r)] we can study the system Availability associated to the states S={0, 1, 2}, A_0(t|r), A_1(t|r), A_2(t|r); S is divided into two disjoint sets S=S_1(up states {0, 1})∪ S_2(down states {2}); A_i(t|r) [availability associated to state i], [S(t)=i]∈S={0, 1, …, n_U, n_U +1, N}, is the probability that the system is working [in the up states, S_1], when it entered the state i∈S at time r. Following the same lines, we can write the following fundamental system of the Availability Integral Theory, AIT) [holding whichever is the distributions of the time to failures and times to repair of the various units]; the stochastic process ruling the transitions is named G-Process: all the quantities are computed using the kernels $b_{i,j}$(s|r).

$$A_0(t|r) = \overline{W_0}(t|r) + \int_r^t b_{0,1}(s|r)A_1(t|s)ds + \int_r^t b_{0,2}(s|r)A_2(t|s)ds$$

$$A_1(t|r) = \overline{W}_1(t|r) + \int_r^t b_{1,0}(s|r)\, A_0(t|s)ds + \int_r^t b_{1,2}(s|r)A_2(t|s)ds$$

$$A_2(t|r) = \int_r^t b_{2,1}(s|r)A_2(t|s)ds + \int_r^t b_{2,0}(s|r)A_0(t|s)ds \quad (8)$$

When t→∞ all the availabilities $A_0(t)$, $A_1(t)$, $A_2(t)$ approach the same value A_{SS}=MUT/MTBF =MUT/(MUT+MDT), the steady state Availability (proved in the author's books).

Notice the differences with the EJTAS paper December 2023 "A New Approach… of Biomedical Equipment" where we find (…, excerpt 3)
Same problems are found in the Excerpt 1.
For a general SMP we can derive that
A_{SS}=MUT/MTBF =MUT/(MUT+MDT) (9)

where MUT is the Mean Up Time, a suitable mean of the $MTTF_i$, from i∈S_1={0, 1, …, n_U} to S_2={n_U+1,…, N} in the steady state, and MDT is the Mean Down Time, a suitable mean of the $MTTR_j$, from j∈ S_2={n_U+1,…, N} to S_1={0, 1, …, n_U} in the steady state.

Excerpt 3. From "A New Approach…"
Statistics and RIT
RIT (G-Processes) can be used for parameters estimation and Confidence Intervals (CI), (n 1981, 1982, 1995, 2010, 2015, 2016), in particular for Control Charts (Deming, 1986, 1997, Shewhart 1931, 1936, Galetto 2004, 2006, 2015). In fact, any Statistical (or Reliability) Test can be depicted by an "Associated Stand-by System" whose transitions are ruled by the kernels bk,j(s); we can write the fundamental system of integral equations for the reliability tests, whose duration t is related to interval 0-----t; the collected data tj can be viewed as the times of the various failures (of the units comprising the System) [t0=0 is the start of the test, t is the end of the test and g is the number of the data]

$$R_j(t|t_j) = \bar{W}_j(t|t_j) + \int_{t_j}^t b_{j,j+1}(s|t_j)R_{j+1}(t|s)ds$$

$$\text{for } i = 0, 1, \dots, \quad g-1, \; R_g(t|t_g) = \bar{W}_g(t|t_g) \quad (10)$$

R_j (t|t_j) is the probability that the stand-by system (statistical test or CC) does not enter the state g, at time t, when it starts in the state j at time tj, W¯j (t | t_j) is the probability that the system does not leave the state j, b_(j,j+1) (s|t_j)ds is the probability that the system makes the transition j → j+1.

The reliability system (10) can be written in matrix form,

$$R(t|r) = \bar{W}(t|r) + \int_r^t B(s|r)R(t|s)ds \quad (11)$$

At the end of the reliability test, at time t, we know the data (the times of the transitions t_j) and the empirical sample D={x_1, x_2, …, x_g}, with $x_j=t_j - t_{j-1}$ is the length between the transitions; the transition instants are $t_j = t_{j-1} + x_j$ giving D*={t_1, t_2, …, t_{g-1}, t_g, t}; t is the duration of the test.

We consider now that we want to estimate the unknown MTTF=θ=1/λ of each item comprising the stand-by system: each datum is a measurement from the exponential pdf; we compute the determinant $\det B(s|r;\, \theta, D^*) = (1/\theta)^g \exp[-T(t)]$ of the integral system (11), where T(t) is the "Total Time on Test" $T(t)=\sum_1^g x_i$. At the end time t, the integral equations, constrained by the constraint D*, provide the equation

$$(\partial \ln \det B(s|r;\, \theta, D^*))/\partial\theta = \theta/g - T(t) = 0 \quad (12)$$

In the case of exponential distribution, it is exactly the same result as the one provided by the MLM Maximum Likelihood Method.

If the data are normally distributed, $X \sim N(\mu_X, \sigma_X^2) = 1/(\sqrt{2\pi}\,\sigma_X)e^{-(x-\mu_X)^2/(2\sigma_X^2)}$, with sample size n, then we get the usual estimator $\bar{X} = \sum X_i/n$ such that $E(\bar{X}) = \mu_X$.

The same happens with any other distribution provided that we can write the kernel $b_{i,i+1}(s)$.

The reliability function $R_0(t|\theta)$, [formula (10)], with the parameter θ, of the "Associated Stand-by System" provides the *Operating Characteristic Curve* (OC Curve, reliability of the system) [8-23, 30, 35] and allows to find the Confidence Limits (θ_L Lower and θ_U Upper) of the "unknown" mean θ, to be estimated, for any type of distribution (Exponential, Weibull, Rayleigh, Normal, Gamma, …); by solving, with unknown θ, the two equations $R_0(t_0|\theta) = 1-\alpha/2$ and $R_0(t_0|\theta) = \alpha/2$ we get the two values (θ_L, θ_U) such that

$$R_0(t_o|\theta_L = 1/\lambda_U) = \alpha/2 \;\; and \;\; R_0(t_o|\theta_U = 1/\lambda_L) = 1-\alpha/2 \quad (13)$$

where t_o is the "total of the length of the transitions $x_i=t_j - t_{j-1}$ data of the empirical sample D" and CL=1-α is the Confidence Level. CI=θ_L-------θ_U is the Confidence Interval of θ.

For example, from the Reliability R_0 (λt_0) of a "4 units Stand-by system" with MTTF=θ=123 days and t_0 is the total time on test of the 4 units, by R_0 (λ_L t_0)=0.9 and R_0 (λ_U t_0)=0.1 we can derive θ_L=62.5 days=1/λ_U and θ_U=200 days=1/λ_L, with CL=0.8. It is quite interesting that the book Meeker et al., "Statistical Intervals: A Guide for Practitioners and Researchers", John Wiley & Sons (2017) use the same ideas of FG (shown in the formula 13) for computing the CI; the only difference is that the author FG defined the procedure in 1982, 35 years before Meeker.

The same procedure can be used for normal data as those of the paper "The mixed CUSUM-EWMA (MCE) Control Chart as a new alternative in the Monitoring of a Manufacturing Process" published in the Brazilian Journal of Operations & Production Management, pp. 1-13, DOI: 10.14488/BJOPM.2019.v16.n1.a1, written by 6 authors.

Consider the 30 data and the authors' Control Chart

69.80	69.50	68.80	70.90	69.20	70.40	71.00	71.30	70.00	70.10
72.10	69.90	70.10	70.30	71.20	70.80	70.70	69.95	71.20	71.35
71.35	69.90	70.25	70.50	70.28	71.30	70.20	70.35	70.15	70.10

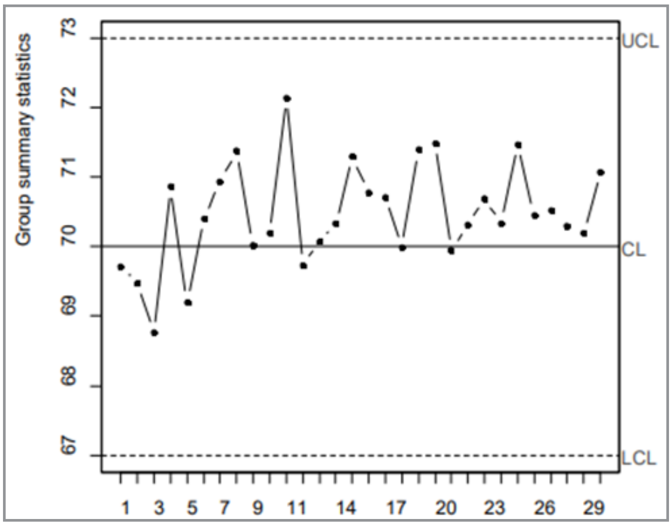


Figure 6: Shewhart I-CC from Brazilian Journal of Op. & Prod. Management (fig. 12 in their paper).

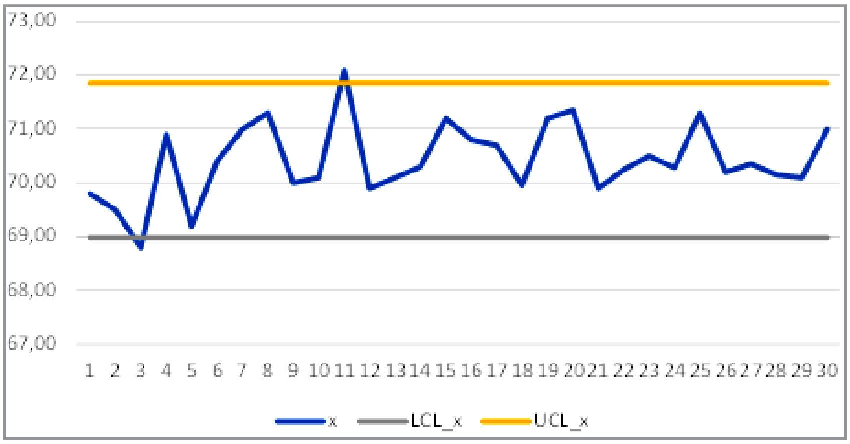


Figure 7: Shewhart I-CC computed by F. Galetto; notice the OOC points.

Notice: the process appears IC because in the fig. 6 the Control Limits do not depend on the collected data; actually, they are the Probability Limits of the Probability Interval. The process is OOC! It is very clear that the Process is Out Of Control (OOC), due to two causes: a) two points out of the Control Limits, and b) the points from 8 to 30 have a mean larger than the mean of the first seven points; notice that the process is OOC in two ways: a) because one point is below LCL and one point is above UCL, b) because the mean of the last 23 points is statistically different form the mean of the first 7 points.

The Brazilian Journal … Management, did not publish the letter sent by FG to the Editors.

Control Charts for Process Management

Statistical Process Management (SPM) entails statistical Theory and tools used for monitoring any type of a process, industrial or not. The Control Charts are the tool used for monitoring a process, to assess two states: the first, when the process, named IC (In Control), operates under the common causes of variation (variation is always naturally present) and second, named OOC (Out Of Control), when the process operates under some assignable causes of variation. The CCs, using the observed data, allow us to decide if the process is IC or OOC.

Control Charts were very considered by Deming (1986, 1997) and Juran (1988) after Shewhart invention (1931, 1936). We start with Shewhart ideas (see the excerpt 4). He wrote on page 294, where $\bar{X}$ is the "Grand Mean", computed from D, σ is "estimated standard of each sample" (with sample size n), $\bar{\sigma}$ is the "estimated mean standard deviation of all the samples".

Excerpt 4. From Shewhart book (1931)

From Excerpt 4, we clearly see that Shewhart, the inventor of the CCs, used the "Normal Approximation (Central Limit Theorem)" [8-16] and the data to compute the Control Limits, LCL (Lower Control Limit) and UCL (Upper Control Limit) both for the mean μ_X (the 1st parameter of the Normal pdf) and for σ_X (the 2nd parameter of the Normal pdf). Similar ideas can be found in Dore, 1962, Belz, 1973, Ryan, 1989, Rao, 1965, Cramer, 1961, Mood, 1963, Rozanov, 1975 (where we see the idea that CCs can be viewed as a Stochastic Process). See also F. Galetto [19, 30, 35].

Compare Excerpt 4 (where LCL, UCL depend on the data) with Excerpt 5 (where LCL, UCL depend on the Random Variables) and appreciate the profound difference: this is the cause of the many errors in the CCs for TBE (Time Between Events (see the "Garden…"). Notice that an author wrote several papers… Notice the wrong statement (with k=3) "*The control limits of the standard Shewhart chart ($\bar{X}$ chart or the X chart) are given by* $UCL_I=\mu_Y+3\sigma_Y$ *and* $LCL_I=\mu_Y-3\sigma_Y$ *where* μ_Y *and* σ_Y *are the specified IC mean* *and standard deviation of the charting statistic* Y_i". Notice that, as per Excerpt 5, $L=\mu_Y-3\sigma_Y$------$U=\mu_Y+3\sigma_Y$ is $P[L=\mu_Y-3\sigma_Y \le Y \le \mu_Y+3\sigma_Y=U]=0.9973$ (14). The same error is in other books (e.g. Montgomery D., 1996-2019, page 192-3). The right ideas are in Galetto F. (2006, 2015, 2016). RIT will show clearly the drawbacks of those many authors (Galetto 1981, 2006, 2015, 2016).

Excerpt 5. From Control Charts, Synthetic (2021), a paper in the "Garden…". Notice that one of the authors wrote several papers…

Notice, in the Excerpt 5, the statement "… in case of individual observations (i.e. n=1)… the Control Limits…".

It is very interesting that a Peer Reviewer chosen by the Editors of Quality and Reliability Engineering International (QREI) suggested (February 2024) the author to read the following paper in order to learn the way to compute the Control Limits for Individual Control Charts with Exponentially distributed data: Khakifirooz, M., Tercero-Gómez, V. G. and Woodall, W. H. (2021). The role of the normal distribution in statistical process monitoring, Quality Engineering 33(3), 497–51

We anticipate our conclusion about the Excerpt 6:

the 3 authors statement (in the Excerpt 6) "We can see from this chart that there are nine false alarms. (see the figure 1, in the Excerpt 6)" IS WRONG.

The TRUE Control Limits (by RIT!) for the chart in (Figure in the Excerpt 6) are actually:
LCL=0.103 and UCL>>100

From the figure 1 we cannot read the value of the data, BUT surely there are NO … false alarms (above the TRUE UCL) (figure 1, in the Excerpt 6).

IF we had the data, we could assess that there could be Out Of Control, BELOW the LCL… The Peer Reviewer did not know the TRUE Theory, as did not the authors.

Notice the wrong LCL=1.022-2.06 and UCL=1.022+2.06 in the Figure 1 (in the Excerpt 6): they are computed with the wrong formula given in the Excerpt 5 (as though the Exponential data were Normal data!): Nonsense!

All the people involved did not know that also the differences |xi+1-xi| are exponentially distributed.

The 3 authors write (authors' statements):
Four examples involving transformations Example based on simulation. We can demonstrate through a simple example how the use of a power transformation can result in poor process monitoring performance. We generated a set of 100 independent exponential random variables which, without loss of generality, were assumed to have a mean of one. The X-chart is shown in Figure 1. We can see from this chart that there are nine false alarms. After the 0.2777 power transformation recommended by Nelson (1994) and others, we have the Xchart in Figure 2, for which there are no signals. If we transform the upper and lower control limits of Figure 2 back to the original units, then we have 7.456 and 0.0000192, respectively. The probability of exceeding this upper control limit for the exponential distribution with mean one is 0.00058, while the probability of falling below the lower control limit is very low, 0.0000192. This numerical example illustrates why the results of Santiago and Smith (2013)

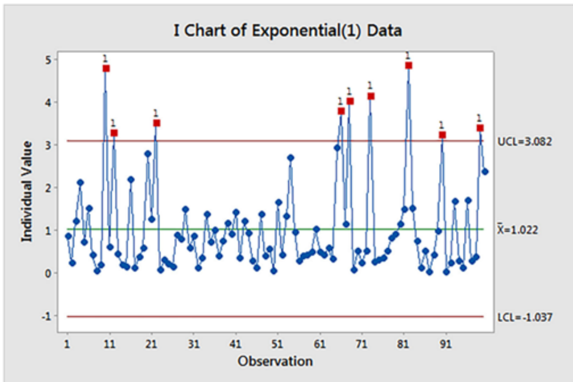
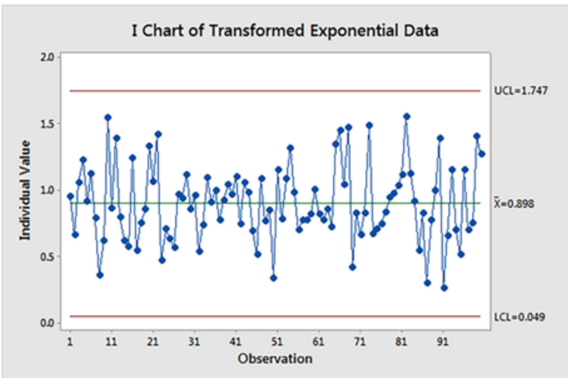


Figure 1. *X*-chart for exponential data with a mean of one.



ure 2. *X*-chart for exponential data after power transformation.

Excerpt 6: From the paper the role of the normal distribution in statistical process monitoring

Generally, the data plotted are the means $\bar{x}(t_i)$, determinations of the Random Variables $\bar{X}(t_i)$, i=1, 2, ..., n (n=number of the samples) computed from the collected data x_{ij}, j=1, 2, ..., k (k=sample size), determinations of the RVs $X(t_{ij})$ at very close instants t_{ij}, j=1, 2, ..., k. In other applications, the data plotted are the Individual Data $x(t_i)$, determinations of the Individual Random Variables $X(t_i)$, i=1, 2, ..., n (n=number of the collected data), modelling the measurement process of the "Quality Characteristic" of the product: this model is very general because it is able to consider every distribution of the Stochastic Process $X(t)$.

Shewhart on page 289 of his book (1931) writes "… we saw that, no matter what the nature of the distribution function of the quality is, the distribution of the arithmetic mean approaches normality rapidly with increase in n (his n is our k, the sample size), and in all cases the expected value of means of samples of n (our k) is the same as the expected value of the universe" (Central Limit Theorem in Excerpt 4). Let k be the sample size: the RVs $\bar{X}(t_i)$ are assumed to follow a normal distribution; $\bar{X}(t_i)$ [i^th rational subgroup] is the mean of RVs IID $X(t_{ij})$ j=1, 2, ..., k, (k data sampled, at very near times t_{ij}), we assume here that it is distributed as [probability density function (pdf) of "transitions from a state to the subsequent state" of a subsystem] $\bar{X}(t_i)\sim N(\mu_{\bar{X}(t_i)}, \sigma^2_{\bar{X}(t_i)})$ [experimental mean $\bar{x}(t_i)$] with mean $\mu_{\bar{X}(t_i)}$ and variance $\sigma^2_{\bar{X}(t_i)}$. $\bar{\bar{X}}$ is the "grand" mean and $\sigma^2_{\bar{\bar{X}}}$ is the "grand" variance: $\bar{\bar{X}}\sim N(\mu_{\bar{\bar{X}}}, \sigma^2_{\bar{\bar{X}}})$ [experimental "grand" mean $\bar{\bar{x}}$]. In Fig. 8 the distribution, the determinations of the RVs $\bar{X}(t_i)$ and $\bar{\bar{X}}$ are shown. The function connecting the points x_{ij} is called a "sampled trajectory" of the stochastic process X(t).

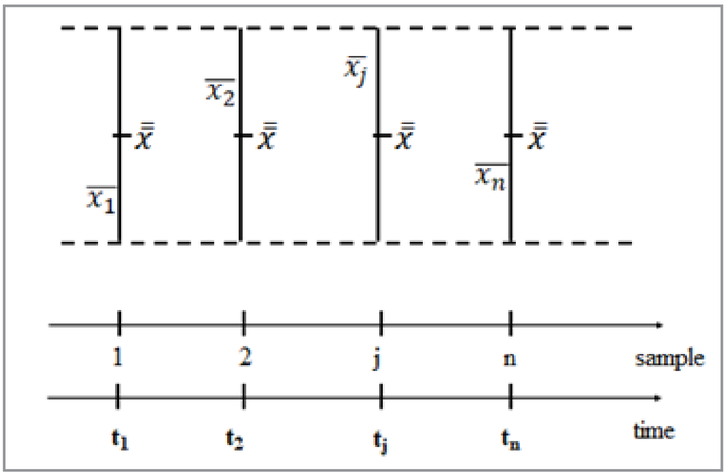


Figure 8: The Individuals xij, the "means" x ̅i of the process and the "grand mean" x ̿.

When the process is OOC (Out of Control, i.e. assignable causes of variation) some of the means μ_(X ̅(t_i)), estimated by the experimental means $\bar{x}_i = \bar{x}(t_i)$, are "statistically different)" (Galetto 1981, 2006, 2015, 2016).

We said that (14) is a Probability Interval; IF we put $\bar{\bar{x}}$ in place of μ_Y and $\bar{s}/\sqrt{k}$ in place of σ_Y we get the CI of $\mu_{\bar{\bar{X}}}$ when a sample size k is considered for each $\bar{X}(t_i)$, with CL=0.9973. The quantity $\bar{s}$ is the mean of the standard deviations of each sample. This allows us to compare each (subsystem) mean $\mu_{\bar{X}(t_q)}$, q=1,2, …, n, to any other (subsystem) mean $\mu_{\bar{X}(t_r)}$ r=1,2, …, n, and to the (Stand-by system) grand mean $\mu_{\bar{\bar{X}}} = \mu$. If two of them are different the process is OOC. The quantities $LCL_X = \bar{\bar{x}} - 3\bar{s}/\sqrt{k}$ and $LCL_X = \bar{\bar{x}} + 3\bar{s}/\sqrt{k}$ are the limits of the Control Limits of the CC. When the Ranges Ri=max(x_{ij})-min(x_{ij}) are considered for each sample we have $LCL_X = \bar{\bar{x}} - A_2\bar{R}$, $UCL_X = \bar{\bar{x}} + A_2\bar{R}$ and $LCL_R = D_3\bar{R}$, $UCL_R = D_4\bar{R}$, where the coefficients A_2, D_3, D_4 are tabulated and depend on the sample size k [26-35].

The interval LCL_X-------UCL_X is the "Confidence Interval" with "Confidence Level" 1-α=0.9973 for the unknown mean $\mu_{X(t)}$ of the Stochastic Process X(t) (Galetto 1981-2022).

The interval LCL_R---------UCL_R is the "Confidence Interval" with "Confidence Level" CL=1-α=0.9973 for the unknown Range of the Stochastic Process X(t) (Galetto 1981-2022). Notice that the Control Interval [Confidence Interval] UCL_X-LCL_X=U-L [Probability Interval, formula (14)] for normally distributed data and that LCL_X can be obtained from L by substituting μ with ; the same for UCL_X and U.

The error highlighted, the confusion between the Probability Interval and the Control Limits (Confidence Interval!) has NO consequences for decisions WHEN the data are Normally distributed, as hypothesised by Shewhart. On the contrary, it has BIG consequences for decisions WHEN the data are non-normally distributed as in the Excerpt 6. Notice!

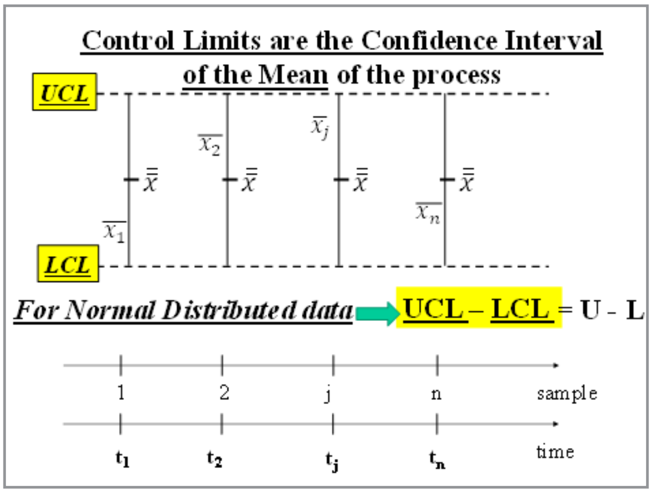


Figure 9: Control Limits LCLX----UCLX=L----U (=Probability interval), for Normal data.

Control Charts for TBE data

We consider now, again, TBE (Time Between Event) data, exponentially or Weibull distributed. Quite a lot of authors (see Appendix, "Garden …") compute wrongly the Control Limits.

The previous section formulae are used also for NON_normal data (see Excerpt 6): for that, the NON_normal data are transformed "with suitable transformations" in order to "produce Normal data" (see Excerpt 6) and to apply those formulae (above).

Sometimes we have few data and then we use the so called "individual control charts" I-CC. The I-CC are very much used for exponentially distributed data: they are named "rare events Control Charts for TBE (Time Between Events) data", I-CC_TBE (see Excerpt 6).

The author (FG) knew about the wrong way of dealing with I-CC_TBE since 1996 by reading the Montgomery book where he transformed the into Weibull data (approximately normal) following Nelson L. S. (J. Qual. Techn., 1994): he acted wrongly in all the later editions of the book. Any scholar who wants to learn Control Charts both with normal distribution and TBE can usefully read the book "Statistical Process Management".

Several authors did the same as Montgomery did. See the Journal Operation Research and Decisions where the 3 authors, in their paper "An EWMA Control Chart for the exponential distribution" made the same error transforming the data into Weibull with β=1.36; the data are the "Urinary Tract Infection" (UTI) taken from a paper of two Minitab authors (Santiago & Smith) in their "Control charts based on the Exponential Distribution", Quality Engineering;: their T Chart (figure 11) shows the process IC: wrong decision; making the transformation we could draw the I-CC as in figure 12 where the Control Interval=UCL-LCL=U-L=the Probability Interval. The process is again IC: wrong decision. It is NOT so if we analyse directly the TBE (figure 13).

Using RIT (the Reliability Integral Theory of F. Galetto) we get the Figure 13 (vertical axis logarithmic, to let the OOC points evident). The process is OOC.

The problem with the authors in the "Garden…" is that they do not care of Theory: they do not consider that THEORY MATTERS, in every field!

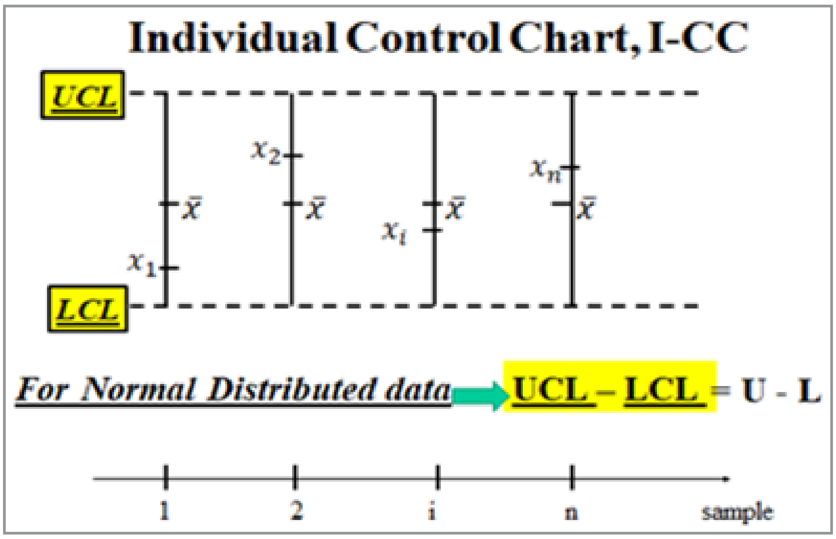


Figure 10: Individual Control Chart (sample size k=1). Control Limits LCL----UCL=L----U (Probability interval), for Normal data.

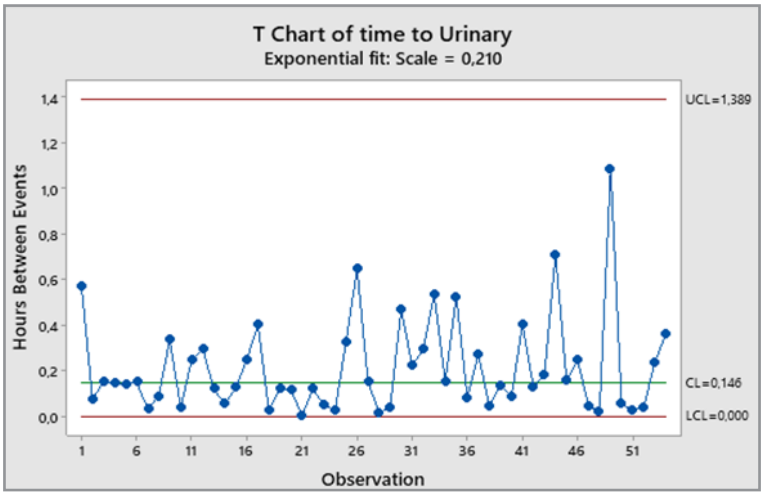


Figure 11: Chart of Minitab authors' paper data (Urinary), Minitab 19 used.

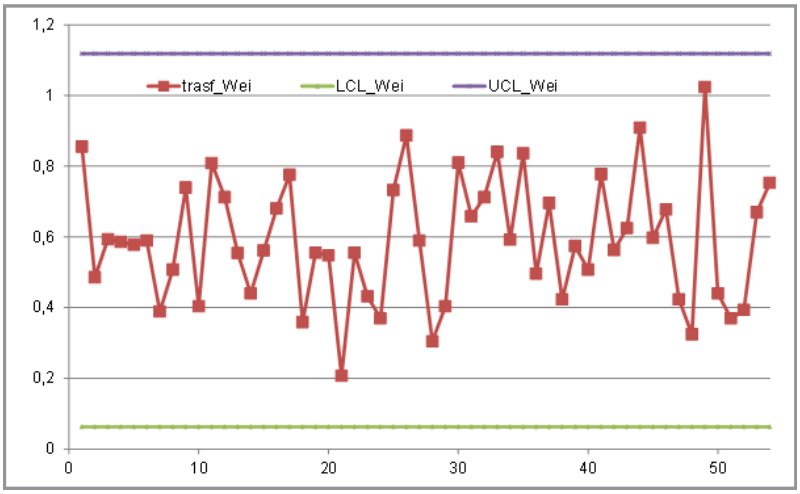


Figure 12: I-CC of the UTI, transformed into Weibull data.

On the contrary, making the same error as Montgomery, the 3 authors transform the data into Weibull with β=1.36 and then they make an EWMA Chart with "double Control Limits" with the formulae $LCL_1 = \theta_0^* c_{L1}$ $LCU_1 = \theta_0^* c_{U1}$ $LCL_2 = \theta_0^* c_{L2}$ $LCU_2 = \theta_0^* c_{U2}$, where $\theta_0^* = \theta_0^{1/3.6}$ with θ_0 the "target of an IC process":

$$c_{L1} = \Gamma\left(1+\frac{1}{3.6}\right) - k_1\sqrt{\frac{\lambda}{2-\lambda}\{1-(1-\lambda)\}^{2t}}\sqrt{\Gamma\left(1+\frac{2}{3.6}\right)-\Gamma\left(1+\frac{1}{3.6}\right)^2}$$

$$c_{U1} = \Gamma\left(1+\frac{1}{3.6}\right) + k_1\sqrt{\frac{\lambda}{2-\lambda}\{1-(1-\lambda)\}^{2t}}\sqrt{\Gamma\left(1+\frac{2}{3.6}\right)-\Gamma\left(1+\frac{1}{3.6}\right)^2}$$

$$c_{L2} = \Gamma\left(1+\frac{1}{3.6}\right) - k_2\sqrt{\frac{\lambda}{2-\lambda}\{1-(1-\lambda)\}^{2t}}\sqrt{\Gamma\left(1+\frac{2}{3.6}\right)-\Gamma\left(1+\frac{1}{3.6}\right)^2}$$

$$c_{U2} = \Gamma\left(1+\frac{1}{3.6}\right) - k_2\sqrt{\frac{\lambda}{2-\lambda}\{1-(1-\lambda)\}^{2t}}\sqrt{\Gamma\left(1+\frac{2}{3.6}\right)-\Gamma\left(1+\frac{1}{3.6}\right)^2}$$

Excerpt 7. From the paper "An EWMA Control Chart …" (the k coefficients are to be suitably found…). Notice the errors in the formulae.

See now the wrong I-CC in the figures 11, 12, 14; only the CC in the fig. 13 is right.

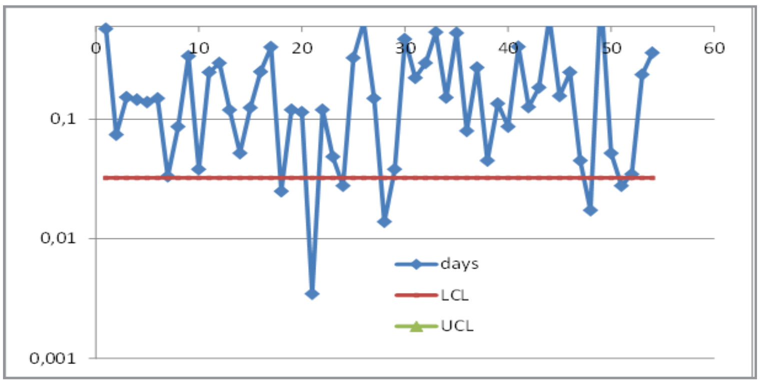


Figure 13: I-CC of the UTI, y-axis logarithm mic. RIT used (F. Galetto).

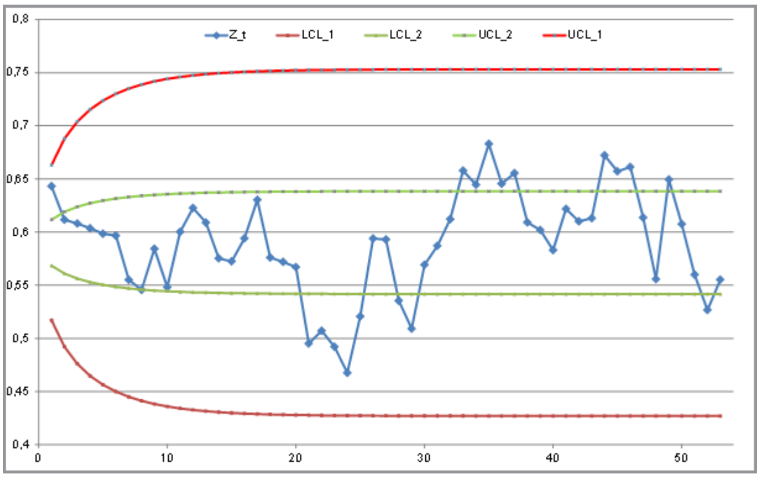


Figure 14: EWMA (F. Galetto) of the UTI.

In the paper mentioned there is a figure: we did not find the data used for the target θ_0 and the coefficients k. Hence, we used the target θ_0=0.59, coefficients k=-2.7, -0.8, 0.8, 2.7 and λ=0.2; we got the figure 14.

Those authors made a mess; the Peer Reviewers did not analyse correctly the paper, and the Editors did not do their Job. The same as for all the papers in the "Garden …". Remember Juran who mentioned the FG paper, at the Plenary Session of the EOQC Conference … [36].

Results

Now it is time to see the wrong formulae used by the "Garden …" authors. A smal sample in in the Excerpt 8.

Typical statement by ALL ...

A uniform model the exponential TBE charts is that the occurrence of events is modelled by a Poisson process, and the time between events X_i (i=1, 2, …) re independent and identically distributed random variables with pdf $f(x) = \theta^{-1}\exp(-x/\theta)\ for\ x \geq 0$, 0 otherwise, where θ is the "mean time between events".
The Control Chart plots the quantity produced before observing an event;
The Control Limits can be calculated as

$$LCL = \theta\ln(1-\alpha/2), \qquad UCL = \theta\ln(\alpha/2)$$

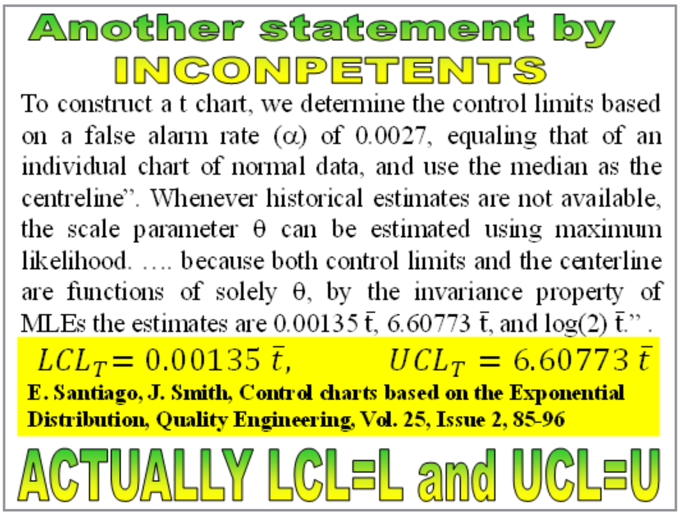
Liu J., Xie M., Sharma P., "A Comparative Study of Exponential Time Between Event Charts", *Quality Technology & Quantitative Management*, 2006 Issue 3, pp. 347-359

ACTUALLY LCL=L and UCL=U

Suppose LCL and UCL denote the lower and upper control limits of the Phase II t_r-chart respectively. Then for a given false alarm rate (FAR) α_0, they can be obtained from $P(T_r < LCL|IC) = P(T_r > UCL|IC) = \alpha_0/2$ according to the equal tail probabilities approach. Thus, we have (see also Kumar and Baranwal (2019))

$$LCL = \frac{\chi^2_{2r,\alpha_0/2}}{2\lambda_0} = \frac{A_1}{\lambda_0} \text{ and } UCL = \frac{\chi^2_{2r,1-\alpha_0/2}}{2\lambda_0} = \frac{A_2}{\lambda_0} \quad (1)$$

where $A_1 = \frac{\chi^2_{2r,\alpha_0/2}}{2}$, $A_2 = \frac{\chi^2_{2r,1-\alpha_0/2}}{2}$ are the design constants and λ_0 is the known or specified IC rate parameter value. The $\chi^2_{2r,a}$ denotes the a-th quantile of the chi-square distribution with $2r$ degrees of freedom. The center line (CL) of the t_r-chart can be considered as the median of the IC distribution of T_r and it is given by CL $= \frac{\chi^2_{2r,0.5}}{2\lambda_0}$.



Excerpt 8: Typical statements in the "Garden full of errors …" where the authors name LCL, UCL what actually are the Probability Limits L and U.

All the authors in the "Garden …" make the same error: they confuse the Probability Interval with the Control Interval in CCs (Confidence Interval!). The same happens for MINITAB, JMP, SAS, … software [37-40].

Now we see how RIT solves the I-CC_TBE with exponentially distributed data. Before we computed the Confidence Interval is CI=θ_L--------θ_U of the parameter θ, using all the data with t_O the "total of the data of the empirical sample D (n=20)" and Confidence Level CL=1-α. When we deal with a I-CC_TBE we have to consider the figure 10 and compute the LCL and UCL through the empirical mean t ̄_O (mean observed time to failure t ̄_O=t_O/n) we only have to solve the two following equations with unknown LCL and UCL

$$R_0(\bar{t}_O|LCL) = 1-\alpha/2\,, \qquad R_0(\bar{t}_O|LCL) = \alpha/2 \qquad (15),$$

similar to (13). For exponentially distributed data (15) become

$$exp[-\bar{t}_O/LCL] = 1-\alpha/2 \text{ and } exp[-\bar{t}_O/LCL] = \alpha/2 \qquad (16).$$

The two equations (16) show clearly the errors of the authors in the "Garden …". See on the left.

See the case by the Peer Reviewer chosen by the Editors of Quality and Reliability Engineering International about the Control Limits for Individual Control Charts with Exponentially distributed data and compare the results: Khakifirooz, M., Tercero-Gómez, V. G. and Woodall, W. H. (2021). The role of the normal distribution in statistical process monitoring, Quality Engineering 33(3), 497–51 Some papers from the "Garden …" Let's see some other few cases from the "Garden …". Consider the paper Box-plot based Control Charts [by Chakraborti (same author in excerpt 5.) et al.), Quality and Reliability Engineering International, 2011.

Notice QREI again], where the lifetime data ("valves TTF") the same as in Montgomery, 2013, page 334) are analysed; the authors use the median (instead of the mean) and the interquartile range (instead of the ranges). The two authors define the control limits with a form similar to Shewhart (but significance level α_0=0.01): the process (figure 15) is found IC, as did Montgomery [41-44].

Using the T Chart of Minitab (which makes use of the wrong formulae, devised by Santiago & Smith) we can find the figure 16: the process is found IC (as in figure 15, Chakraborti, and as Montgomery).

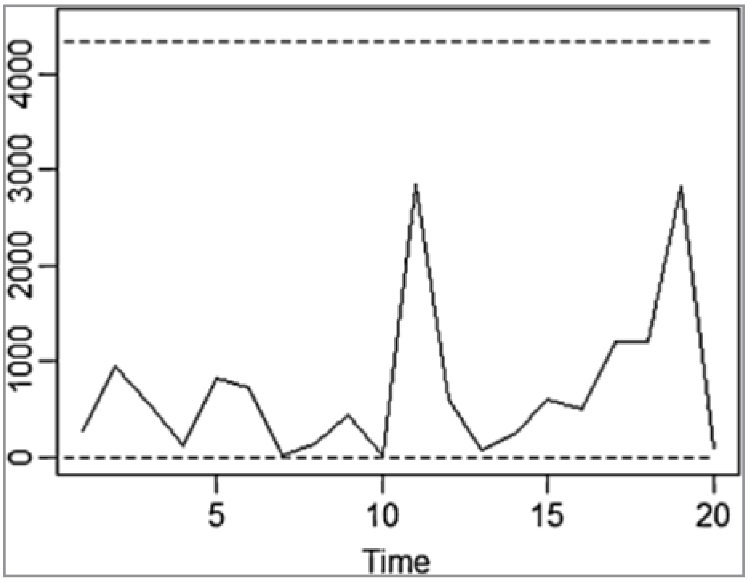


Figure 15: I-CC of Montgomery data analysed by Chakraborti (with α_0=0.01).

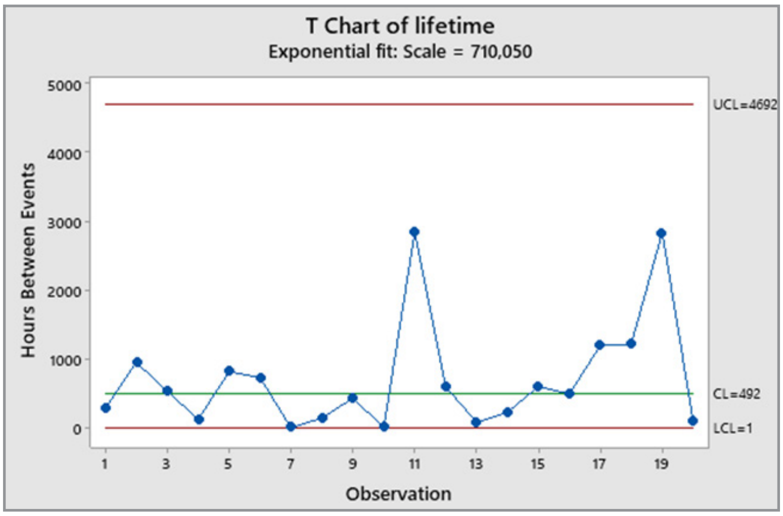


Figure 16: T Chart of Table 1 data. Minitab 19&20&21 used (F. Galetto).

By using Minitab, one finds the figure 17 (with wrong OOC as in the Excerpt 8, as happened in the Excerpt 6): UCL and LCL are wrong, while the dotted line (found with RIT) is the correct LCL. Compare figures 16 and 17: only the dotted line is the right correct LCL, allowing taking correct decisions: huge costs of DIS-quality applications/decisions by Minitab Clients, caused by Minitab wrong methods.

The process is OOC. The reader can see easily from figures 17, 18. The ranges too are OOC.

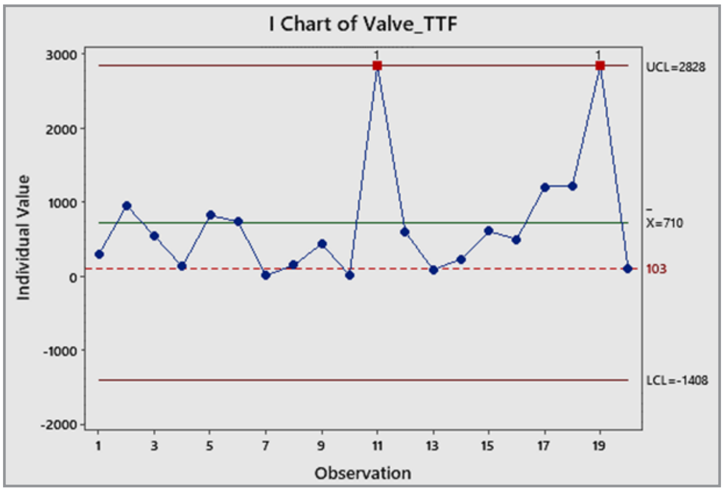


Figure 17: (F. Galetto) I Chart (Control charts) for valves data (Minitab 19&20&21 used). The dotted line is the right correct LCL when RIT is used; the UCL is wrong.

It should be clear that Managers, Professors and Scholars must use the Theory. The author, for many years, has been showing the many drawbacks present in various books and papers: unfortunately, he had little success; only few understood (one was Juran at 1989 EOQC Conference, Vienna).

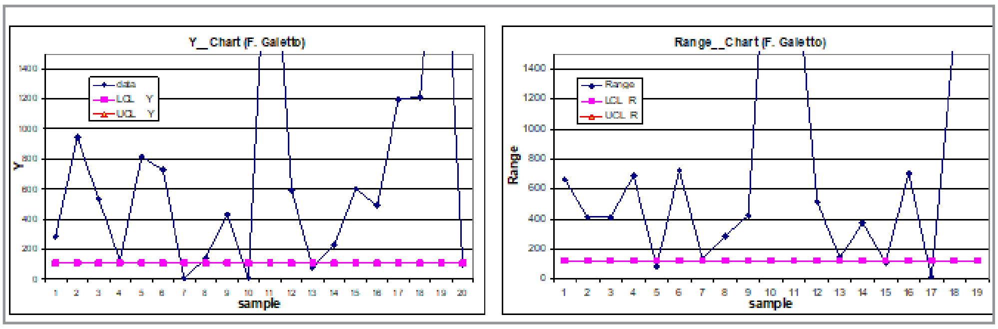


Figure 18: (F. Galetto) Scientific Control Charts for valves data [related to the data and control charts in Mont-gomery books]. RIT is used.

Now we see another paper in the "Garden…" (found online, 2021) "Improved Phase… for Monitoring TBE" [Chakraborti (same author above et al.) published by QREI (again). The two authors provide a wrong solution (found neither by the Peer Reviewers nor by the Editor!). Nevertheless, they write in their Acknowledgements: … The authors would like to thank Dr. Douglas Montgomery, Co-editor, for his interest and encouragement. In the authors' Abstract, we read

... In this paper, phase I control charts are considered for the observations from an exponential distribution with an and out-of-control performance of the proposed chart. It is seen that ***the proposed charts are considerably more in-control robust than*** *two competing charts and have comparable out-of control properties. Copyright © 2014 John Wiley & Sons, Ltd.*

and

Table ... shows a set of 30 failure time data generated from a Poisson distribution with a mean of 0.1. For these data, n = 30, l=8, m= 15, and u = 23. ... The center line for the proposed two-sided control chart is CL=X(15)=6.91, and the lower and UCL are given by LCL=-53.9213 and UCL=47.2320. ... we set the LCL as LCL=0. It can be seen from Figure (our 20) that ***the eleventh observation 52.32 plots outside the UCL****, which indicates an OOC situation that needs further investigation. Note that for these data, neither the Dovoedo and Chakraborti, nor the Jones and Champ control chart indicates any OOC situation.*

See their Concluding remarks.

We agree that "Further work is necessary on the OOC performance of these charts": the further Work must be to STUDY (see Deming!). The wrong CC (in figure 19) shows a "false" OOC situation and various "false" IC…

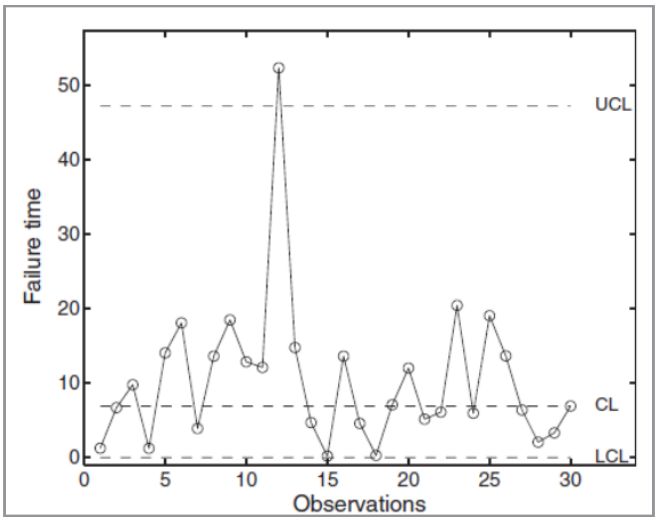


Figure 19: Control Chart from "Improved Phase… for Monitoring TBE".

Using RIT as done previously the n=g*=30 TBE can be considered as the "transition times" between states of a stand-by system of 30 units: the Up-states are 0, 1, …, 29, and 30 is the Down-state; ti is the "time to failure " from state i-1 to state i. $R_0(t|\theta)$ is the system reliability for the interval 0----t, given θ, and it is, as well, the Operating Characteristic Curve of the reliability test, given t. At the end of the test, we know tO the observed Total Time on Test [45].

We want to analyse if the "individual" TBE are significantly different from the "mean observed time to failure" (t_o)¯=tO/n. The Control Limits are the values satisfying the two equations (13) with tO replaced by (t_o)¯=tO/n, that is two equations (15 and 16) for any single unit; so, we have 30 Confidence Intervals [all equal, by solving formulae (16)], given (t_o)¯and CL=1-α.

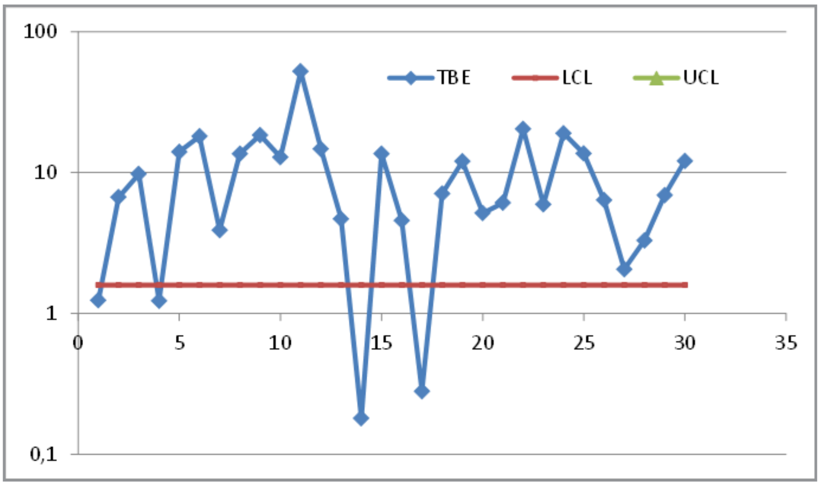


Figure 20: Control Chart of the data from "Improved Phase… for Monitoring TBE"; vertical axis logarithmic; UCL is >100. RIT used (F. Galetto).

Compare the figures 19 and 20: it is clear that the I-CC_TBE from "Improved Phase… TBE" presents 5 errors about OOC; the paper is wrong [46-50].

Also consider the paper Some effective control chart procedures for reliability monitoring published in Reliability Engineering & System Safety. Again, WRONG Control Limits! The authors Xie et al. the "Time between failures of a component". They do not realise that at least 20% of the data are OOC (figure 21), a very good result for a PR paper! All the people involved did not know the Theory. "It is necessary to understand the theory of what one wishes to do or to make." (Deming 1996) T Charts and the "Garden…" methods make the users to take wrong decisions...

Also, see a paper in (Multidisciplinary Open Access) IEES Access 2017, "EWMA Control Chart For Rayleigh Process With Engineering Applications (Alduais, Khan)". At the end of the Abstract, we read the fantastic statements "An application of the REWMA chart on simulated data also reveals that the proposed chart is highly sensitive to smaller and persistent shifts in the scaling parameter of Rayleigh distribution. Finally, an example from real-life has been presented to illustrate the importance of the suggested chart."

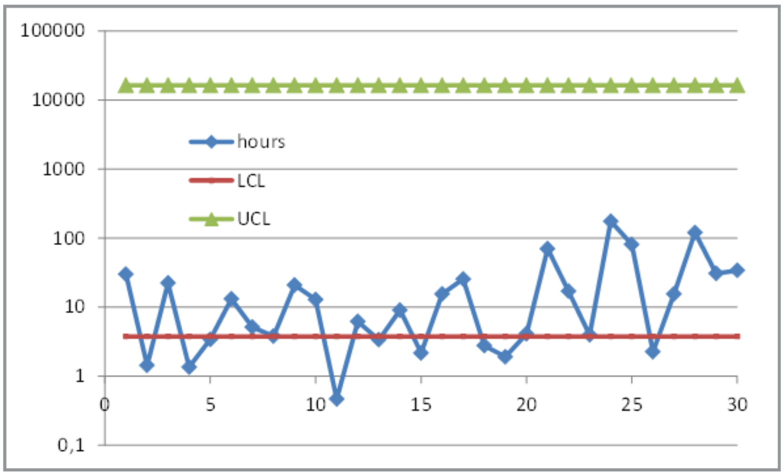


Figure 21: I-CC_TBE of Xie TBF data in "Some effective … for reliability Monitoring"; vertical axis logarithmic; RIT used (F. Galetto).

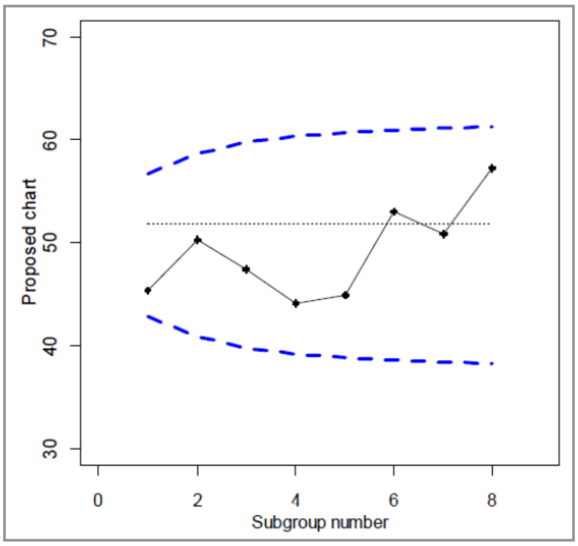


Figure 22: Proposed CC, ball bearing data [EWMA of Vˆ_SQR (i)=√(x_ij^2/6) of 8 samples, size 3)].

They consider the TTF (Time to failure, Rayleigh distributed) of 24 bearings (8 samples of size 3). The process of the 8 samples is IC (figure 22) by their "theory". On the contrary, the process is OOC [using RIT], both for the 24 Individuals (figure 23) and the 8 samples (figure 24).

The two authors claim in their Conclusions: "Simulation analysis also indicates the considerable improvement of the REWMA chart over the existing procedure in detecting shifts of smaller sizes in the study parameter".

We think that the readers agree will not agree on that, by seeing the application (real) on the Ball Bearing failure data: the authors "detect shifts" but do not detect OOC… (figures 23, 24).

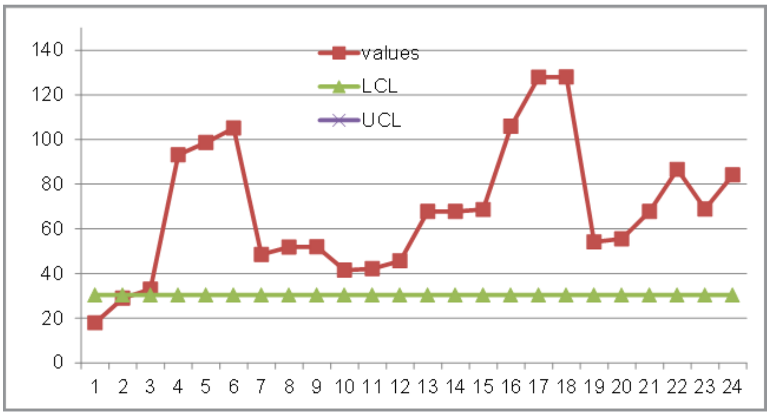


Figure 23: CC of the 24 Individuals TTF, RIT used.

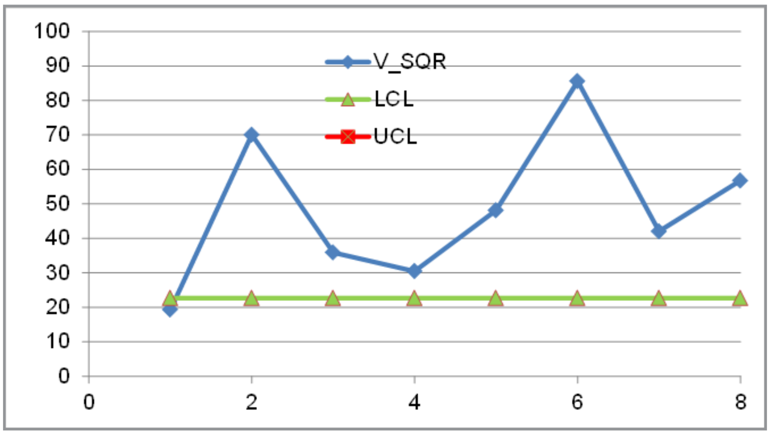


Figure 24: CC for ball bearing data [Vˆ_SQR (i) of the 8 samples], RIT used.

Last case: two papers (The length-biased weighted exponentiated inverted Weibull distribution, Cogent Mathe-matics, 2016, The Weighted Exponentiated Inverted Weibull Distribution, Journal of Informatics and Mathe-matical Sciences, 2017),

Distance between cracks in a pipe data-set

30.94	18.51	16.62	51.56	22.85	22.38	19.08	49.56
17.12	10.67	25.43	10.24	27.47	14.70	14.10	29.93
27.98	36.02	19.40	14.97	22.57	12.26	18.14	18.84

Goodness of fit summary of distance between cracks in a pipe data set.

Fitting models:	WEIW	EIW	LBEIW	LBIW	Weibull
Parameter Estimates:	$\beta^\wedge = 2.8639$ $\theta^\wedge = 394.0386$	$\beta^\wedge = 2.7347$ $\theta^\wedge = 2384.5601$	$\beta^\wedge = 3.3891$ $\theta^\wedge = 9508.9505$	$\beta^\wedge = 1.3484$ $\theta^\wedge = 26.0230$	$\beta^\wedge = 2.3089$ $c^\wedge = 0.4815$
K.S statistic	0.0865	0.0891	0.1031	0.5129	0.1436
P-value	0.9850	0.9822	0.9376	0.0000	0.6532

the estimates are wrongly written for Weibull $\hat{\beta}=2.3089$, $\hat{\theta}=26.0230$ and $\hat{c}=0$

Excerpt 9: From the paper "The length-biased weighted exponentiated inverted …"

The papers deal with the same data, on the "distance of cracks in a pipe data-set": same subject and the same real data as an application: they are in Excerpt 9, with the estimates of the density g(x;β,θ,c)=(βθ^((1+c-1/β)))/(Γ(1+c-1/β)) x^(-(1+c)β) {e^(-x^(-β)) }^θ. The estimates of the parameters are (by the authors): $\hat{\beta}=1.4256$, $\hat{\theta}=100.7943$ and $\hat{c}=1.4857$. Notice that there is NO Confidence Interval… The authors do not provide any way to do that… When c=0 we get Length-Biased Exponentiated Inverted Weibull pdf (LBEIW), with estimates of the parameters (by the authors): $\hat{\beta}=3.3891$, $\hat{\theta}=9508.9505$, $\hat{c}=0$. NO Confidence Interval and not any way to find it…

A question arises: do the data of Excerpt 9 show a process In Control? In the papers there is no way to assess that. Using RIT, we find that the process is OOC for the 24 Individuals (fig. 25). Again, Authors, Peer Reviewers and Editors were wrong!

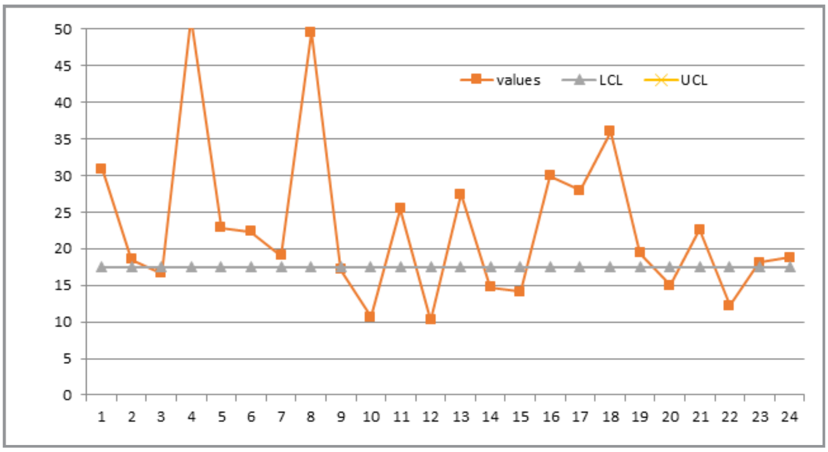


Figure 25: CC of the Individuals (from the paper “The length-biased weighted exponentiated inverted …”), RIT used.

Also consider the paper On designing a new control chart for Rayleigh distributed processes with an application to monitor glass fiber strength published in Communication in Statistics-Simulation and Computation, January 2020. Again, WRONG Control Limits! The authors M. Pear Hossain et al. consider the “data on strength of 15 cm glass fibers”. They write:

Illustrative example (section 8)

To illustrate the developed VR chart, we use the data on strength of 15cm glass fibers. This data has been collected from the National Physical Laboratory in England (Smith and Naylor 1987). For illustration purpose, first, we check that the data follows Rayleigh distribution or not using Kolmogorov-Smirnov test. We fail to reject the null hypothesis that data follows Rayleigh distribution at 5% level of significance with p-value 0.144.

The arrangement of the sample batches is given in Table 6 (see our Excerpt 10). They use α=0.0027 …

Table 6. Strength of 15 cm glass fiber.

	Observation				
Sample Number	1st	2nd	3rd	4th	5th
1	1.2	1.1	0.7	1.51	1.25
2	1.2	1.1	0.75	1.28	0.4
3	1.17	1.3	0.8	1.42	1.35
4	1.38	1.22	0.94	1.43	1.28
5	1.4	1.21	1.37	0.81	1.29
6	1.4	0.92	1.37	1.06	1.29
7	1.09	0.92	1.35	1.13	0.95
8	1.08	0.86	1.61	1.14	0.98
9	1.06	0.83	1.53	1.15	1.03

Excerpt 10: From the paper “On designing a new control chart … to monitor glass fiber strength.”

Notice the authors’ statement “We fail to reject the null hypothesis that data follows Rayleigh distribution at 5% level of significance with p-value 0.144.”

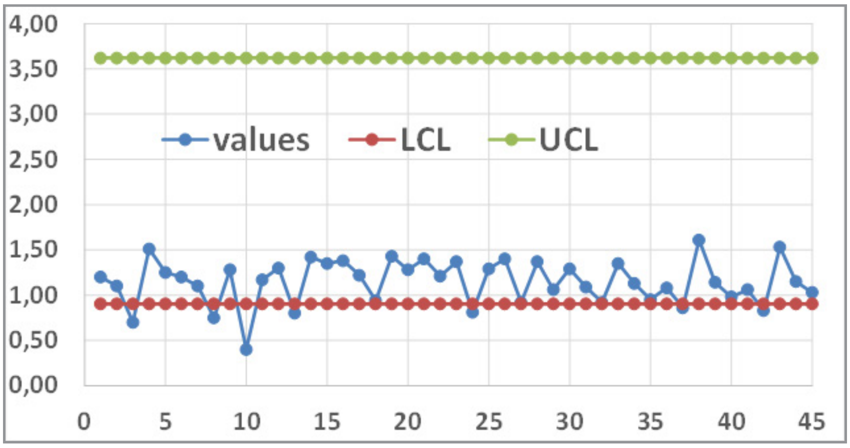


Figure 26: CC of the Individuals (from the paper “On designing a new control chart … to monitor glass fiber strength”), RIT used.

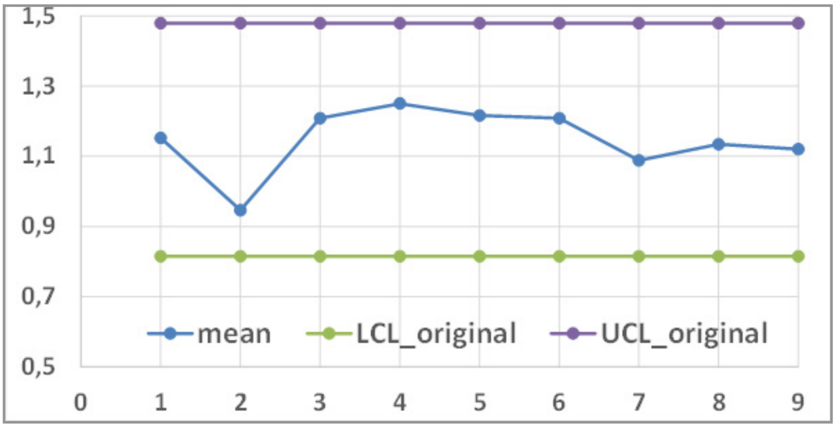


Figure 27: CC of the samples with sample size 5 (from the paper "On designing a new control chart … to monitor glass fiber strength"), Normal distribution used.

According the Rayleigh distribution can be considered a Weibull distribution with β=2 (shape parameter). Ana-lysing the data in Excerpt 10, we find that β=5.59, with a Confidence Interval CI=[3.81, 8.86] at CL=99.5%; the value 2 is not comprised in the CI: hence the distribution is not the Rayleigh distribution (also for CL=95%).

All the authors' considerations are not valid for their Illustrative example (section 8), that is our Excerpt 10; they find that the "process is IC".

Analising the data with RIT we get the figure 26: the process is OOC, using the correct distribution and directly the data in our **Excerpt 10.** Analising the data with the Normal distribution (a Weibull with β=5.59 can be approximated by the Normal) we get the figure 27: now the process is IC … as it was found by the authors with the Rayleigh distribution!

RIT and the Duane method

We found this method in the software Minitab 19&20&21.
Minitab provides the data on "repairable air-conditioners" and a graphical picture of them [see figure 28], and computes, the mean number of failures up to time t [M(t) function], of the 13 repairable systems: M(t)=E[N(t)]; they do not give any "theory" to interpret the results; they only inform us that (1) M(t) is interpolated by a model named "power law" $(t/\eta)^{\beta}$, with β=shape parameter and η=scale parameter, and (2) the MLM (Maximum Likelihood Method) is used. No "Reliability Theory" is provided by Minitab: this is extremely dangerous and costing [51, 52]. They say (with figures):

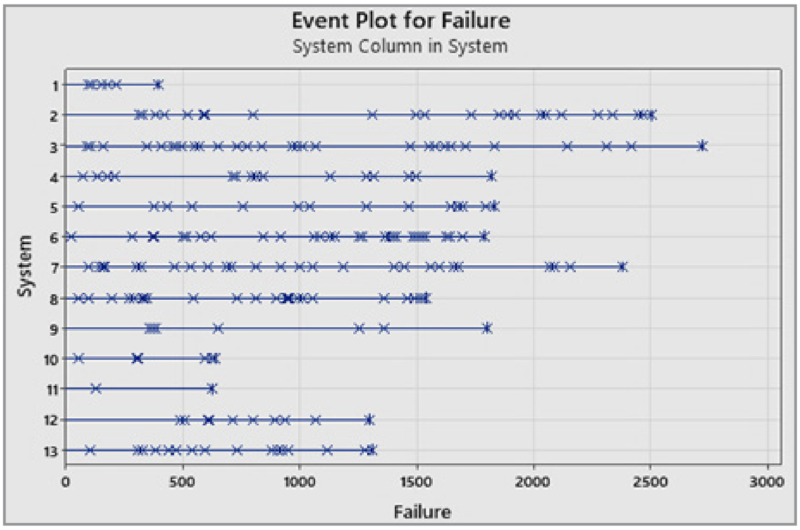


Figure 28: 13 repairable air-conditioners.

AIRCONDITIONERRELIABILITY.MTW **Parametric Growth Curve: Failure**
Model: Power-Law Process, Estimation Method: Maximum Likelihood, **Parameter Estimates**

			95% Normal CI	
Parameter	**Estimate**	**StandardError**	**Lower**	**Upper**
Shape	1.10803	0.067	0.984256	1.24738
Scale	128.763	22.489	91.4369	181.325

Test for Equal Shape Parameters Bartlett's Modified Likelihood Ratio Chi-Square

Test Statistic	10.88
P-Value	0.539
Df	12

Figure 29: Statistical Output for 13 repairable air-conditioners (Minitab 21 used).

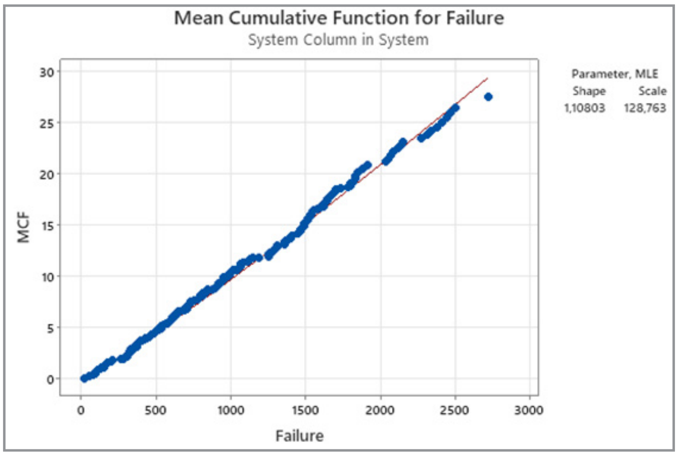


Figure 30: Graphical Output for 13 repairable air-conditioners data (Minitab 21 used).

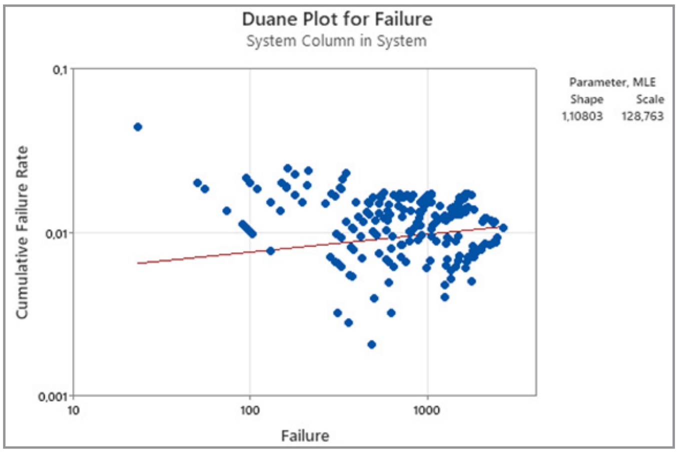


Figure 31: Duane plot for 13 repairable air-conditioners data (Minitab 21 used).

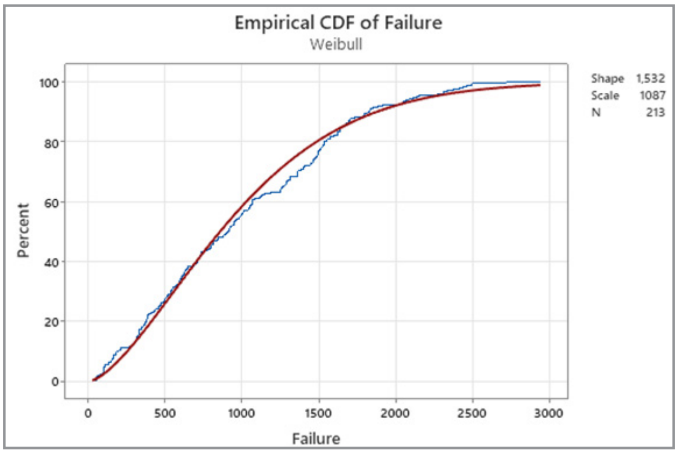


Figure 32: Distribution of repairable air-conditioners data tij (Minitab 21 used).

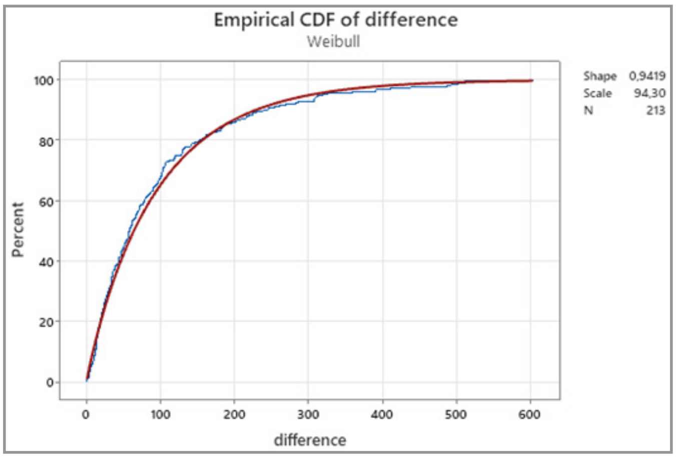


Figure 33: Distribution of repairable air-conditioners differences dij (Minitab 21 used).

We can compare the figure 30 [the M(t)] with the 31 [the "cumulative failure rate"]; how it is related to "our" failure rate, as defined in our theory? Think about that ... See the figures 29, 30. The figure 31 is the Duane Model!
The figures 32 and 33 show the distribution of times tij, and their differences dij, respectively.

From figures 30, 31, 32 we see that the shape parameter β of M(t) is estimated by Minitab as β_{PL}=1.10803, where PL stands for "Power Law". Notice that this estimate tells us that "there is no aging"; moreover, the figures 32 and 33 describe a completely different aging process of the air-conditioners! β_W=1.532 (aging) and β_d=0.9219 (no aging). Where is the TRUTH?
It is in the given Theory, RIT.

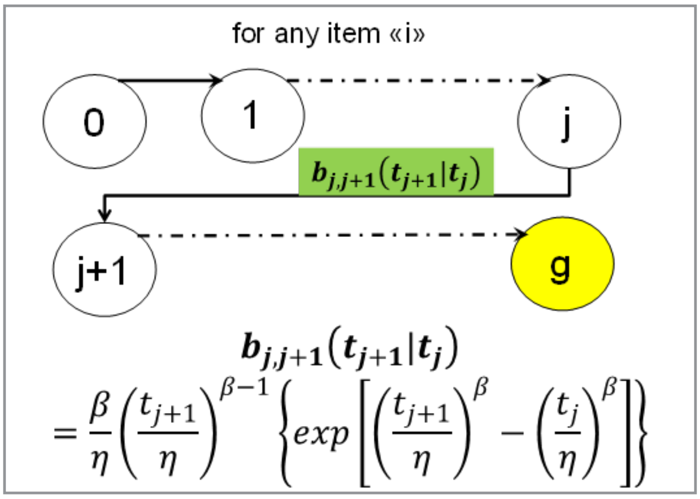


Figure 34: Transition Diagram of a repairable unit (BAO) and probability density of transitions (RIT).

The fundamental system (integral equations) for reliability tests (duration 0-----t) [t_0=0 is the start of the test and t is the end of the test], with t_j times of failures is given in (10), with the kernels of figure 28; at the end t of the reliability test, we know the empirical sample D={t_1, t_2, …, t_{g-1}, t_g, t}; t_g is the last failure. To estimate the parameters β and , from the equations we compute the determinant of the integral system (in matrix form) detB(s|r) [depending on β and η]. We have, for the system (air-conditioner) 1, with failures time $t_{1,j}$, and g_1 failures, the formula (identical to the Likelihood)

$$\det[B(|;\beta_1,\eta_1,D = \frac{\beta_1^{g_1} e^{-(\frac{t_{1g_1}}{\eta_1})^{\beta_1}}[t_{11}t_{12}t_{13}......t_{1g_1}]^{\beta_1-1}}{\eta_1^{\beta_1 g_1}}$$

The values maximising $\det[B(|;\beta_1,\eta_1,D)$, *for the item 1*, are $\beta_1 = \frac{g_1}{\sum_1^{g_1}\ln(t_{1g1}/t_{1j})}$ and $\eta_1 = \frac{t_{1g_1}}{g_1^{1/\beta_1}}$ Similar results are found for all the 13, identical and repaired, air conditioners.

Same results can be found with the MLM.
From the reliability system of 13 items, we get the estimations β_{all} and η_{all} of the parameters β and η:

$\beta_{all} = 0.99$ and $\eta_{all} = 92.38$

The CI of β_{all} is 0.858-------1.121, with CL=95%.
Notice: β_{al}=0.99 is slightly in the (Minitab) CI of β (0.984256-------1.24738, with CL=95%), AND the (Minitab) □PL=1.10803 is slightly in the CI of β_{all} (0.858-------1.121, with CL=95%). The contrary would happen by choosing CL=90%!
We cannot have "enough confidence" that the (Minitab) β_{PL}=1.10803 AND β_{all}=0.99 are "equivalent"!
Minitab provides wrong results for repairable systems and Duane analysis: Minitab lacks scientificity and generates huge costs for Companies using them, due to their wrong analyses [53-55].

The wrong "Duane method" is based on the wrong "Duane Axiom": "the MTBFc (the Mean Time Between Failures, instantaneous cumulated) is the ratio of the total cumulated time by the tested items, tc, to the total number of failures M(tc) experienced in the total time test interval t0-----tc". So, they write with α=0.2 ÷ 0.4, and t0 the "total time cumulated at the beginning of the total time test interval t0-----tc" where MTBF=MTBF0

For the Weibull distribution, we have $h(t)=(\beta/\eta)(t/\eta)^{\beta-1}$ and (by the absurd "Duane Axiom") $MTBF=1/h(t)=t^{1-\beta}\,\eta^{\beta}/\beta$, α=1-β with $MTBF_0/t_0^{1-\beta}$=constant.

The position MTBF=1/h(t) is an absolute NONSENSE, as shown before.

Discussion and Conclusions

Applying the G-Process we could show the way to solve various cases of practical interest: analysis of repairable systems reliability and availability, statistical estimation (and Confidence Interval evaluation) of the parameters of distributions, correct computation of Control Limits of the Control Charts, especially for Individual CC with TBE exponentially distributed and of the Douane method [56-60].

We introduced the Stochastic G-Processes which rule the relationships between the reliabilities Ri(t|s). The stochastic processes [HMP, NHMP, SMP, RP, A&RP] used for reliability analyses (to the author knowledge) are particular cases of the G-Process. We showed various cases (from papers) where errors were present due to the lack of knowledge of RIT [61].

The author many times tried to compel several scholars to be scientific: he did not have success (Galetto 1981-2023). Only Juran appreciated the author's ideas when he mentioned the paper "Quality of methods for quality is important" at the plenary session of EOQC Conference, Vienna [62-64].

For the control charts, it came out that RIT proved that the T Charts, for rare events and TBE (Time Between Events), used in the software Minitab, SixPack, JMP or SAS are wrong. So doing the author increased the h-index of the mentioned authors publishing wrong papers. See Appendix.

We suggest the readers to consider the various excerpts, especially those related to CCs: many authors have been diffusing wrong concepts for years and years…

RIT allows the scholars (managers, students, professors) to find sound methods also for the ideas shown by Wheeler in Quality Digest documents [65-67].

We proved also that Minitab software provides wrong analysis repairable systems Reliability (Minitab says "the items are aging", while they are actually GAN after any failure).

We informed the authors and the Journals who published wrong papers by writing various letters to the Editors…: no "Corrective Action", a basic activity for Quality. The same happened for Minitab: so, people continue taking wrong decisions…

Deficiencies in products and methods generate huge cost of DIS-quality (poor quality) as highlighted by Deming and Juran. Any book and paper are a product (providing methods). The books present financial considerations about reliability: their wrong ideas and methods generate huge cost for the Companies using them. The methods given here provide the way to avoid such costs, especially when RIT gives the right way to deal with Preventive Maintenance (risks and costs), Spare Parts Management (cost of unavailability of systems and production losses), Inventory Management, cost of wrong analyses and decisions.

We think that we provided the readers with the belief that Quality of Methods for Quality is important and with several ideas and methods to be meditated in view of the applications, generating wealth for the companies using them.

There is no "free lunch": metanoia and study are needed and necessary.

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Data Availability Statement

"MDPI Research Data Policies" at https://www.mdpi.com/ethics.

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Conflicts of Interest

"The author declares no conflicts of interest."

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Abbreviations

LCL, UCL: Control Limits of the Control Charts (CCs)
L, U: Probability Limits related to a probability 1-α
θ: Parameter of the Exponential Distribution
θL-----θU : Confidence Interval of the parameter θ
RIT: Reliability Integral Theory

Appendix A

The "Garden of flowers" (anthology, florilegium)

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There is no “free lunch”: metanoia and study are needed and necessary.